International Library of Technology
313

Structural Engineering Loads, Materials, Beams

156 Illustrations

By
EDITORIAL STAFF
INTERNATIONAL CORRESPONDENCE SCHOOLS

LOADS IN STRUCTURES
PROPERTIES OF SECTIONS
MATERIALS OF STRUCTURAL ENGINEERING
BEAMS AND GIRDERS

Published by
INTERNATIONAL TEXTBOOK COMPANY
SCRANTON, PA.
1927

PRESS OF
INTERNATIONAL TEXTBOOK COMPANY
SCRANTON, PA.
313 91945

PREFACE

The volumes of the International Library of Technology are made up of Instruction Papers, or Sections, comprising the various courses of instruction for students of the International Correspondence Schools. The original manuscripts are prepared by persons thoroughly qualified both technically and by experience to write with authority, and in many cases they are regularly employed elsewhere in practical work as experts. The manuscripts are then carefully edited to make them suitable for correspondence instruction. The Instruction Papers are written clearly and in the simplest language possible, so as to make them readily understood by all students. Necessary technical expressions are clearly explained when introduced.

The great majority of our students wish to prepare themselves for advancement in their vocations or to qualify for more congenial occupations. Usually they are employed and able to devote only a few hours a day to study. Therefore every effort must be made to give them practical and accurate information in clear and concise form and to make this information include all of the essentials but none of the nonessentials. To make the text clear, illustrations are used freely. These illustrations are especially made by our own Illustrating Department in order to adapt them fully to the requirements of the text.

In the table of contents that immediately follows are given the titles of the Sections included in this volume, and under each title are listed the main topics discussed.

INTERNATIONAL TEXTBOOK COMPANY

B

CONTENTS

NOTE.—This volume is made up of a number of separate parts, or sections, as indicated by their titles, and the page numbers of each usually begin with 1. In this list of contents the titles of the parts are given in the order in which they appear in the book, and under each title is a full synopsis of the subjects treated.

LOADS IN STRUCTURES

PROPERTIES OF SECTIONS

BEAMS AND GIRDERS, PART 1—(*Continued*)

BEAMS AND GIRDERS, PART 2

BEAMS AND GIRDERS, PART 3

LOADS IN STRUCTURES

FLOOR, ROOF, AND WIND LOADS

DEAD LOAD

1. The weight of the material used in the permanent structure of a building produces loads on the floor systems, the columns, and the foundations. These loads are called the **dead loads** and include the weight of the structural framework, walls, floors, partitions, and roofs. In fact, the weight of every piece of material used in the construction of the building is included in the dead load.

Before the dead load can be computed, the weight of various materials must be known, and those in common use are given in the following tables. The units in which these weights are expressed are those most often employed in making estimates of loads in engineering calculations. Thus, Table I gives the weight, per cubic foot, of the materials usually measured by that unit, together with the weight, per cubic inch, of a few often measured in inches; while Table II gives the weights of such materials as are used in the construction of floors, roofs, ceilings, etc., where the quantities are generally expressed in square feet.

2. Weight of Fireproof Floors.—If fireproof floors are of standard construction, their weight may be determined from the weights given by the manufacturers of the particular type to be used; they may be found for most systems in the tables in *Fireproofing*. Where the fireproof-floor system is of special construction, that is, different from the standard commercial construction, a careful estimate of the dead load

TABLE I

WEIGHT OF MATERIALS

Name of Material	Average Weight	
	Pounds per Cu. In	Pounds per Cu Ft.
Aluminum	.096	166
Asphalt pavement composition		130
Bluestone		160
Brass	.302	523
Brickwork, in lime mortar		120
Brickwork, in cement mortar		130
Bronze	.319	552
Cement, Portland		80 to 100
Cement, Rosendale		56 to 60
Concrete, in cement		140
Copper, cast	.319	550
Earth, dry and loose		72 to 80
Earth, dry and moderately rammed		90 to 100
Gneiss, common		168
Gneiss, in loose piles		96
Granite		165 to 170
Gravel		117 to 125
Iron, cast	.260	450
Iron, wrought	.277	480
Lead, commercial cast	.412	712
Limestone		170
Marble		164
Masonry, granite or limestone		165
Masonry, granite or limestone rubble		150
Masonry, granite or limestone dry rubble		138
Masonry, sandstone		145
Mortar, hardened		90 to 100
Quartz, common pure		165
Sand, pure quartz, dry		90 to 106
Sandstone, building, dry		144 to 151
Slate		160 to 180
Snow, fresh fallen		5 to 12
Steel, structural	.283	490
Terra cotta		110
Terra-cotta masonry work		112
Tile		110 to 120

NOTE —While it is not necessary for the student to memorize all of this table, it is well to keep in mind the weights of the materials printed in *Italics*.

TABLE II

WEIGHT OF MATERIALS

Name of Material	Average Weight per Square Foot Pounds
Corrugated galvanized iron No. 20, unboarded	$2\frac{1}{4}$
Copper, 16-ounce, standing seam	$1\frac{1}{4}$
Felt and asphalt, without sheathing	2
Glass, $\frac{1}{8}$ inch thick	$1\frac{3}{4}$
Hemlock sheathing, 1 inch thick	$2\frac{1}{2}$
Lead, about $\frac{1}{8}$ inch thick	6 to 8
Lath-and-plaster ceiling (ordinary)	6 to 8
Mackite, 1 inch thick, with plaster	10
Neponset roofing felt, 2 layers	$\frac{1}{2}$
Spruce sheathing, 1 inch thick	2
Slate, $\frac{3}{16}$ inch thick, 3 inches double lap	$6\frac{3}{4}$
Slate, $\frac{1}{8}$ inch thick, 3 inches double lap	$4\frac{1}{2}$
Shingles, 6 inches by 18 inches, one-third to weather	2
Skylight of glass, $\frac{3}{16}$ inch to $\frac{1}{4}$ inch, including frame	4 to 10
Slag roof, 4-ply	4
Tin, IX	$\frac{3}{4}$
Tiles, $10\frac{1}{2}$ inches by $6\frac{1}{4}$ inches by $\frac{5}{8}$ inch; $5\frac{1}{4}$ inches to weather (plain)	18
Tiles, $14\frac{1}{2}$ inches by $10\frac{1}{2}$ inches; $7\frac{1}{4}$ inches to weather (Spanish)	$8\frac{1}{2}$
White-pine sheathing, 1 inch thick	$2\frac{1}{2}$
Yellow-pine sheathing, 1 inch thick	4

per square foot of floor surface should be made. The volume of all materials that are measured by the cubic inch or cubic foot should be obtained by the rules and methods in *Geometry and Mensuration*, and the load obtained by multiplying by the unit weights of the materials found in Table I. The area covered by materials, such as flooring, sheathing,

roof covering, etc., that are measured by the square foot, should be computed and multiplied by the weight, per square foot, given in Table II, to obtain the load.

In making calculations for the dead load of floors where the section through the floor shows irregularities in thickness and consequently in volume and weight, it is necessary to consider the section through a **panel**, which is the space between two floor beams. The section of the floor is taken

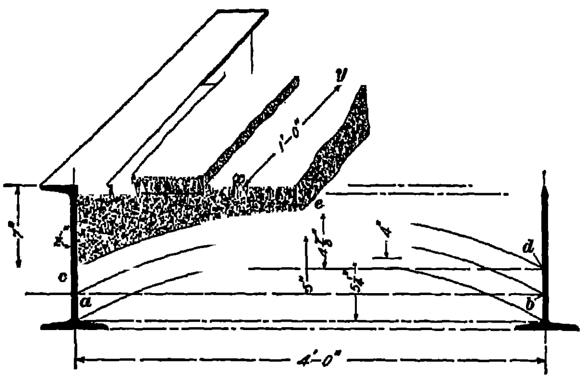


FIG. 1

1 foot in depth, as designated at xy, Fig. 1, so that when the entire weight of the section has been obtained the average weight per square foot can be found by dividing by the panel distance, or the distance between the floor beams.

EXAMPLE.—What is the amount of dead load per square foot of floor surface on the floor system shown in Fig. 1?

SOLUTION.—The sectional area of the brick arch is practically equal to the product of the length of the arc on the center line ab by the thickness of the arch, which in this instance is 4 in. The length of the chord of the arc ab is $47\frac{1}{2}$ in., while the rise is 5 in. From these dimensions the length of the arc on the center line ab may be found by substituting in the formula $l = \frac{4\sqrt{c^2 + 4h^2} - c}{3}$, given in *Geometry and Mensuration*, in which c equals the chord and h the rise of the arc, the value of l is found to equal

$$\frac{4\sqrt{(47.5 \times 47.5) + (4 \times 5 \times 5)} - 47.5}{3} = 48.8883 \text{ in.}$$

Then the sectional area of the brick arch equals $\frac{48.8883 \times 4}{144} =$ 1 358 sq. ft. Since the calculation is for a portion of the floor system 1 ft. in depth or length, the area of the section of the arch also equals, numerically, the cubical contents, so that the weight of the brick arch 1 ft. wide is equal to 1.358 multiplied by 120, the weight per cubic foot of brickwork laid in lime mortar, obtained from Table I, or 162.96 lb.

The area of the section of the concrete is equal to the area of a rectangle, in this case 7 in. × 47½ in., from which must be deducted the area of the segment of the circle included between the arc *c e d* and the chord *c d*. In order to obtain the area of this segment calculate the radius of the arc *c e d* by applying the formula $r = \frac{c^2 + 4h^2}{8h}$, given in *Geometry and Mensuration*. The quantities c and h represent, as before, the chord and the rise and are equal, respectively, to 47.5 in. and 4 875 in. Substituting these values in the formula,

$$r = \frac{(47.5 \times 47.5) + (4 \times 4.875 \times 4.875)}{8 \times 4.875} = 60.29 \text{ in.}$$

The area of the segment is equal to the area of the sector minus the area of the triangle formed by the chord and the radii, or, as designated in Fig. 2, the area of the shaded portion is equal to the area *c e d o* minus the area of the triangle *c d o*. The area of the sector may be found by the formula $a = \frac{l r}{2}$, given in *Geometry and Mensuration*, in which l equals the length of the arc and r the radius The arc *c d* has a smaller rise than that of arc *a b* and will therefore be shorter. Its length, found by the formula just given, is 48.82 in. Inserting this value and that of the radius in the formula,

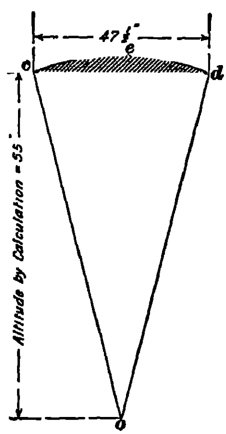


FIG 2

$$a = \frac{48\ 82 \times 60.29}{2} = 1{,}471.68 \text{ sq. in.}$$

The area of the triangle to be deducted from the sector is equal to one-half the product of the base and the altitude. From Figs 1 and 2, the base equals 47 5 in. and the altitude equals the radius minus the rise of the arc, or 60 29 − 4 875 = 55 415 in., and consequently the area of the triangle *c d o*, as designated in Fig. 2, is $\frac{47.5 \times 55\ 415}{2} = 1{,}316\ 1063$ sq. in Since the area of the sector *c e d o* equals 1,471 68 sq in. and the triangle *c d o* has an area of 1,316 1063 sq. in , the area of the segment *c e d* equals the difference between these

quantities, or 155.57 sq. in.; hence, the area of concrete is (7 × 47½) − 155 57 = 176.93 sq. in. The weight, from Table I, is 140 lb. per cu. ft; since the length of the concrete section is 1 ft., its weight equals (176.93 ÷ 144) × 140 = 172 lb. The steel beam shown in Fig 1 weighs 40 lb. per lineal ft. From these calculations, the entire weight of a panel section of the floor system for 1 ft. in length or per lineal foot may be itemized as follows:

Weight of brick arch	162.96 lb.
Weight of concrete	172.00 lb.
Weight of steel beam	40.00 lb.
Total	374.96 lb.

This amount is the dead load on 4 sq. ft., and hence the dead load per square foot is 374.96 ÷ 4 = 93.74 lb. Ans.

3. Where the floor construction is of uniform weight and thickness throughout, as in mill construction, designated in Fig. 3, the calculations for the dead load can be made directly for 1 square foot of floor surface. The size of the girders or floor beams is seldom known before the dead load has been determined, so that it is necessary to assume their size

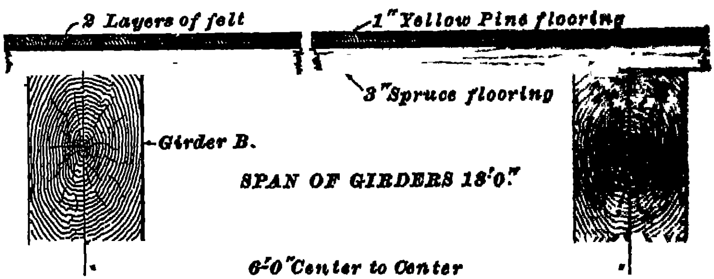


Fig. 3

and to add the weight of the assumed girders or beams in calculating the dead load. When considering the amount of dead weight supported by a beam or girder, it is customary to consider the same as made up of one-half the panel situated on either side of the beam. After the dead load has been found and the size of the girder accurately determined, the assumed weight can be checked by the actual weight.

EXAMPLE.—In Fig. 3, what is the total dead load on the girder *B*?

SOLUTION.—The weight of the materials per square foot may be obtained from Table II and be tabulated as follows:

Yellow-pine flooring, 1 in. thick	4 lb. per sq. ft.
Two layers of felt	½ lb. per sq. ft.
Rough spruce flooring, 3 in. thick	6 lb. per sq. ft.
Assume the weight of the girder	8 lb. per sq. ft.
Total dead load of floor surface	18½ lb. per sq. ft.

The area of the floor carried by the girder is $6 \times 18 = 108$ sq. ft. Then, $108 \times 18\frac{1}{2} = 1{,}998$ lb., the entire dead load on the girder *B*. Ans.

4. Dead Load on Roof Trusses.—This includes the weight of the roof covering, sheathing, and the weight of the roof trusses. The weight of the roof covering and the sheathing may be calculated from the unit weights given in Table II. The weight of the roof trusses, or *principals*, as they are termed, is not known until they have been designed and must be assumed in the original calculation. The weight of roof trusses depends on the material of which they are constructed, the span, and the distance they are placed apart and also on the rise and the type of construction, though these two latter factors are neglected in the usual empirical formulas.

The approximate weight of wooden and iron or steel roof trusses may be determined by the following formula:

$$W = aDL\left(1 + \frac{L}{10}\right) \qquad (1)$$

in which W = approximate weight of truss, in pounds;
a = constant, for wood .50, for iron or steel .75;
D = distance, in feet, from center to center of trusses;
L = span of the truss, in feet.

This formula may be expressed as follows:

Rule.—*Multiply the constant for the material of which the truss is composed by the distance, in feet, from center to center of trusses by the span of the truss, in feet; the product of this result and 1 plus one-tenth of the span of the truss, in feet, is the approximate weight of the truss, in pounds.*

5. After the weight of the principals, or roof trusses, has been determined by formula **1**, it is usual, in order to find the panel loads, or weight created at the connections of the truss, to determine what weight per square foot of roof surface it is necessary to add to the weight of the covering in order to provide for the weight of the principals or trusses. This weight may be found by dividing W, as determined from formula **1**, by the actual area on the slope of the roof supported by one truss, or, it may be determined directly from the following formula, which is evolved from formula **1** by dividing by DL sec x; this expression represents twice the length of the slope multiplied by the distance from center to center of trusses. The value w, or the approximate weight of the truss in pounds per square foot of roof surface, can be obtained from the following formula:

$$w = \frac{a\,(10 + L)}{10 \sec x} \qquad \textbf{(2)}$$

in which a and L have the same values as above and x equals the angle of rafter member with horizontal.

This may be stated in the form of a rule as follows:

Rule.—*Multiply the constant by 10 plus the span of the truss, in feet; divide this product by 10 times the secant of the angle that the rafter member makes with the horizontal, which gives the approximate weight of the truss, in pounds per square foot of roof surface.*

EXAMPLE.—Determine the weight, per square foot, that it is necessary to add to the weight of the roof covering to provide for the weight of the principals, when the steel trusses have a span of 72 feet and a rise of 18 feet.

SOLUTION.—The roof slope has a pitch of 6 in. for every foot horizontal and the angle x of the slope with the horizontal is found from Table VII to be 26° 33′. The sec x = hypotenuse ÷ adjacent side or $\sec x = \frac{\sqrt{36^2 + 18^2}}{36} = 1.118$. The hypotenuse in this case is identical with the roof slope, the length of which is found by the formula: $\text{hypotenuse}^2 = \left(\frac{\text{span}}{2}\right)^2 + \text{rise}^2 = 36^2 + 18^2$. The hypotenuse is therefore $\sqrt{36^2 + 18^2}$, which value has been inserted in the formula.

The secant x may also be found by means of the formula: $\sec x = \frac{1}{\cos x}$; $\cos 26° 33'$ being .8945, $\sec x = \frac{1}{.8945} = 1.118$. Substituting the values of a, L, and $\sec x$ in formula **2**.

$$w = \frac{.75\,(10 + 72)}{10 \times 1.118} = 5.5 \text{ lb. Ans.}$$

From formula **2**, the following table has been calculated. It gives the weight that it is necessary to add to a square

TABLE III

WEIGHT OF ROOF TRUSSES

$w = \frac{a\,(10 + L)}{10 \sec x}$ = Form. 2 Character of Truss	Span Feet	Pounds per Square Foot of Roof Surface			
		$\frac{1}{2}$ Pitch	$\frac{1}{3}$ Pitch	$\frac{1}{4}$ Pitch	$\frac{1}{6}$ Pitch
Wood	30	1.417	1.63	1.79	1.90
	35	1.588	1.87	2.01	2.13
	40	1.764	2.08	2.24	2.37
	45	1.941	2.29	2.46	2.61
	50	2.115	2.49	2.68	2.85
	55	2.293	2.70	2.91	3.08
	60	2.470	2.91	3.13	3.32
	65	2.646	3.12	3.35	3.56
	70	2.823	3.33	3.58	3.80
	75	2.999	3.54	3.80	4.03
	80	3.176	3.75	4.03	4.27
Iron or steel	30	2.117	2.45	2.69	2.85
	35	2.382	2.81	3.02	3.20
	40	2.647	3.12	3.35	3.56
	45	2.911	3.44	3.69	3.92
	50	3.176	3.74	4.02	4.28
	55	3.440	4.05	4.37	4.62
	60	3.705	4.37	4.70	4.98
	65	3.965	4.68	5.03	5.34
	70	4.235	5.00	5.37	5.70
	75	4.499	5.31	5.70	6.05
	80	4.764	5.63	6.05	6.41

foot of roof covering in order to provide, in the amount of the unit dead load, for the weight of the principals or trusses.

The term pitch used in Table III may have more than one interpretation; it may be the quotient $\frac{\text{rise}}{\text{span}}$ or $\frac{\text{rise}}{\frac{\text{span}}{2}}$. Throughout this Course it will mean: $\frac{\text{rise}}{\text{span}}$. For instance, $\frac{1}{4}$ pitch means a rise one-fourth of the total span; $\frac{1}{6}$ pitch a rise one-sixth of the total span, etc.

EXAMPLES FOR PRACTICE

1. A 2″ × 3″ wrought-iron bar is 36½ inches long. What is its weight? Ans. 60.66 lb.

2. The outside diameter of a cast-iron column is 10 inches, and the thickness of the material composing the column is ¾ inch. What is its weight per foot of length? Ans. 68 lb.

3. The wall of a brick building, laid in cement mortar, is 24 inches thick, 36 feet high, and 100 feet long; in it are located 20 window openings, 2 feet 6 inches wide by 6 feet high. What is the weight of this wall? Ans. 858,000 lb.

4. What is the weight of a structural steel angle 6″ × 6″ × ½″ × 20′ long? Ans. 390 54 lb.

5. The roof of a building is made of No. 20 corrugated galvanized iron, laid on 1-inch spruce boarding. What is the weight of the roof covering per square foot? Ans. 4¼ lb.

6 What will be the difference in weight per square foot between a 4-ply slag roof, laid on 3-inch tongued-and-grooved yellow-pine planking, and a $\frac{3}{16}$-inch slate roof laid on 2-inch hemlock sheathing, covered with Neponset roofing felt, two layers thick? Ans. 3¾ lb.

7. The span of a steel roof truss is 40 feet, and its rise 10 feet. Referring to Table III, what weight per square foot of roof surface should be assumed so as to allow for the weight of the principal or roof truss? Ans. 3.35 lb

LIVE LOAD

6. Besides the dead load, which includes the weight of all the material used in the structure itself, there is a load due to the weight of people, machinery, and merchandise; this load is called the **live load.** The live load comprises people in the building, furniture, movable stocks of goods, small

safes, and varying weights of any character. Large safes and extremely heavy machinery require some special provision, usually embodied in the construction. Table IV gives the live loads per square foot, recommended as good practice in conservative building construction.

TABLE IV

Character of Building	Pounds
Dwellings	70
Offices	70
Hotels and apartment houses	70
Theaters	120
Churches	120
Ballrooms and drill halls	120
Factories	from 150 up
Warehouses	from 150 to 250 up

The load of 70 pounds will probably never be realized in dwellings; but inasmuch as a city house may, at times, be used for some purpose other than that of a dwelling, it is not generally advisable to use a lighter load. In the case of a country house, a hotel, or a building of like character, where economy demands it, and its actual use for a long time, for some fixed purpose, is almost certain, a live load of 40 pounds per square foot of floor surface is ample for all rooms not used for public assembly.

For rooms thus used, a live load of 80 pounds will be sufficient, experience having demonstrated that a floor cannot be crowded to more. If the desks and chairs are fixed, as in a schoolroom or church, a live load of more than 40 to 50 pounds will never be attained. Retail stores should have floors proportioned for a live load of 100 pounds and upwards. Wholesale stores, machine shops, etc., should have the floors proportioned for a live load of not less than 150 pounds per square foot. The floors of printing houses and binderies, especially where the accumulation of heavy stock, such as

bound volumes and calendered paper, is likely to occur, should be proportioned for a live load of at least 250 pounds per square foot. Special provision should be made in floor systems for heavy presses, trimmers, and cutters and the beams should be proportioned for twice the static load likely to occur from such machines.

The static load in factories seldom exceeds 40 to 50 pounds per square foot of floor surface, and, therefore, in the majority of cases, a live load of 100 pounds, including the effects of vibrations due to moving machinery, is ample. The conservative rule is, in general, to assume loads not less than the above, and to proportion the beams so as to avoid excessive deflection. Stiffness is as important a factor as strength.

7. Warehouse Floors.—In the design of warehouse floors, the character of the material to be stored should always be considered and the data should be obtained regarding the manner of storing, the bulk of the packages, and the weight of the load per square foot. With the view of furnishing reliable data to manufacturers, architects, and engineers, the Boston Manufacturers' Mutual Fire Insurance Company has prepared from its extensive experience the following table, which gives the greatest possible loads that can be placed on warehouse floors, with the usual system of loading, and the space that the merchandise occupies. Where the floor space and the cubical contents of the load are given in the table, the height of the load above the finished floor may be obtained by dividing the volume of the load by the floor area covered. For instance, the floor space occupied by white linen rags in a bale is 8.5 square feet, and the cubical contents are 39.5 cubic feet; then the height of the loading is $39.5 \div 8.5 = 4.65$ feet. It is unusual and hardly possible, in the absence of hoists, to place such materials on the floor more than one bale in thickness, and the same thing applies to merchandise in barrels on the side and end. Where no data other than the weight per cubic foot is given in the table, it signifies that the possible height of the load is only

limited by the height of the room, or the *headroom*, as it is called. With a live load of such merchandise, the floor system must be designed for the maximum load, which consists of the weight of the merchandise covering the entire floor area to a depth of not less than 6 feet.

The building ordinances of the principal American cities are particularly emphatic with reference to warehouse floors. For instance, the building laws of Greater New York stipulate that, in all warehouses, storehouses, factories, workshops, and stores where heavy materials are kept and stored, or machinery introduced, the weight that each floor will safely sustain upon each superficial foot, or upon each varying part of such floor, shall be estimated by the owner or occupant, or by a competent person employed by the owner or occupant.

Such estimate shall be reduced to writing or printed forms, stating the material, size, distance apart, and span of beams and girders, posts or columns to support floors, and its correctness shall be sworn to by the person making the same; it further being required that this estimate shall be filed in the office of the Department of Buildings.

But if the commissioners of buildings shall have cause to doubt the correctness of said estimate, they are empowered to revise and correct the same, and for the purpose of such revision the officers and employes of the Department of Buildings may enter any building and remove so much of any floor or other portion thereof as may be required to make necessary measurements and examination.

When the correct estimate of the weight that the floors in any such buildings will safely sustain has been ascertained as herein provided, the Department of Buildings shall approve the same. Thereupon the owner or occupant of said building, or any portion thereof, shall post a copy of such approved estimate in a conspicuous place on each story or varying parts of each story of the building to which it relates. No person shall place on any floor of any building any greater load than the safe load as correctly estimated and ascertained.

TABLE V

WEIGHTS OF MERCHANDISE FOR CALCULATING LIVE LOADS

Materials		Measurements		Approximate Weights		
		Floor Area Sq. Ft.	Contents Cu Ft	Total Pounds	Pounds per Sq Ft.	Pounds per Cu Ft
Cotton, etc.	Bale	8.1	44.2	515	64	12
	Bale of compressed	4.1	21.6	550	134	25
	Bale of American Cotton Co.	4.0	11.0	263	66	24
	Bale of Planters Compress Co.	2.3	7.2	254	110	35
	Bale of jute	2.4	9.9	300	125	30
	Bale of jute lashings	2.6	10.5	450	172	43
	Bale of manila	3.2	10.9	280	88	26
	Bale of hemp	8.7	34.7	700	81	20
	Bale of sisal	5.3	17.0	400	75	24
Cotton goods	Bale of unbleached jeans	4.0	12.5	300	72	24
	Piece of duck	1.1	2.3	75	68	33
	Bale of brown sheetings	3.6	10.1	235	65	23
	Case of bleached sheetings	4.8	11.4	330	69	30
	Case of quilts	7.2	19.0	295	41	16
	Bale of print cloths	4.0	9.3	175	44	19
	Case of prints	4.5	13.4	420	93	31
	Bale of tickings	3.3	8.8	325	99	37
	Skein of cotton yarns					11
	Burlaps			130		30
	Jute bagging	1.4	5.3	100	70	24
Grain	Wheat in bags	4.2	4.2	165	39	39
	Wheat in bulk					44

Grain	Wheat in bulk					39
	Wheat in bulk, mean					41
	Flour in barrels on side	4 10	5 40	218	53	40
	Flour in barrels on end	3.10	7.10	218	70	31
	Corn in bags	3 60	3.60	112	31	31
	Corn meal in barrels	3 70	5 90	218	59	37
	Oats in bags	3.30	3 60	96	29	27
	Bale of hay	5 00	20 00	284	57	14
	Hay, Derrick compressed	1.75	5 25	125	72	24
	Straw	1 75	5 25	100	57	19
	Tow	1.75	5.25	150	87	29
	Excelsior	1.75	5 25	100	57	19
Paper	Calendered book					50
	Supercalendered book					69
	News paper					38
	Straw board					33
	Leather board					59
	Writing					64
	Wrapping					10
	Manila					37
Rags in bales . .	White linen	8 50	39 50	910	107	23
	White cotton	9 20	40.00	715	78	18
	Brown cotton	7 60	30.00	442	59	15
	Paper shavings	7 50	34.00	507	68	15
	Sacking	16 00	65 00	450	28	7
	Woolen	7 50	30 00	600	80	20
	Jute butts	2 80	11 00	400	143	36

TABLE V—(*Continued*)

Materials		Measurements		Approximate Weights		
		Floor Area Sq Ft	Contents Cu Ft	Total Pounds	Pounds per Sq. Ft.	Pounds per Cu. Ft.
Wool	Bale, East India	3.0	12	340	113	28
	Bale, Australian	5.8	26	385	66	15
	Bale, South American	7.0	34	1,000	143	29
	Bale, Oregon	6.9	33	482	70	15
	Bale, California	7.5	33	550	73	17
	Bag of wool	5.0	30	200	40	7
	Sack of scoured wool					5
Woolen goods . .	Case of flannels	5.5	12.7	220	40	17
	Case of flannels, heavy	7.1	15.2	330	46	22
	Case of dress goods	5.5	22	460	84	21
	Case of cassimeres	10.5	28	550	52	20
	Case of underwear	7.3	21	350	48	16
	Case of blankets	10.3	35	450	44	13
	Case of horse blankets	4.0	14	250	63	18
Miscellaneous . .	Box of tin	2.7	5	139	99	278
	Box of glass					60
	Crate of crockery	9.9	39.6	1,600	162	40
	Cask of crockery	13.4	42.5	600	52	14
	Coal, anthracite, broken					54
	Coal, anthracite, moderately shaken .					58
	Coal, anthracite, solid					93
	Coal, anthracite, heaped bushel . . .					80
	Coal, bituminous, solid					84

Miscellaneous . .	Coal, bituminous, broken, loose . . .					54
	Coal, bituminous, heaped bushel . .					74
	Coke, loose					30 to 50
	Bale of leather	7.3	12.2	190	26	16
	Bale of goat skin	11.2	16.7	300	27	18
	Bale of raw hides	6.0	30.0	400	67	13
	Bale of raw hides, compressed	6.0	30.0	700	117	23
	Bale of sole leather	7.3	12.2	190	26	16
	Pile of sole leather					17
	Barrel of granulated sugar	3.0	7.5	317	106	42
	Barrel of brown sugar	3.0	7.5	340	113	45
	Cheese					30
	Hogshead of bleaching powder	11.8	39.2	1,200	102	31
	Hogshead of soda ash	10.8	29.2	1,800	167	62
	Box of indigo	3.0	9.0	385	128	43
	Box of cutch	4.0	3.3	150	38	45
	Box of sumac	1.6	4.1	160	100	39
	Caustic soda in iron drum	4.3	6.8	600	140	88
	Barrel of starch	3.0	10.5	250	83	23
	Barrel of pearl alum	3.0	10.5	350	117	33
	Box of extract logwood	1.1	.8	55	52	70
	Barrel of lime	3.6	4.5	225	63	50
	Barrel of American cement	3.8	5.5	325	86	59
	Barrel of English cement	3.8	5.5	400	105	73
	Barrel of plaster	3.7	6.1	325	88	53
	Barrel of rosin	3.0	9.0	430	143	48
	Barrel of lard oil	4.3	12.3	422	98	34
	Rope					42

EXAMPLE —What will be the entire live load coming on a large girder supporting a portion of a church floor, if the floor area to be supported is 600 square feet?

SOLUTION.—From the list given in Table IV, 120 lb. is usually considered safe for a live load in a church. Therefore, 600 × 120 = 72,000 lb., the total live load on the girder. Ans.

EXAMPLES FOR PRACTICE

1. What will be the entire live load on the floor of a church 50 feet by 120 feet? Ans. 720,000 lb.

2. What live load will a joist in a city dwelling be required to bear, the distance between centers being 14 inches, and the span of the joist 20 feet? Ans. 1,633 lb.

3. A steel beam, supporting a portion of the floor in an office building, sustains an area of 80 square feet. What will be the live load coming on the beam? Ans. 5,600 lb.

4. A warehouse used for the storage of South American wool is 40 feet wide and 80 feet long inside. The girders extend across the building and divide it lengthwise into 5 bays; provided the floor construction and the girders weigh 20 pounds per square foot of surface, what is the total dead and live load on each girder? Ans. 104,320 lb.

8. In proportioning the live loads on floors, the architectural engineer cannot always exercise his own judgment, for if the building is to be erected in a large city, the live load must comply with the building laws. As these are not uniform in the several cities the following table is given to show the stipulated live loads in the four largest cities in the United States:

TABLE VI

THE ALLOWABLE LIVE LOADS ON FLOORS IN DIFFERENT CITIES

Character of Building	Pounds per Square Foot			
	New York	Philadelphia	Chicago	Boston
Buildings for public assembly .	90	100	120	150
Buildings for ordinary stores, light manufacturing, and light storage	120	100	120	
Dwellings, apartment houses, hotels, tenement houses, or lodging houses	60	40	70	50
Office buildings, first floor .	150	100	100	100
Office buildings, above first floor	75	100	100	100
Public buildings, except schools				150
Roofs, pitch less than 20° . .	50	25	30	25*
Roofs, pitch more than 20° . .	30	25	30	25*
Schools or places of instruction	75			80
Stables or carriage houses less than 500 square feet in area	75	40		
Stables or carriage houses more than 500 square feet in area . .	75	100		
Stores for heavy materials, warehouses, and factories . .	150		150	250
Sidewalks	300			

NOTE.—In this table the values given for roofs are not live loads, for a snow load can hardly be classified with live loads. The roof loads, in the last column, marked with the asterisk (*), do not include the wind load, and the building laws of Boston require that a proper allowance for the wind load exerting a pressure of 30 pounds per square foot of vertical surface shall be made in designing roofs.

SNOW AND WIND LOADS

9. In calculating the weight on roofs, there are two other loads always to be considered when obtaining the stresses on the various members of the truss; these are *snow* and *wind loads*. When the roof is comparatively flat, that is, when the rise of the roof is under 12 inches per foot of horizontal distance, the **snow load** is estimated at 12 pounds

per square foot; for roofs that have a steep slope, or a rise of more than 12 inches per foot of horizontal distance, it is good practice to assume the snow load to be 8 pounds per square foot. In northern climates, such as Canada, snow loads 50 per cent. greater than the above should be assumed.

10. Wind Pressure.—The wind pressure depends on the velocity with which the air is moving. United States Government tests have determined that the pressure per square foot on a vertical surface is approximately represented by the formula

$$p = .004\,V^2 \qquad (3)$$

in which p = pressure, in pounds per square foot, of vertical surface;

V = velocity of wind, in miles per hour.

This formula may be expressed in a rule thus:

Rule.—*The wind pressure, in pounds per square foot of vertical surface, is obtained by multiplying the square of the velocity of the wind, in miles per hour, by .004.*

The velocity of the wind varies from a pleasant breeze of 2 or 3 miles per hour to a violent hurricane or tornado of 100 or more miles per hour. Careful records, extending over a period of years, show that the velocity of the wind seldom attains 100 miles per hour; probably not more than once in the lifetime of the structure. In cyclonic storms, the velocity of the wind greatly exceeds 100 miles per hour, and structures cannot be built that will withstand their fury.

By applying formula **3**, it will be found that a wind having a velocity of 100 miles per hour will exert a pressure of 40 pounds per square foot of vertical surface and this is the pressure usually assumed by conservative engineers in providing for the resistance to wind pressure on structures. However, where the surface is of great area, as the side of a large office building, a pressure of 30 pounds per square foot is considered ample; for, the average unit pressure on a large surface is never so great as the maximum unit pressure on a small surface.

11. Curved surfaces, such as would be presented by circular towers and stacks, and flat surfaces not in a vertical plane, as roofs, are subjected to less pressure than flat vertical surfaces. The pressure on a cylindrical surface is about one-half the pressure on a flat surface having the same width as the diameter of the cylinder and the same height.

12. On roofs the wind pressure is always assumed as acting normal (that is, perpendicular) to the slope. In Fig. 4, the outline *a b c* of a roof is shown; the force *d* is normal to the slope *a b*, and represents the assumed pressure of the wind on the roof. The wind generally acts in a horizontal direction, as shown by the full arrow *e*. The reason for assuming the force *d* instead of *e* as the active wind pressure is for convenience in determining the stresses in the members of a roof frame or truss, which is explained in *Graphical Statics*. At present it may suffice to say that in order to estimate the effect of the force *e*, it is supposed to consist of two

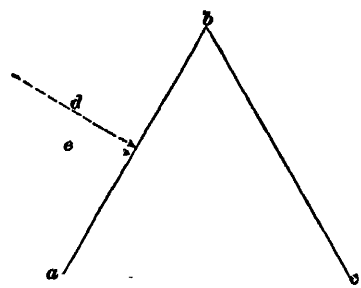


FIG 4

FIG 5

components, one of which is the force *d* and the other a force acting upwards and in a direction parallel with the slope. The latter is not taken into consideration. The wind, blowing with a horizontal pressure of 40 pounds, strikes the roof at an angle; consequently, the pressure *d*, normal to the slope, is considerably less than 40 pounds, unless the slope of the

roof is very steep. Referring to Figs. 4 and 5, it is clear that the horizontal force e of the wind on the slope of the roof, shown in Fig. 4, is almost as intense as on a vertical surface; on the extremely flat roof in Fig. 5, however, the wind exerts hardly any force normal to the slope, because it strikes the slope at such an acute angle, and therefore has a tendency to slide along its surface. The more acute the angle between the lines e and d, the greater the pressure normal to the slope; whereas, the greater the angle, the less the pressure normal to the slope, until they approximate a right angle with each other, when the pressure on the roof may be disregarded. In the design of roof trusses a horizontal wind pressure of 40 pounds per square foot is usually assumed.

13. All necessary data for calculating the wind pressure on a roof are given in Table VII, but it may be interesting to the student to know how the results given therein are obtained, and therefore the following explanation is offered. The pressure normal to the slope is generally determined by what is known as *Hutton's formula.* This formula is trigonometric, and may be expressed as follows:

$$p = p' \sin x^{1.84 \cos x - 1} \qquad (4)$$

in which p = pressure, in pounds per square foot, normal to slope of roof;

p' = wind pressure, in pounds per square foot, of vertical surface;

x = internal angle of roof with the horizontal.

This formula may be expressed as follows:

Rule.—*The pressure, in pounds per square foot, normal to the slope of the roof is equal to the product of the wind pressure, in pounds per square foot of vertical surface, and the sine of the angle that the roof slope makes with the horizontal, having an exponent of 1.84 times the cosine of the same angle, minus 1.*

For further explanation and application of the formula, assume a roof slope of 30° and the usual horizontal wind pressure of 40 pounds per square foot. The formula becomes, on substitution, $p = 40 \sin 30^{\circ\,1.84 \cos 30^\circ - 1}$, and

since sin 30° equals .5000 and cos 30° equals .86603, by further substitution $p = 40 \times .5000^{1.84 \times .86603 - 1}$, or $p = 40 \times .5000^{.5935}$. Referring to any logarithmic table, the calculation for this last expression is as follows:

$$\text{Log } .5000 = \bar{1}.69897 \text{ and } \bar{1}.69897 \times .5935 = -.1787$$

Then,

$$\begin{aligned} \log 40 &= 1.60206 \\ &\ -.17870 \\ &\ \ 1.42336 \end{aligned}$$

The number corresponding to this logarithm is 26.5+, the value of p or the pressure in pounds per square foot normal to the slope of the roof.

The following table, which gives the normal wind pressure on roofs of different slopes, has been calculated for a horizontal wind pressure of 40 pounds per square foot for the benefit of those who are not familiar with logarithms:

TABLE VII

WIND PRESSURE NORMAL TO THE SLOPE OF ROOF

Rise	Angle of Slope With Horizontal	Pitch, Proportion of Rise to Span	Wind Pressure Normal to Slope Pounds per Sq. Ft.
4 inches per foot horizontal. .	18° 25′	$\frac{1}{6}$	16.8
6 inches per foot horizontal. .	26° 33′	$\frac{1}{4}$	23.7
8 inches per foot horizontal. .	33° 42′	$\frac{1}{3}$	29.1
12 inches per foot horizontal. .	45° 0′	$\frac{1}{2}$	36.1
16 inches per foot horizontal. .	53° 7′	$\frac{2}{3}$	38.7
18 inches per foot horizontal. .	56° 20′	$\frac{3}{4}$	39.3
24 inches per foot horizontal. .	63° 27′	1	40.0

14. A diagram of the formula for obtaining the wind pressure normal to the slope, such as shown in Fig. 6, is interesting and provides a convenient means of determining the amount of the pressure for any slope. This may be found by projecting the point which represents the upper end of the slope on the arc horizontally, until it intersects

the curve of normal pressure; the multiplier directly under this point multiplied by the assumed wind pressure on a vertical surface will give the normal pressure for the given

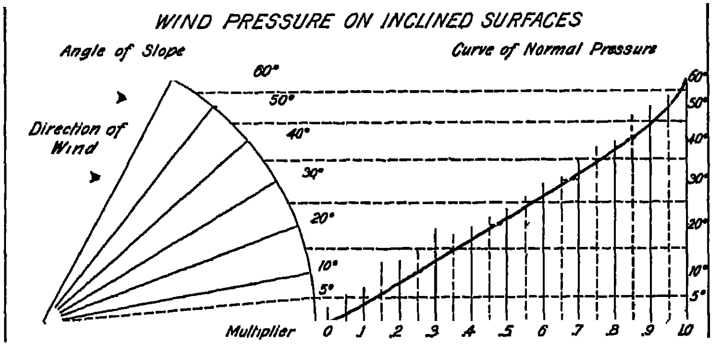


FIG. 6

slope. For instance, assume that it is desired to obtain the normal pressure on a roof whose slope forms an angle of 35°

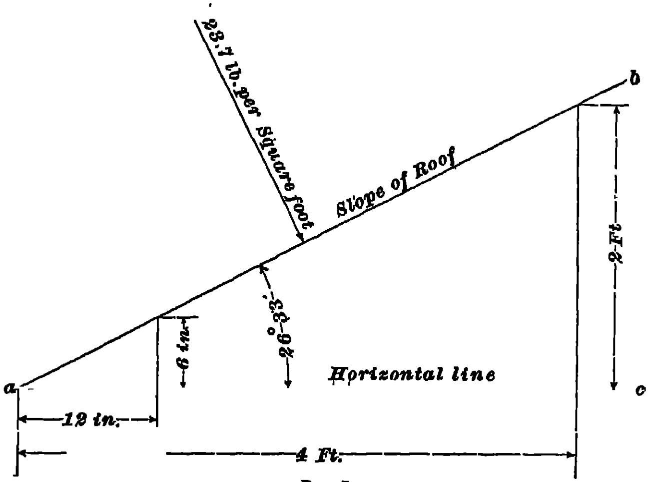


FIG 7

with the horizontal; mark off 35° on the arc and project this point horizontally until it intersects the curve of normal pressure, then note the value of the multiplier directly under

the point of intersection, which in this case is .75. Assuming a wind pressure of 40 pounds per square foot on a vertical surface, the normal pressure is found to be $40 \times .75 = 30$ pounds per square foot.

15. In order to more fully explain Table VII assume the conditions shown in Fig. 7. The rise in the slope *a b* is 6 inches for every 12 inches on the horizontal line *a c*; for instance, at 4 feet from *a* on the horizontal line *a c*, the rise is four times 6 inches, or 2 feet, the angle included between the line of slope *a b* and the horizontal base line *a c* is 26° 33′, and the pressure normal to the slope, according to Table VII, is 23.7 pounds per square foot. Since the rise at the center is equal t› one-half the length of one-half the span, the total rise is one-quarter of the span. Under these conditions, the pitch of the roof, that is, the ratio of the rise to the span, is $\frac{1}{4}$, and the roof is said to be $\frac{1}{4}$ *pitch.*

EXAMPLE.—(*a*) What will be the dead load per square foot of roof surface, on a roof with a 12-inch rise per foot horizontal, the span of the trusses being 50 feet, the roof covering 1-inch white-pine sheathing, 2 layers of Neponset roofing felt, and $\frac{1}{8}$-inch slate 3-inch lap? (*b*) What will be the wind pressure per square foot normal to the slope? (*c*) If the roof trusses are placed 12 feet apart, what will be the entire dead load on one truss? Fig. 8 shows a plan with elevation and detail section of the roof.

SOLUTION —(*a*) It is first necessary to obtain the length of the line of slope *a b*; this is done by calculating the hypotenuse of the triangle, or by laying the figure out to scale and measuring. In this case it is found that *a b* measures about 35 ft. 4 in , equal to 35 33 ft. The area of the roof supported by one truss is $2 \times 35.33 \times 12 = 847.92$ sq. ft. By referring to Table III, it is seen that the approximate weight of a roof truss of $\frac{1}{2}$ pitch and with a span of 50 ft. is 3.176 lb per sq. ft. of roof surface. Using the approximate value of 3.2 lb., the dead load per square foot of roof surface is, then, as follows:

Weight of supporting truss	3.2 lb per sq. ft.
Weight of white-pine sheathing, 1 inch thick	2.5 lb. per sq. ft.
Weight of 2 layers of Neponset roofing felt	.5 lb per sq. ft.
Weight of slate ($\frac{1}{8}$ inch thick)	4 5 lb. per sq. ft.
Total	10.7 lb. per sq. ft.

The weight of the purlins supporting the sheathing has not been estimated in the above, it being safe in this case to assume that the

weight used for the principals, or trusses, is sufficient to cover this item. A snow and accidental load of 12 lb. per sq. ft. of roof surface should also be added to the dead load to get the entire vertical load on the roof.

(*b*) The wind pressure normal to the slope of this roof, according to Table VII for a ½-pitch roof is 36.1 lb., say 36 lb per sq. ft. Ans.

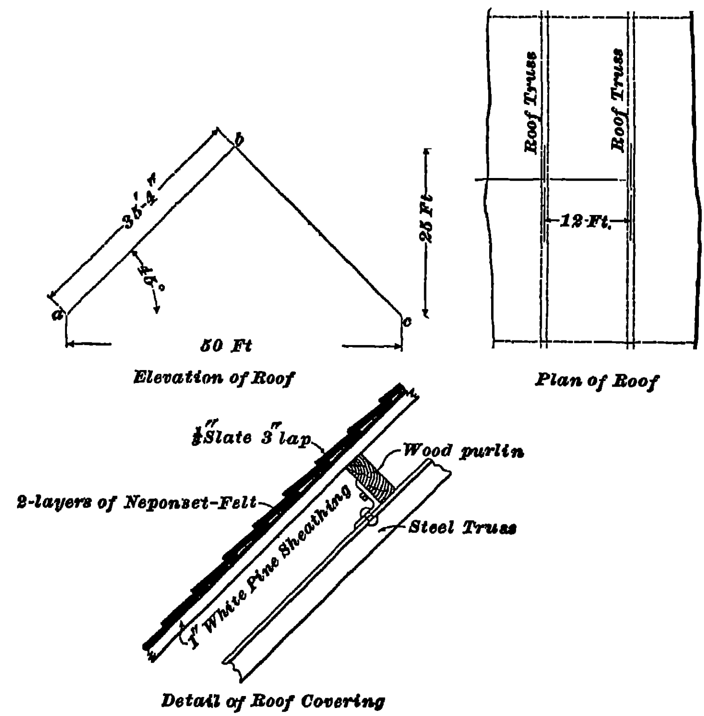


FIG. 8

(*c*) The area of the roof supported by one truss is, as previously found, 847 92 sq. ft., and the dead load, 10.7 lb. per sq ft. Then, 847 92 × 10 7 = 9,072 74 lb. to be supported by one truss, not including the snow load. Ans.

16. Engineering, it must be remembered, is not an exact science, the results obtained depending more or less on the judgment and experience of the designer. When, for instance, the wind is blowing a hurricane, snow never lodges on a roof, the slates, shingles, and sheathing being

themselves exposed to sudden removal. If, therefore, the full wind pressure be assumed, the snow load may, in most cases, be neglected, especially if it is desired to build an economical roof. However, it is not well for the student to make such assumptions until his experience and judgment are sufficiently developed to enable him to make true deductions.

EXAMPLES FOR PRACTICE

1. With the wind blowing at a velocity of 36 miles per hour, what is the pressure, in pounds per square foot of vertical surface?

Ans. 5.18 lb.

2. The area of one slope of a ½-pitch roof is 800 square feet What is the entire pressure on the slope of the roof, provided the maximum horizontal wind pressure is taken at 40 pounds per square foot?

Ans. 28,880 lb.

3. In a ¼-pitch roof the trusses are 20 feet apart, and the length of the roof slope is 40 feet. What wind load is there on each roof truss, if the horizontal pressure is 40 pounds per square foot? Ans. 18,960 lb.

4. The purlins supporting a ¾-pitch roof are placed 6 feet apart, and the trusses are 12 feet from center to center. What is the maximum load due to the wind on each purlin, providing the greatest horizontal pressure is 40 pounds per square foot? Ans. 2,830 lb.

5. The angle that the slope of a roof makes with the horizontal is 40°. Provided the wind exerts a pressure of 30 pounds per square foot of vertical surface, what is the pressure normal to the slope?

Ans. 25.50 lb.

ACCIDENTAL AND SUDDENLY APPLIED LOADS

17. Careful designers sometimes make allowance for the **accidental load** caused by a heavy body falling on the floor, or by a mass of snow dropping from one roof to another. The latter may usually be ignored, because it is taken care of in the factor of safety, within the limit of which every member in the structure is designed.

Members subjected to suddenly applied loads are seldom encountered in building construction, and still less frequently are members required to resist the effect of impact, or the blow imparted by a falling load. The beams supporting the mechanism at the heads of elevator shafts are at times

subjected not only to suddenly applied loads but also to falling loads; therefore, they should always be proportioned to withstand at least a suddenly applied load.

18. Where a load is placed suddenly on a beam, the stress produced is twice as great as if the same load had been at rest; that is, a beam to sustain a suddenly applied load should have twice the transverse strength required to sustain the same load at rest.

A falling load produces a much greater stress on a beam than a load suddenly applied, owing to the impact produced by the momentum of the falling body. It is usual in considering the effect of a falling load on a beam to determine the amount of a statical load that would produce the same results; the formula used to accomplish this is as follows:

$$W_1 = W\left(1 + \sqrt{\frac{2ah}{d} + 1}\right) \qquad (5)$$

in which W_1 = static load that would produce same stress in beam as falling load;

W = falling load;

h = height load falls, in inches;

d = deflection of beam, in inches;

a = constant.

The value of d may be determined as described in *Beams and Girders*, Part 1, while the value of a must be determined by the formula

$$a = \frac{1}{1 + .489\frac{W_2}{W}} \qquad (6)$$

in which W_2 = combined weight of beam and dead load that it supports;

W = amount of falling load, as before.

This may be expressed by a rule as follows:

Rule.—*The value of the constant is equal to 1 divided by the sum of 1 and .489 times the quotient obtained by dividing the combined weight of the beam and the dead load that it supports, by the falling load.*

19. Formula **5** may be stated as follows:

Rule.—*To the quotient of 2 times the constant times the distance the load falls, in inches, divided by the deflection of the beam in inches, add 1; the square root of this result is added to 1 and their sum multiplied by the falling load gives the static load that will produce the same strain in a beam as the falling load.*

EXAMPLE.—The drop test of a fireproof-floor system is to be made by letting a weight of 300 pounds fall through a distance of 3 feet. Provided the deflection of the floor beams under the falling load is $\frac{1}{8}$ inch and the weight of the beam with the dead load is 3,000 pounds, what static concentrated load is it necessary to figure on, that it may equal the falling load in its effect?

SOLUTION.—From formula **6**, the value of a equals $\dfrac{1}{1 + .489\,\dfrac{W_2}{W}}$,

or by substitution, $a = \dfrac{1}{1 + 489 \times \frac{3000}{300}} = .1698.$

The value of a having been found, the equivalent static load required by the example may be obtained from formula **5** where $W_1 = W\left(1 + \sqrt{\dfrac{2\,a\,h}{d} + 1}\right)$, or by substitution,

$$W_1 = 300\left(1 + \sqrt{\frac{2 \times .1698 \times 36}{.125} + 1}\right) = 3{,}282 \text{ lb.} \quad \text{Ans.}$$

EXAMPLES FOR PRACTICE

1. The static load from an elevator car on the steel beams at the head of the shaft is 4,000 pounds; what load should be figured on to compensate for the weight being suddenly applied in starting the car? Ans. 8,000 lb.

2. The beams supporting a loading platform have a span of 20 feet and are frequently subjected to a load of 500 pounds falling a distance of 2 feet Provided the deflection of the beam is $\frac{3}{16}$ inch, what static load will be equivalent to the effect of the falling load if the beam and the dead load on it weigh 1,000 pounds? Ans. 6,210 lb

THE DISPOSITION OF LOADS

20. In warehouses built especially for the storage of heavy merchandise, where the floors are likely at any time to be fully loaded, the beams, girders, columns, and foundations are always proportioned for the entire live and dead load on all floors. However, where the building exceeds four or five stories in height and is used for any other purpose than for storage, as, for instance, a modern office building, it is customary to assume that certain members, while proportioned for the entire dead load, carry only a certain percentage of the live loads.

In an office building, or similar structure, it is highly improbable that all the floors or all parts of the same floor will be fully loaded at the same time, and in view of this fact it is considered good practice, while proportioning the floor beams for the full live load, to calculate only 90 per cent. of the live load on the girders. It is usual to proportion the columns supporting the roof and the top floor for the full live load. The live loads on the columns, in each successive tier, from the floor above is reduced 5 per cent. until 50 per cent. of the live load is reached, when such reduced loads are used for all remaining floors to the basement. The economy obtained by this disposition of the live load is best observed from Table VIII, which gives the distribution of the assumed live loads on the columns in the several tiers of an eighteen-story office building.

While this system of graduating the live loads on the columns from floor to floor is generally practiced, the amount of reduction at each floor is a matter that depends on the judgment of the designer. The percentage of reduction is often fixed by the building laws of the city, with which the designer must comply. The reduction of 5 per cent. at each floor, the economy of which is shown in the table, is conservative and in most cases will be found to be in accordance with the building departments of the principal American cities.

TABLE VIII

SHOWING REDUCTIONS OF LIVE LOADS FROM FLOOR TO FLOOR

Floors	Live Load in Lb. per Sq. Ft. on Each Floor	Live Load in Lb. per Sq. Ft of Floor on Columns From Floor Above at 5% Reduction	Live Load in Lb. per Sq. Ft. of Floor on Columns From All Floors Above If No Reduction Were Made	Live Load in Lb per Sq Ft. of Floor on Columns From All Floors Above Increment of Reduction	Theoretical Percentage of Saving Instituted by the Reduction of 5% at Each Floor
	a	$a_1 = a - .05a$	Σa	Σa_1	$\frac{\Sigma a - \Sigma a_1}{\Sigma a}$
Roof	20	20.00	20	20.00	
18	60	60.00	80	80.00	
17	60	57.00	140	137.00	2.1
16	60	54.15	200	191.15	4.4
15	60	51.44	260	242.59	6.7
14	60	48.87	320	291.46	8.9
13	60	46.43	380	337.89	11.1
12	60	44.11	440	382.00	13.2
11	60	41.90	500	423.90	15.2
10	60	39.80	560	463.70	17.2
9	60	37.81	620	501.51	19.1
8	60	35.92	680	537.43	21.0
7	60	34.12	740	571.55	22.8
6	60	32.41	800	603.96	24.5
5	60	30.79	860	634.75	26.2
4	60	30.00	920	664.75	27.7
3	60	30.00	980	694.75	29.1
2	60	30.00	1,040	724.75	30.3
1	60	30.00	1,100	754.75	31.4

21. In the design of the type of building known as **skeleton construction**, that is, one in which all floors and walls are supported on beams and girders that transmit the loads to columns and, in turn, are supported on ample

foundation footings, it is necessary to fix on the general arrangement, disposition, and approximate dimensions of the component parts before the dead load can be computed. After the calculations are made and the structural details are designed, the actual dead load should be checked to see whether it approximates the assumed load. If any considerable variation is found it can be provided for by increasing or diminishing the weight or thickness of the rolled steel shapes making up the structural members, the sizes of which have already been determined.

Where permanent partitions exist, they should always be figured in the dead load; and where they are directly above a beam or a girder, the member should be proportioned to sustain the additional weight without appreciable deflection. Where movable partitions occur or where there is a probability of the location of permanent partitions being changed, it is usual to add 20 pounds per square foot of floor surface to the dead load to take care of such contingencies.

The foundations of an office building should be proportioned for the entire dead load and none of the live load, the latter being provided for by making the unit pressure on the footings and piers well within the safe unit bearing value of the soil. In this way unequal settlement is prevented, as explained in *Foundations*.

PROPERTIES OF SECTIONS

CENTER OF GRAVITY

1. Introduction.—The calculations involved in the design of such built-up members of a building as steel columns and plate girders—members that are formed by the combination of several of the simple sections produced by the rolling mills—require a knowledge of certain mathematical properties of the simpler sections, together with the methods by which these properties may be calculated. In many cases, the exact determination of the required properties is based on complicated mathematical principles; there are, however, numerous formulas and practical methods by means of which the values for all sections used in ordinary practice may be determined, either exactly or with a degree of approximation sufficiently close for all practical purposes.

2. The **center of gravity** of a body, or of a system of bodies, is that point from which, if the body or system were suspended, it would be in equilibrium. If the body or system were suspended from any other point than the center of gravity, and in such a manner as to be free to turn about the point of suspension, the body would rotate until the center of gravity reached a position directly under the point of suspension.

3. Center of Gravity of Plane Figures.—If a plane figure has an *axis of symmetry*, this axis passes through its center of gravity. If the figure has two axes of symmetry, its center of gravity is at their point of intersection.

The *center of gravity of a triangle* lies on a line drawn from a vertex to the middle point of the opposite side, and at a distance from that side equal to one-third of the length of the line; or it is at the intersection of lines drawn from the vertexes to the middle of the opposite sides.

The perpendicular distance of the center of gravity of a triangle from the base is equal to one-third of the altitude.

The *center of gravity of a parallelogram* is at the intersection of its two diagonals; consequently, it is midway between its sides.

The *center of gravity of an irregular four-sided figure* may be found by first dividing it, by a diagonal, into two triangles and joining the centers of gravity of the triangles by a straight line; then, by means of the other diagonal, divide the figure into two other triangles, and join their centers of gravity by another straight line; the center of gravity of the figure is at the intersection of the lines joining the centers of gravity of the two sets of triangles.

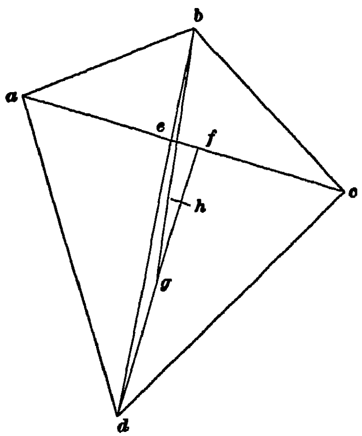


FIG. 1

Another method by which to locate the center of gravity of an irregular four-sided figure is illustrated in Fig. 1. Draw the diagonals *a c* and *b d*, and from their intersection *e*, measure the distance to any vertex, as *a e*. From the opposite vertex, lay off this distance, as at *c f*. Then from *f*, draw a line to one of the other vertexes, as *f d*, and bisect this line as at *g*. Connect *g* and *b* and lay off one-third of its length from *g* at the point *h*. This point is the center of gravity of the figure.

The distance of the *center of gravity of the surface of a half circle* from the center is equal to the product of the radius multiplied by .424.

THE NEUTRAL AXIS

4. Explanation of Neutral Axis.—When a beam is subjected to a bending stress, the fibers on the concave side of the beam are shortened while those on the convex side are lengthened; hence, there is a compressive stress on the concave side of the beam and a tensile stress on the convex side. Therefore, there will be a plane near the center of the beam in which the fibers are neither lengthened nor shortened and in which there is no stress; this plane is termed the **neutral plane,** or **neutral surface.** A **cantilever** beam, or beam supported at one end, is shown in Fig. 2, having a load W applied at the end. In this case the upper side of the beam is lengthened and the lower side is shortened, while in a simple beam, or one having a support at each end, the reverse is the case. A line dc representing the intersection of the neutral plane $a\,b\,c\,d$ with a cross-section of the beam is termed the **neutral axis.** This axis always passes through the center of gravity of the cross-section of the beam until the elastic limit is reached.

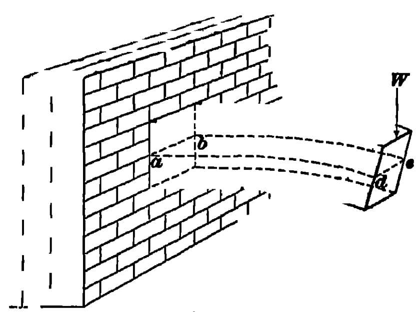


FIG 2

LOCATION OF NEUTRAL AXIS

5. Mechanical Method.—It is evident, from Fig. 2, that the neutral axis is perpendicular to the direction in which the load acts on the beam; therefore, to find the neutral axis of a section with reference to a set of loads applied in a given direction, it is only necessary to pass a

line through the center of gravity perpendicular to the direction of the load.

A simple approximate method of locating the center of gravity and neutral axis of a section is shown in Fig. 3. Draw the outline of the section, either full size or to some

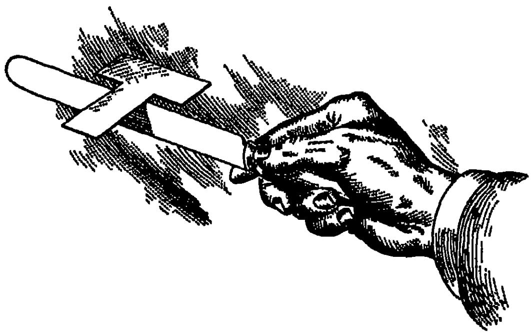


FIG. 3

convenient scale, on a piece of heavy cardboard. Cut out the section and balance it carefully over a knife edge, as shown in the figure; the line along which it rests on the edge of the knife is a line passing through the center of gravity, and by locating two such lines in different directions, the center of gravity will be found at their point of intersection.

6. Locating the Neutral Axis by Means of the Principle of Moments.—A convenient method of locating the neutral axis is based on the principle that the moment of any figure, with respect to a given line as an axis or origin of moments, is equal to the sum of the

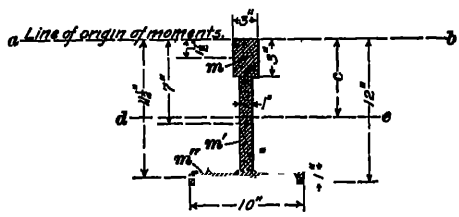


FIG. 4

moments of its separate parts with respect to the same axis. The moment of a figure about a given axis is the product of its area by the perpendicular distance from the center of gravity of the figure to the given axis. Thus, in Fig. 4, take

the line *a b* as the line of origin of moments. Divide the figure into the three rectangles *m*, *m′*, *m″*. In accordance with the principles stated in Art. **3**, the center of gravity of each of these rectangles is midway between its edges; the distances of the respective centers from the axis are, therefore, $1\frac{1}{2}$, 7, and $11\frac{1}{2}$ inches. The areas of the figures are, respectively, 3×3 or 9 square inches, 8×1 or 8 square inches, and 10×1 or 10 square inches. The moments of these areas about the axis *a b* are:

Section *m*	$9 \times 1\frac{1}{2}$	=	13.5
Section *m′*	8×7	=	56.0
Section *m″*	$10 \times 11\frac{1}{2}$	=	115.0
	Total,		184.5

The area of the whole section is equal to $9 + 8 + 10 = 27$ square inches, the sum of the areas of the rectangles; the distance c of its neutral axis from the line of the origin of moments is, therefore, $184.5 \div 27 = 6.83$ inches, or nearly $6\frac{27}{32}$ inches.

It is not necessary that the line of the origin of moments should coincide with an edge of the figure, as in Fig. 4, since any other line parallel with the direction of the required neutral axis gives the same results; in most cases, however, it will be found more convenient to take the axis about which the moments are calculated on one of the extreme edges of the section.

Since the section shown in Fig. 4 is symmetrical with respect to an axis perpendicular to the neutral axis, it is evident that its center of gravity is on their intersection. If, however, there were no axis of symmetry, the center of gravity could have been located by taking a second line perpendicular to *a b* as an origin of moments and finding the neutral axis parallel to it. The intersection of this neutral axis with the one first found is the center of gravity of the section. In accordance with the principles illustrated in this example, we have the following rule:

Rule.—*To find the neutral axis of any section, first divide it into a number of simple parts, each of whose areas and centers*

of gravity can be readily found; then find the sum of the moments of the areas of each of these parts with respect to an axis parallel to the required neutral axis; finally, divide this sum by the sum of the areas of the parts of the section. The result will be the perpendicular distance from the axis of the origin of moments to the required neutral axis.

7. Application of the Rule to a Built-Up Section. Fig. 5 shows a section of the rafter member of a large roof truss formed of a $\frac{3}{8}'' \times 16''$ web-plate and a $\frac{3}{8}'' \times 12''$ flange plate, the two joined by two $4'' \times 4'' \times \frac{1}{2}''$ angles. What is the distance from the neutral axis of the section to the top edge of

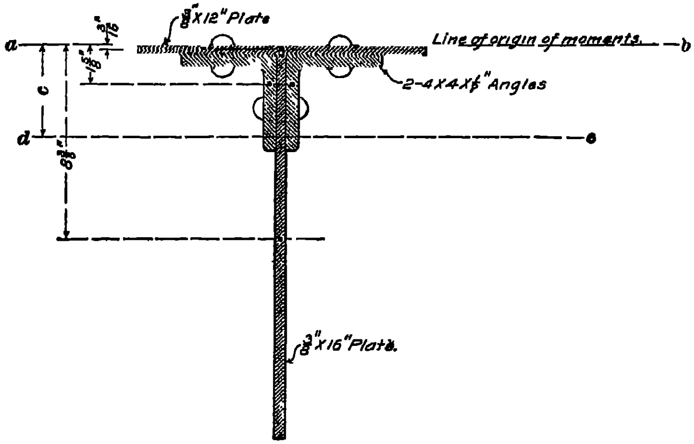


Fig 5

the flange plate? By means of the principles given, the centers of gravity of the two rectangular plates are easily located as shown. The centers of gravity of the angles might also be located by applying the rule given in the preceding article; this, however, is unnecessary, since the center of gravity can be obtained directly by reference to the tables of the properties of rolled sections. Referring to Table V, the center of gravity of a $4'' \times 4'' \times \frac{1}{2}''$ angle is found to be 1.18 inches from the back of a flange, thus giving us the distance $1.18 + .375 = 1.555$, or about $1\frac{9}{16}$ inches from the top edge of the flange plate to the axis through the centers of gravity of

the angles. From the same table it is also found that the area of the section of a $4'' \times 4'' \times \frac{1}{2}''$ angle is 3.75 square inches.

The area of the section of the flange plate is $\frac{3}{8} \times 12 = 4.5$ square inches, and of the web-plate, $\frac{3}{8} \times 16 = 6$ square inches; the area of the whole section is, therefore, $2 \times 3.75 + 4.5 + 6 = 18$ square inches.

The moments of the areas of the separate sections, with respect to the line *a b*, are as follows:

Flange plate,	$4.5 \times \frac{3}{16} =$	.84
Two angles,	$2 \times 3.75 \times 1\frac{14}{25} =$	11.70
Web-plate,	$6 \times 8\frac{3}{8} =$	50.25
	Total,	62.79

The distance *c* from the top edge of the flange plate to the neutral axis *d e* of the section is, therefore, $62.79 \div 18 = 3.48$ inches.

8. Graphical Method of Locating the Neutral Axis. Let it be required to determine the position of the neutral axis of the cast-iron beam section shown in Fig. 6. First, divide the depth of the section into any number of parts—as has been done in this case by the dotted lines *w′ x*, *y z*, etc.—whose areas and centers of gravity can readily be found. Then compute the area and locate the center of gravity of each part. In Fig. 6, the area of the top slice is 12 square inches, the area of each of the slices in the web of the section is 3 square inches, and the area of the bottom flange is 28 square inches. Assume some scale whose unit of length represents 1 square inch; for example, in this case, let $\frac{1}{16}$ inch represent 1 square inch. Then, commencing at some point *a*, lay off on the line *a l* lengths *a b*, *b c*, *c f*, etc., which represent the respective areas of the successive parts into which the section of the beam has been divided, beginning at the top part. Thus, with the scale of $\frac{1}{16}$ inch $= 1$ square inch, the line *a b*, which represents an area of 12 square inches, is $\frac{12}{16} = \frac{3}{4}$ inch long; each of the lines *b c*, *c f*, etc. is $\frac{3}{16}$ inch long; and the line *k l* is $\frac{28}{16} = 1\frac{3}{4}$ inches long.

From the points *a* and *l*, draw lines at any convenient angle

to al intersecting at the point m. Then from the points b, c, f, etc., draw the lines bm, cm, fm, etc. Through the center

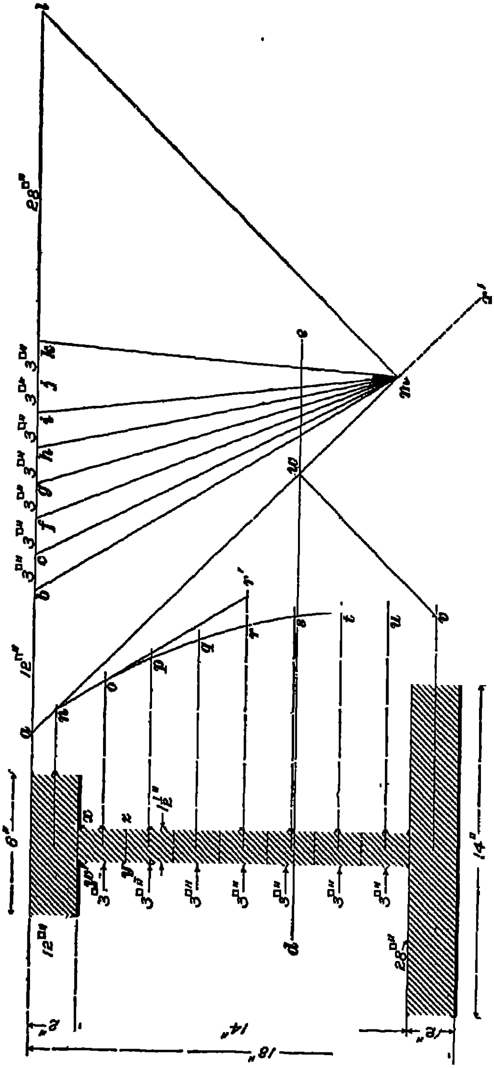


FIG. 6

of gravity of each of the parts of the section, draw indefinite lines parallel to al.

From the point n, where the line through the center of gravity of the top section intersects the line $a\,m$, draw the line $n\,o$ parallel to the line $b\,m$ until it intersects the line passing through the center of gravity of the second slice in the point o; draw the line $o\,p$ parallel to $c\,m$; $p\,q$ parallel to $f\,m$; $q\,r$ parallel to $g\,m$; $r\,s$ parallel to $h\,m$; $s\,t$ parallel to $i\,m$; $t\,u$ parallel to $j\,m$; and $u\,v$ parallel to $k\,m$.

From the point v, which is the point at the intersection of the line $u\,v$ with the line drawn through the center of gravity of the last elementary section, draw the line $v\,w$ parallel to $m\,l$; then its intersection w with the line $a\,m$ is a point on the required neutral axis. If the line $a\,m$ is so short that the line $v\,w$ fails to cut it, it may be extended indefinitely, as shown at $m\,x'$, so as to make it intersect with the line $v\,w$. Having found the point w, draw the horizontal line $d\,e$ through it. This line is the required neutral axis of the figure, and passes through its center of gravity.

This method of determining the position of the neutral axis and center of gravity may be applied to any irregular-shaped section, and in many cases may be found more convenient than the mathematical method.

When, as in Fig. 6, the section is made up of several regular parts whose centers of gravity can be readily located, it is not necessary to subdivide any one of these parts. Thus, the center of gravity of the web of the beam is located on the horizontal line through r, and its area is represented by the distance $b\,k$. We can therefore draw from n a line parallel to $b\,m$ until it intersects the horizontal line through the center of gravity of the web member; then, from this point, draw a line parallel to $k\,m$ until it intersects the horizontal line through the lower section at the point v. The point v, as thus located, is identical with the point previously found when the web section was divided into the small parts, and the line drawn from v parallel to $l\,m$ until it intersects $a\,m$, locates the point w on the neutral axis, as before. If, however, the center of gravity of the web cannot be readily located, it is better to divide it into small parts, as in the first method.

THE MOMENT OF INERTIA

9. The term **moment of inertia** is a mathematical expression that depends on the distribution of either the material of a body or the area of a surface with respect to a given axis. In other words, the moment of inertia is a term that expresses the relative value of all of the infinitely small portions that make up the entire section of a beam or column. The portions of the area farthest from the neutral axis of a section are much more valuable in resisting stress than those portions adjacent to the axis, so that some value must be obtained that will express the efficiency of the entire section to resist transverse stress when compared with any other section. This value is called the moment of inertia. As applied to the area of a plane figure, the moment of inertia, with respect to an axis lying in the same plane, is numerically equal to the sum of the products obtained by multiplying each of the elementary areas of which the figure is composed by the square of its distance from the given axis.

By **elementary area** is meant an area smaller than any with which we are accustomed to deal in ordinary calculations; it is, therefore, impossible to find an exact expression for the moment of inertia of a figure by the methods of calculation in ordinary use. By means of calculus, however, exact formulas have been deduced, by which the moments of inertia of many of the simple geometrical forms, with respect to axes through their centers of gravity, have been found. There are also a number of approximate methods by which the moment of inertia of an irregular section, to which these formulas do not apply, may be found. Further, the moment of inertia of any section that can be divided into parts, the moments of each of which, with respect to an axis through its center of gravity, can be found, is easily calculated by one of the following principles.

COMPUTING THE MOMENT OF INERTIA

10. Analytical Method.—To illustrate a simple approximate method of computing the moment of inertia of a figure, consider the relation of the small **I** section shown in Fig. 7 to the axis *de* through its center of gravity. Divide the section

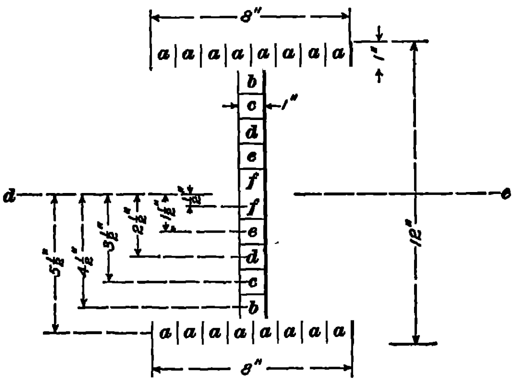


FIG 7

into a number of little squares (in this case, each with an area of 1 square inch) and consider the distance of each square from the axis to be the distance from the axis to its center of gravity. The products of the area of each square, multiplied by the square of its distance from the axis, are as follows:

Squares a, $1 \times (5\tfrac{1}{2})^2 = \tfrac{121}{4}$
Squares b, $1 \times (4\tfrac{1}{2})^2 = \tfrac{81}{4}$
Squares c, $1 \times (3\tfrac{1}{2})^2 = \tfrac{49}{4}$
Squares d, $1 \times (2\tfrac{1}{2})^2 = \tfrac{25}{4}$
Squares e, $1 \times (1\tfrac{1}{2})^2 = \tfrac{9}{4}$
Squares f, $1 \times (\tfrac{1}{2})^2 = \tfrac{1}{4}$

Adding these products for all the squares, we have:

16 squares a, $\tfrac{121}{4} \times 16 = 484$
2 squares b, $\tfrac{81}{4} \times 2 = 40\tfrac{1}{2}$
2 squares c, $\tfrac{49}{4} \times 2 = 24\tfrac{1}{2}$
2 squares d, $\tfrac{25}{4} \times 2 = 12\tfrac{1}{2}$
2 squares e, $\tfrac{9}{4} \times 2 = 4\tfrac{1}{2}$
2 squares f, $\tfrac{1}{4} \times 2 = \tfrac{1}{2}$
Total, $566\tfrac{1}{2}$

which is the sum of the products of each of the small areas multiplied by the square of its distance from the axis. This result, however, is only a rough approximation to the moment of inertia, owing to the fact that the assumed areas are so large. The actual value of the moment of inertia of the section, as will be shown subsequently, is $568\frac{2}{3}$.

11. Rules and Formulas for Moments of Inertia. In Table I exact formulas are given for computing the moment of inertia of such regular figures as are most often used in structural design; it also gives approximate formulas for computing this factor for common rolled sections. The tables of properties of rolled sections published by the rolling mills give accurate values of the moment of inertia of all the principal sections used in the construction of buildings, so that it is not generally necessary to make the calculations for these sections; the approximate formulas in Table I are, however, sometimes useful in making calculations when the tables published by the rolling mills are not at hand.

Rule.—*To find the moment of inertia, with respect to any axis of any figure whose moment of inertia, with respect to a parallel axis through its center of gravity, is known, add its moment of inertia, with respect to the axis through its center of gravity, to the product of its area multiplied by the square of the distance from its center of gravity to the required axis.*

This rule may be expressed by the formula

$$I' = I + a x^2 \qquad (1)$$

in which I' = required moment of inertia;
I = moment of inertia of the section, with respect to the axis through its center of gravity and parallel to the given axis;
a = area of the figure;
x = distance from its center of gravity to the required axis.

The moment of inertia, with respect to an axis through its center of gravity, of any section that can be divided

into a number of parts, the moments of inertia of each of which, with respect to an axis through its center of gravity parallel to the given axis, is known, is equal to the sum of the moments of inertia of its parts with respect to the given axis. Since the moment of inertia of any figure, with respect to any axis, is expressed by the formula $I' = I + a\,x^2$, if we denote the sum of the moments of the separate figures making up a section, with respect to an axis through the center of gravity of that section, by $\Sigma\,I'$ (in which the Greek letter Σ, read *sigma*, means *sum of*), we have

$$I_s = \Sigma\,I' = \Sigma(I + a\,x^2) \qquad (2)$$

which is a general formula often used to denote the moment of inertia I_s of any built-up section.

12. Graphical Methods of Computing Moments of Inertia.—There are several graphical methods of computing the moment of inertia, one of which is an extension of the graphical method of locating the center of gravity and neutral axis that was described in Art. 8 and illustrated by Fig. 6. Thus, let it be required to determine the moment of inertia, with respect to the axis *d e*, of the beam section shown in Fig. 6. Using the same scale as that to which the section was drawn, compute or measure the area of the figure enclosed by the lines *n o p q* . . . *v w n*; multiply this area by the area of the section—shown graphically by the length of the line *a l*—and the product will be the moment of inertia of the section, with respect to the axis *d e*, through its center of gravity. For example, suppose that the section shown in Fig. 6 has been drawn to a scale of $\frac{1}{4}$ inch = 1 inch. Using this scale, and computing the area of the figure enclosed by the lines *n o p q* . . . *v w n*, we find it to be 43.36 square inches. The area of the section is 61 square inches; therefore, according to the rule, its moment, with respect to the axis *d e*, is $43.36 \times 61 = 2{,}644.96$.

For finding the moment of inertia, it is necessary to divide the section into a number of parts, for it is evident that the area of the figure enclosed by the lines *n r′ v w n* is considerably less than that of the figure *n o p q* . . . *v w n*,

obtained by dividing the web into the small sections and drawing the lines of the diagram for each.

The area of the figure *nopq . . . vwn* may be computed by extending the horizontal lines through o, p, q, etc., so as to divide it into a series of triangles and trapezoids. The dimensions of these can be readily measured, and their areas can be calculated by means of the principles of mensuration.

This method of computing the moment of inertia will be found convenient in the case of very irregular sections, to which the methods previously given can be applied only with considerable difficulty. The accuracy of the result will, in general, be greater when the section is drawn to a large scale and divided into a comparatively large number of parts.

EXAMPLE 1.—What is the moment of inertia of the section of a 10″ × 16″ beam about an axis through its center of gravity parallel to its shorter side?

SOLUTION.—From Table I, the formula for the moment of inertia of a solid rectangle is $I = \frac{b d^3}{12}$. Substituting the given dimensions, we have

$$I = \frac{10 \times 16^3}{12} = 3{,}413\tfrac{1}{3}. \quad \text{Ans.}$$

EXAMPLE 2.—Using the formula given in Table I, compute the least moment of inertia of the section of a cast-iron column shown in Fig. 8.

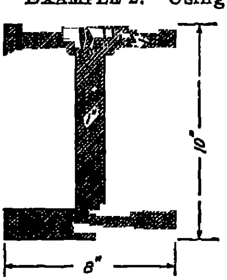


FIG. 8

SOLUTION.—It can readily be seen that the least moment of inertia will be with reference to an axis parallel with the web. Hence, substituting in the formula, we have

$$I = \frac{2 s b^3 + h t^3}{12}$$

$$= \frac{2 \times 1 \times 8^3 + (8 \times 1^3)}{12} = 86. \quad \text{Ans.}$$

EXAMPLE 3.—Referring to the rules and formulas in Art. **11**, compute the moment of inertia, with respect to the axis de through its center of gravity, of the section shown in Fig. 7.

SOLUTION.—This section is made up of three rectangles, the moments of inertia of which, with respect to the given axis, can be found by means of formula **1**. The moment of inertia of one of the flanges, with respect to an axis through its center of gravity parallel to de is $I = \frac{8 \times 1^3}{12} = \frac{2}{3}$. The area of this figure is $8 \times 1 = 8$ sq in.,

and the distance of its center of gravity from de is $5\frac{1}{2}$ in.; therefore, its moment of inertia, with respect to de, is $I' = \frac{2}{3} + 8 \times (5\frac{1}{2})^2 = 242\frac{2}{3}$. The axis through the center of gravity of the web section coincides with the axis de; hence, the moment of inertia of this section, with respect to de, is

$$I' = \frac{1 \times 10^3}{12} = 83\frac{1}{3}$$

Then, the moment of inertia I_x of the whole section $= \Sigma I' = 242\frac{2}{3} + 242\frac{2}{3} + 83\frac{1}{3} = 568\frac{2}{3}$. Ans.

EXAMPLE 4.—What is the moment of inertia, with respect to the axis de, of the column section shown in Fig. 9?

SOLUTION.—The moment of inertia of one of the flange plates, with respect to an axis through its center of gravity, parallel to the axis de, is $\frac{12 \times (\frac{3}{8})^3}{12} = .05$, the area of the plate is $12 \times \frac{3}{8} = 4.5$ sq. in., and the distance of its center of gravity from the axis is $6\frac{3}{16}$ in. Therefore, the moment of inertia of the plate, with respect to the axis de, is $.05 + 4.5 \times (6\frac{3}{16})^2 = 172.33$. From Table V, the area of a $4'' \times 4'' \times \frac{1}{2}''$ angle is 3.75 sq. in., and the distance of its center of gravity from the back of a flange is 1.18 in. The distance of the center of gravity of the angle from the axis de, in accordance with the dimensions given in the figure, is $6 - 1.18 = 4.82$ in. In accordance with the formula $\frac{t(a-c)^3 + ac^3 - (a-t)(c-t)^3}{3}$ given in Table II, for finding the moment of inertia of an angle with equal legs, the moment of inertia of the $4'' \times 4'' \times \frac{1}{2}''$ angle, with respect to the axis through its center of gravity, is

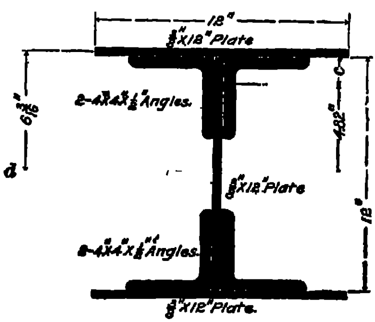


FIG. 9

$$\frac{.5(4 - 1.18)^3 + 4 \times 1.18^3 - (4 - .5)(1.18 - .5)^3}{3} = 5.54$$

The moment of inertia of the angle, with respect to de, is, therefore, $5.54 + 3.75 \times 4.82^2 = 92.65$. The center of gravity of the web-plate lies on the axis de, therefore, the moment of inertia of the plate, with respect to de, is $\frac{\frac{3}{8} \times 12^3}{12} = 54$. The moment of inertia of the whole section, with respect to de, is, therefore, $2 \times 172.33 + 4 \times 92.65 + 54 = 769.26$. Ans.

EXAMPLE 5.—What is the moment of inertia, with respect to the axis de, of the column section shown in Fig. 10?

SOLUTION.—The moment of inertia of one of the cover-plates, with respect to an axis through its center of gravity, parallel to de, is $\frac{12 \times (\frac{1}{2})^3}{12} = .125$; the area of the plate is $12 \times \frac{1}{2} = 6$ sq. in., and the distance of its center of gravity from de is $5\frac{1}{4}$ in; therefore, its moment of inertia, with respect to de, is $I' = .125 + 6 \times (5\frac{1}{4})^2 = 165.5$. From Table IV, the area of a 10-in. 15-lb. channel is 4.46 sq. in., and its moment of inertia, with respect to an axis through its center of gravity, corresponding in this case with the axis de, is 66.9, therefore, the moment of inertia of the whole section, with respect to de, is $2 \times 165.5 + 2 \times 66.9 = 464.8$. Ans.

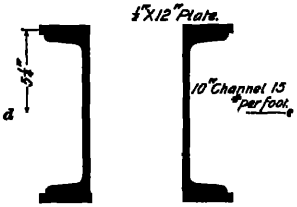


FIG. 10

EXAMPLES FOR PRACTICE

1. What is the moment of inertia of a hollow, square column section 12 inches outside and 10 inches inside? Ans. $894\frac{2}{3}$

2. Find the moment of inertia of a $4'' \times 6'' \times \frac{1}{2}''$ angle, with respect to an axis parallel to its long leg, using the formula given in Table II. Ans. 6.27

3. What is the moment of inertia, with respect to an axis parallel to the web, of a **Z** bar 5 inches in depth, $\frac{1}{2}$ inch thick, and having legs $3\frac{1}{4}$ inches in length? Use the formula given in Table II. Ans. 9.05

SECTION MODULUS

13. The **section modulus** of a section under **transverse stress**, or the stress to which a beam is subjected under a load which tends to bend it, is the moment about the neutral axis of such a portion of the section that if its area is multiplied by the **extreme fiber stress**, the **resisting moment** of this portion will equal the resisting moment of the variable stress in the actual section, from the neutral axis to the extreme fibers. By the term resisting moment is meant the value which expresses the resistance of the beam to the bending moment produced upon it by the loads which it supports. The resisting moment of any beam section is always equal to the section modulus multiplied by the extreme fiber stress, which is the strength value of

the material of which the section is composed, per square inch of area; and the material is only subjected to this extreme stress at the outside edge of the section.

The value designated as the section modulus is more clearly explained by referring to Fig. 11, in which $ABCD$ shows a section of a rectangular beam. $E\,F$ represents the neutral axis, which passes through the center of gravity, and $G\,H$ is a line situated at the edge of the section and represents the position of the extreme fibers. Then $\frac{d}{2}$ is the distance from the extreme fibers to the neutral axis. The stress in the fibers on the neutral axis is zero, but it gradually increases until the edge of the section is reached, where it is maximum. Therefore, while the area is uniform, the stress varies with the distance of the fiber from the neutral axis.

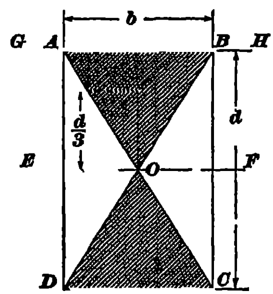


FIG. 11

Draw the diagonals $A\,C$ and $B\,D$; the shaded portions $A\,B\,O$ and $D\,C\,O$ represent the area that, if considered as having a uniform stress equal to the stress in the extreme fibers of the section, offers the same resistance as the entire section having the variable stress. Then the moment of this shaded portion about the neutral axis gives the section modulus of the beam. The combined area of the shaded portions is equal to $\frac{b\,d}{2}$, and the lever arm, which is the distance from the center of gravity of each section to the neutral axis, is $\frac{d}{3}$. The moment, therefore, is $\frac{b\,d}{2} \times \frac{d}{3} = \frac{b\,d^2}{6}$, and the section modulus for a rectangular beam may be expressed by the formula

$$S = \frac{b\,d^2}{6} \qquad (3)$$

in which S = section modulus;

d = depth of beam, in inches;

b = breadth, or width, of beam, in inches.

14. In Fig. 12 is shown the graphical method of obtaining the section modulus for a **T** bar. The neutral axis is located at AB and the greatest distance from AB to the extreme fiber is c; from a and b draw lines to the center of gravity of the section o. Below the neutral axis lay off the line CD parallel to the neutral axis AB and at a distance from it equal to c. Project the points d and e to d' and e' and from these points draw lines to o; also project f and g to f' and g' and from these points draw lines to o. Then the shaded portions $h\,i\,o\,j\,k\,l\,m$ and $a\,o\,b$ represent areas that, if considered as having a uniform stress equal to that in the extreme fiber, will produce a stress equal to the total resisting stress, which is not uniform in the entire section. The moments of these areas about the neutral axis will give the section modulus of the section.

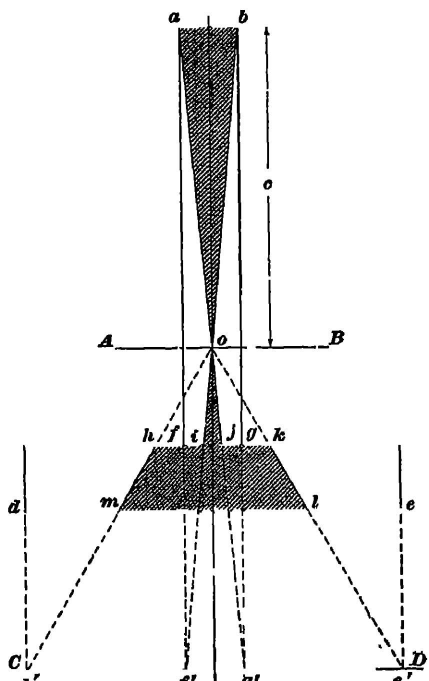


Fig 12

The same method may be applied to an irregular section, as illustrated in Fig. 13. In this case the top and bottom edges of the section are equally distant from the neutral axis. Several points are projected from the surface of the section to the lines CD and EF and from these points lines are drawn to the point O on the neutral axis. Where the lines thus drawn intersect the horizontal lines drawn through the points in question, will be located points on the surface of the shaded portion.

The outline of the figure may then be drawn through the points thus located and the section determined. The moment of this section about the neutral axis gives the section modulus. In considering an irregular section, it is very necessary to apply the graphical method for determining the section modulus, as it is extremely difficult to evolve a convenient formula for obtaining it.

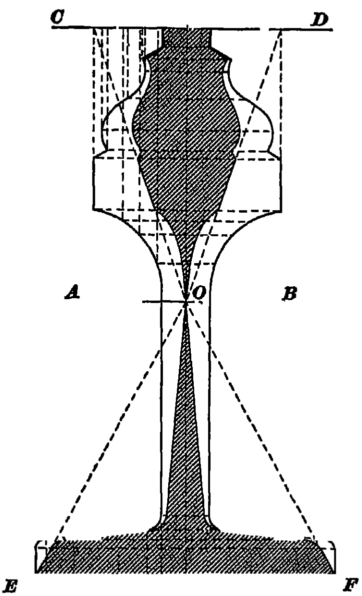


FIG. 18

15. The section modulus for any section whose moment of inertia is known, is found by dividing the moment of inertia by the greatest distance of the neutral axis from the outside fibers of the section. This may be expressed by the formula

$$S = \frac{I}{c} \qquad (4)$$

in which I = moment of inertia, with respect to the neutral axis;

S = section modulus;

c = distance from neutral axis to farthest edge of section.

EXAMPLE 1.—What will be the section modulus of a yellow-pine beam, 10 inches wide by 12 inches in depth?

SOLUTION.— d = 12 in., b = 10 in Therefore, by applying formula **3**, we have

$$S = \frac{b\,d^2}{6} = \frac{10 \times 12^2}{6} = \frac{1{,}440}{6} = 240. \text{ Ans.}$$

EXAMPLE 2.—What is the section modulus of the cast-iron lintel shown in Fig. 14?

SOLUTION.—From Table I, we have the formula $S = \frac{I}{d - c_1}$. As it

is necessary to have the area in order to determine the value of c_1, in the above formula, this will be determined by the formula in the first column, or $A = bs + ht$. Substituting the values, we have $A = 8 \times \frac{3}{4} + 9\frac{1}{4} \times \frac{3}{4} = \frac{207}{16}$. Then substituting in the formula

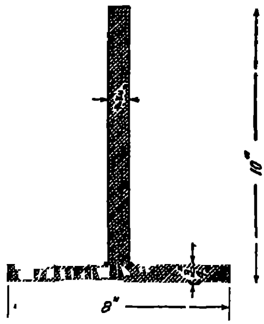


FIG. 14

$$c_1 = \frac{d^2 t + s^2 (b - t)}{2A}, \text{ gives}$$

$$c_1 = \frac{10^2 \times \frac{3}{4} + (\frac{3}{4})^2 (8 - \frac{3}{4})}{2 \times \frac{207}{16}} = 3.056 \text{ in.}$$

$$c = d - c_1 = 10 - 3.056 = 6.944 \text{ in.}$$

The moment of inertia is obtained from the formula

$$I = \frac{t c^3 + b c_1^3 - (b - t)(c_1 - s)^3}{3}$$

which gives

$$I = \frac{\frac{3}{4} \times 6.944^3 + 8 \times 3.056^3 - (8 - \frac{3}{4})(3.056 - \frac{3}{4})^3}{3} = 130.181$$

Then, $$S = \frac{I}{d - c_1} = \frac{130.181}{6.944} = 18.747. \text{ Ans.}$$

EXAMPLES FOR PRACTICE

1. What is the section modulus of a trapezoidal section whose parallel sides are 6 and 9 inches and the distance between them is 6 inches? Ans. $41\frac{5}{8}$

2. Find the section modulus of a hollow, rectangular, cast-iron lintel that measures 10 inches by 12 inches outside and is $\frac{5}{8}$ inch thick. Ans. 89.026

3. The structural member of a skylight has the section of a cross, the vertical web of which is 5 inches in depth; the cross-bar extends $1\frac{1}{4}$ inches each side of the center line of the vertical bar and the center line of the horizontal bar bisects that of the vertical bar. Provided the thickness of the metal is $\frac{3}{8}$ inch, what will be the section modulus? Ans. 1.5662

RADIUS OF GYRATION

16. In computing the strength of columns, frequent use is made of a property of a section that is numerically equal to the square root of the quotient of its moment of inertia, with respect to an axis through its center of gravity, divided by its area. This property is called the **radius of gyration** of the section, with respect to the given axis. It is usually expressed by the letter r, and its value, with respect to a given axis, for any section whose area A and moment of inertia I, with respect to the same axis, are known, is given by the formula

$$r = \sqrt{\frac{I}{A}} \qquad (5)$$

The form in which the radius of gyration appears in most formulas for calculating the strength of columns is its square; hence, it is convenient to express the above relation by the formula

$$r^2 = \frac{I}{A} \qquad (6)$$

which gives directly the value to be substituted in the column formulas. Formulas for computing the least radius of gyration and its square for the sections most often used in the design of structures, are given in Table I. The tables of the properties of rolled sections also give accurate values of r and r^2 for the sections used in the examples given in the following pages.

EXAMPLE.—Compute the square of the radius of gyration, with respect to the axis de, of the column section shown in Fig. 9.

SOLUTION.—By referring to example 4, Art. **12**, the moment of inertia of the section is found to be $I = 769.26$, and the area of the section is $3 \times 4.5 + 4 \times 3.75 = 28.50$ sq. in. Substituting these values in formula **6**, we have

$$r^2 = \frac{769.26}{28.50} = 26.99. \text{ Ans.}$$

EXAMPLES FOR PRACTICE

1. Find the radius of gyration of a round wooden column having a diameter of 12 inches. Ans. 3

2. What is the square of the least radius of gyration of the column section shown in Fig 8? Ans. 3.583

3. Compute the radius of gyration of the section of a hollow cast-iron column, 8 inches by 12 inches outside and $\frac{3}{4}$ inch thick.
Ans. 4.349

17. The diagram shown in Fig. 15 gives the width of flange, weight in pounds, per foot, and thickness of

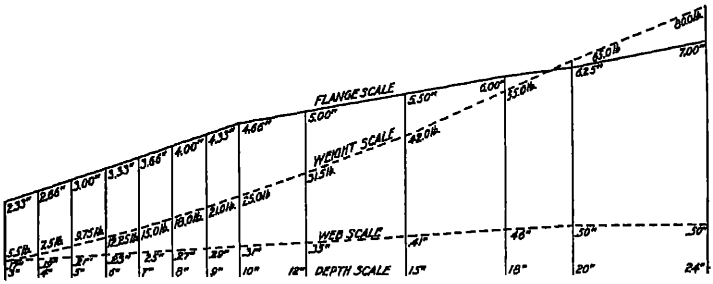


FIG 15

web for beams of the usual depths, these values being for standard beams of minimum weight.

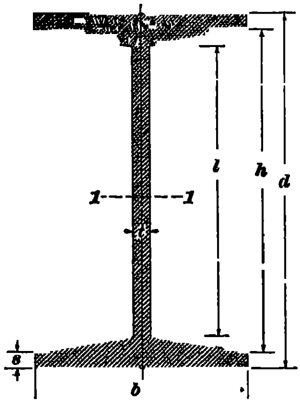


FIG. 16

The following formulas, adopted by the Association of American Steel Manufacturers, are of value in determining the properties of I beams. (See Fig. 16.)

Weight per foot = area × 3.4.

$$\text{Area} = td + 2s(b-t) + \frac{(b-t)^2}{12}.$$

$$\text{Section modulus} = S = \frac{2I}{d}.$$

$$\text{Slope of flange} = g = \frac{h-l}{b-t} = \frac{1}{6}$$

for standard beams.

I = moment of inertia, neutral axis (1–1) parallel to flange.

$I = \frac{1}{12}\left[b\,d^3 - \frac{1}{4g}(h^4 - l^4)\right]$, or $\frac{b\,d^3}{12} - \frac{h^4 - l^4}{8}$ for standard beams.

18. Fig. 17 shows the width of flange, thickness of web, and weight per foot for channel beams of the usual depths

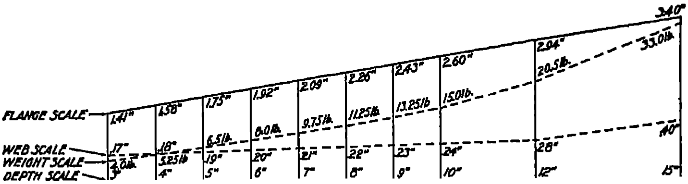


FIG 17

and, as in the diagram for I beams, these values are for channels of minimum weight.

The following formulas are given in connection with Fig. 18.

Weight per foot = area × 3.4.

Area $= t\,d + 2\,s\,(b - t) + \frac{(b - t)^2}{6}$.

Section modulus $= S = \frac{2\,I}{d}$.

Slope of flange $= g = \frac{h - l}{2\,(b - t)}$, or $\frac{1}{6}$ for standard channels.

I = moment of inertia, neutral axis (1–1) parallel to flange.

$I = \frac{1}{12}\left[b\,d^3 - \frac{1}{8g}(h^4 - l^4)\right]$, or $\frac{b\,d^3}{12} - \frac{h^4 - l^4}{16}$ for standard channels.

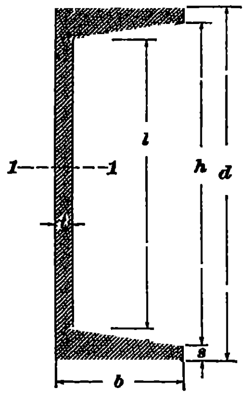


FIG 18

19. Table I gives formulas for computing the area, moment of inertia, section modulus, and radius of gyration of usual sections, also the distance from the neutral axis to the extremities of the section. These formulas are useful in obtaining the properties of sections that are not given in the table of properties for the several standard rolled sections.

20. Table II gives formulas for the moment of inertia of the standard rolled sections with respect to their different

axes, also the distance from the neutral axis to the back of the section; where a third axis is considered it gives the angle between the third axis and the vertical one.

The slope of the flanges of the I beams and channels is $\frac{1}{6}$, that is, $\frac{h-l}{b-t}=\frac{1}{6}$. In the formulas relating to the deck beam, e represents the area of the head.

21. Tables III to IX give the dimensions and properties of structural-steel shapes that conform to the American standards adopted January, 1896. Such tables are found in most of the handbooks published by the different steel manufacturers. The first five columns give the dimensions, area, and weight of the section considered, and as all these sections are of structural steel, there is a fixed relation between the weight and area and one may be obtained from the other. The weight of 1 square inch of structural steel 1 foot long is 3.4 pounds; hence, the weight of 1 foot of any section is equal to its sectional area multiplied by 3.4, as stated in Arts. **17** and **18**, in connection with the weight of I beams and channels.

The values for the section modulus, radius of gyration, and moment of inertia are required in finding the strength of the section under different conditions, and are used in calculating the strength of beams, girders, columns, and posts. The moment of inertia is particularly valuable because the section modulus and radius of gyration may be obtained from it, as explained in Arts. **15** and **16**.

The uses of the coefficients of strength and coefficients of deflection are illustrated in the examples given in Art. **22**.

The values in the columns headed F and F' are the safe uniformly distributed loads, in pounds, including the weight of the beam, for a beam 1 foot long; they have been computed for fiber stresses of 16,000 and 12,500 pounds per square inch, respectively. Hence, the safe load for any span is equal to the coefficient F or F' divided by the span, in feet.

The formulas from which these coefficients were obtained are $F = \frac{2}{3} \times 16{,}000 \times S$ and $F' = \frac{2}{3} \times 12{,}500 \times S$, in which S is the section modulus.

The values for N and N', the coefficients of deflection for uniform and center loads, respectively, were obtained from the formulas $N = \frac{W l^3}{76.8 E I}$ and $N' = \frac{W l^3}{48 E I}$, in which W equals 1,000 pounds, l equals 12 inches, E equals 29,000,000, and I equals the moment of inertia about the axis *1–1*. Therefore, these coefficients represent the deflection, in inches, of a beam 1 foot long having a load of 1,000 pounds. Multiplying the proper coefficient by the cube of the span, in feet, and by the number of 1,000-pound units in the given load, will give the deflection of a beam for any load and span.

Tables XI, XII, and XIII are useful in calculating the strength of these built-up sections when they are used as posts or struts in structural work. The angles given in these tables are not all standard, but the standard sections may be selected by referring to Tables V and VI.

22. The following examples illustrate the application of the tables giving the values for the properties of the several rolled sections.

In making the selection of the beams from the tables, it is customary to select the deepest beam having the least weight and the greatest section modulus. For instance, if a beam is required to have a section modulus of 30, and from the table it is observed that a 10-inch 40-pound beam has a section modulus of 31.7 and that a 12-inch 31.5-pound beam has a section modulus of 36, it is evident that it would be more economical to select the deeper, and consequently, the safer beam, thus saving a weight of 8.5 pounds per foot As steel beams are sold usually by the pound price, such a saving would be great in a large order. This economical practice should always be employed when the several additional inches in the depth of the beam will not interfere with the required headroom in the architectural design.

EXAMPLE 1 —What is the deflection of a 20-inch 65-pound I beam that carries a center load of 28,000 pounds and has a span of 20 feet?

SOLUTION.—The amount of deflection is obtained by multiplying the coefficient of deflection for beams with center loads (column 15,

Table III) by the cube of the span, in feet, and the number of 1,000-lb. units in the load. Hence, the deflection equals

$$.00000106 \times 20^3 \times \frac{28,000}{1,000} = .237 \text{ in. Ans.}$$

EXAMPLE 2.—What size of I beam is required to support a uniform load of 800 pounds per foot over a span of 25 feet, considering a safe unit fiber stress of 16,000 pounds?

SOLUTION.—According to Art. **21**, the safe uniformly distributed load for any span is equal to the coefficient F or F' divided by the span, in feet. By transposing the terms of the equation it is found that F or F' = load $\times$ span. The coefficient required for the load in this case is $800 \times 25 \times 25 = 500,000$. From column 12, Table III, the coefficient of strength for a 15-in. 42-lb. beam is found to be 628,270. The weight of the beam is $42 \times 25 = 1,050$ lb., which requires a coefficient of $1,050 \times 25 = 26,250$; then $628,270 - 26,250$ gives a net coefficient of 602,020. Therefore, a 15-in. 42-lb. beam is the size required If the load on the beam is concentrated at the center, the coefficient of strength required is twice as great. Ans.

EXAMPLE 3.—Two channels, placed back to back with webs vertical, form the support of an overhead mechanism for an elevator. The load is concentrated at the center of the channels and is equal to 10,000 pounds, provided the span of the channels is 8 feet and they are 7 inches in depth and weigh 14.75 pounds per foot, what will be the deflection?

SOLUTION —The deflection is equal to the coefficient multiplied by the cube of the span and the number of 1,000-lb. units in the load. From column N', Table IV, the coefficient of deflection for a 7-in. 14 75-lb. channel is .0000457. As there are two channels, each may be considered as supporting one-half of the load or 5,000 lb. Then the deflection will be equal to

$$.0000457 \times 8^3 \times \frac{5,000}{1,000} = .117 \text{ in. Ans.}$$

EXAMPLE 4.—What is the section modulus of a rectangular bar 3 inches by $\frac{3}{4}$ inch, using the moment of inertia obtained from Table X?

SOLUTION.—The section modulus is determined by the formula $S = \frac{I}{c}$, given in Art. **15**, and from the table the moment of inertia is found to be 1 69 The neutral axis passes through the center of the rectangle and consequently the distance c is equal to $1\frac{1}{2}$ in. Substituting these values in the formula,

$$S = \frac{1.69}{1.5} = 1.127. \text{ Ans.}$$

EXAMPLES FOR PRACTICE

1. Two channels, placed back to back, are to be used to support a uniform load of 1,000 pounds per lineal foot over a span of 20 feet. Using a safe unit fiber stress of 16,000 pounds, what size channels will be the most economical for the purpose?

Ans. Two 12-in. 20½-lb. channels

2. What would be the deflection in the previous example?

Ans. .488 in.

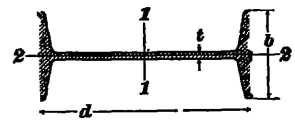
3. What size of I beam should be used to support a center load of 10,000 pounds over a span of 12 feet, using a safe unit fiber stress of 16,000 pounds? Ans. 10-in. 25-lb. I beam

4. An I beam to support a given load is required to have a section modulus of 30; what would be the size and weight of the most economical section? Ans. 12-in. 31½-lb. I beam

5. From the fact that the available headroom is limited, it is necessary to use several 8-inch I beams placed side by side in order to support a uniformly distributed load of 20,000 pounds. The span of the beams is 20 feet. How many beams weighing 25.25 pounds are required to support this load, provided the allowable deflection is ¾ inch, and a safe unit fiber stress of 16,000 pounds is used?

Ans. Three beams

TABLE III
PROPERTIES OF STANDARD I BEAMS

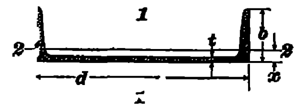


1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Depth of Beam Inches	Weight per Foot Pounds	Area of Section Square Inches	Thickness of Web Inches	Width of Flange Inches	Moment of Inertia, Axis 1-1 Inches	Section Modulus, Axis 1-1 Inches	Radius of Gyration, Axis 1-1 Inches	Moment of Inertia, Axis 2-2 Inches	Radius of Gyration, Axis 2-2 Inches	Increase of Thickness of Web for Each Pound Increase in Weight Inches	Coefficient of Strength: For Fiber Stress of 16,000 Pounds per Square Inch for Buildings	Coefficient of Strength: For Fiber Stress of 12,500 Pounds per Square Inch for Bridges	Coefficient of Deflection: Uniform Load	Coefficient of Deflection: Center Load
d		A	t	b	I	s	r	I'	r'	f	F	F'	N	N'
3	5.50	1.63	.17	2.33	2.5	1.7	1.23	.46	.53	.098	17,650	13,790	.00031253	.00050006
3	6.50	1.91	.26	2.42	2.7	1.8	1.19	.53	.52		19,140	14,950	.00028827	.00046124
3	7.50	2.21	.36	2.52	2.9	1.9	1.15	.60	.52		20,710	16,180	.00026644	.00042630
4	7.50	2.21	.19	2.66	6.0	3.0	1.64	.77	.59	.074	31,810	24,850	.00013009	.00020815
4	8.50	2.50	.26	2.73	6.4	3.2	1.59	.85	.58		33,890	26,480	.00012209	.00019535
4	9.50	2.79	.34	2.81	6.7	3.4	1.54	.93	.58		35,980	28,110	.00011500	.00018400
4	10.50	3.09	.41	2.88	7.1	3.6	1.52	1.01	.57		38,070	29,750	.00010868	.00017389
5	9.75	2.87	.21	3.00	12.1	4.8	2.05	1.23	.65	.059	51,590	40,300	.00006417	.00010267
5	12.25	3.60	.36	3.15	13.6	5.4	1.94	1.45	.63		58,100	45,390	.00005698	.00009117
5	14.75	4.34	.50	3.29	15.1	6.1	1.87	1.70	.63		64,630	50,490	.00005122	.00008195
6	12.25	3.61	.23	3.33	21.8	7.3	2.46	1.85	.72	.049	77,460	60,520	.00003561	.00005698
6	14.75	4.34	.35	3.45	24.0	8.0	2.35	2.09	.69		85,270	66,610	.00003235	.00005177
6	17.25	5.07	.47	3.57	26.2	8.7	2.27	2.36	.68		93,110	72,740	.00002963	.00004741

7	15 00	4.42	25	3 66	36 2	10 4	2 86	2 67	78	.042	110,410	86,260	00002142	.00003427
7	17.50	5 15	35	3 76	39 2	11 2	2 76	2 94	76		119,400	93,290	00001980	.00003168
7	20 00	5 88	.46	3 87	42 2	12 1	2 68	3 24	.74		128,560	100,430	00001839	00002943
8	18 00	5.33	27	4 00	56 9	14 2	3 27	3 78	.84	.037	151,660	118,490	00001364	00002183
8	20 25	5 96	35	4 08	60 2	15 0	3 18	4 04	.82		160,510	125 400	00001289	00002062
8	22 75	6 69	44	4 17	64 1	16 0	3 10	4 36	81		170,970	133,570	00001210	00001936
8	25 25	7.43	53	4 26	68 0	17 0	3 03	4 71	.80		181,430	141,740	.00001140	00001825
9	21 00	6 31	29	4 33	84 9	18 9	3 67	5 16	90	033	201,300	157,260	.00000914	00001462
9	25 00	7 35	.41	4.45	91 9	20 4	3 54	5 65	.88		217,930	170,260	00000844	.00001350
9	30 00	8 82	.57	4 61	101 9	22 6	3.40	6 42	.85		241,460	188,640	00000762	.00001219
9	35 00	10 29	.73	4 77	111.8	24 8	3 30	7 31	.84		264,990	207,020	00000694	00001110
10	25 00	7 37	.31	4 66	122 1	24 4	4 07	6 89	97	029	260,470	203,500	00000635	00001017
10	30 00	8 82	.45	4 80	134 2	26 8	3 90	7 65	93		286 250	223,630	00000578	00000925
10	35 00	10 29	60	4.95	146 4	29 3	3 77	8 52	91		312,390	244,050	00000530	.00000848
10	40 00	11 76	75	5.10	158 7	31 7	3 67	9 50	.90		338,530	264,480	00000489	00000782
12	31 50	9 26	35	5 00	215 8	36 0	4.83	9 50	1 01	025	383,670	299,740	00000360	.00000575
12	35 00	10 29	.44	5 09	228 3	38 0	4 71	10 07	.99		405,800	317,030	.00000340	00000544
12	40 00	11.76	56	5 21	245 9	41 0	4 57	10 95	.96		437,170	341,540	.00000316	.00000505
15	42 00	12.48	.41	5 50	441.8	58 9	5 95	14 62	1 08	020	628,270	490,840	00000176	.00000281
15	45 00	13 24	46	5 55	455 8	60 8	5 87	15 09	1 07		648,310	506,490	00000170	00000272
15	50 00	14 71	56	5 65	483.4	64 5	5 73	16 04	1.04		687,530	537,130	.00000161	.00000257
15	55 00	16 18	66	5 75	511 0	68 1	5 62	17 06	1 03		726,740	567,770	00000152	.00000243
15	60 00	17 65	.76	5 84	538 6	71 8	5 52	18 17	1 01		765,960	598,410	00000144	00000231
18	55 00	15 93	46	6 00	795 6	88.4	7 07	21 19	1 15	016	942,880	736,620	.00000098	.00000156
18	60 00	17 65	56	6 10	841 8	93 5	6 91	22 38	1 13		997,680	779,440	.00000092	00000148
18	65 00	19 12	64	6 18	881 5	97 9	6 79	23.47	1 11		1,044,740	816,200	00000088	00000141
18	70 00	20 59	.72	6 26	921 2	102 4	6 69	24 62	1 09		1,091,800	852,970	.00000084	.00000135
20	65 00	19 08	50	6 25	1,169 5	117 0	7 83	27 86	1 21	015	1,247,490	974,600	00000066	.00000106
20	70 00	20 59	58	6 33	1,219 8	122 0	7 70	29 04	1 19		1,301,110	1,016,490	.00000064	00000102
20	75 00	22 06	65	6.40	1,268 8	126 9	7 58	30 25	1 17		1,353.400	1,057,340	00000061	.00000098
24	80 00	23 32	50	7 00	2,087 2	173 9	9.46	42 86	1 36	.0123	1,855,310	1,449,460	.00000037	.00000060
24	85 00	25 00	57	7 07	2,167 8	180 7	9 31	44 35	1 33		1,926,950	1,505,430	.00000036	.00000057
24	90 00	26 47	63	7 13	2,238 4	186 5	9 20	45 70	1 31		1,989,700	1,554,450	00000035	.00000056
24	95 00	27 94	69	7 19	2,309 0	192 4	9 09	47 10	1 30		2,052,440	1,603,470	.00000034	.00000054
24	100 00	29 41	75	7 25	2,379 6	198 3	8 99	48 55	1 28		2,115,190	1,652,490	00000033	.00000052

TABLE IV

PROPERTIES OF STANDARD CHANNELS

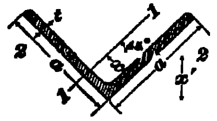


1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
													Coefficient of Strength		Coefficient of Deflection	
Depth of Channel Inches	Weight per Foot Pounds	Area of Section Square Inches	Thickness of Web Inches	Width of Flange Inches	Moment of Inertia, Axis 1-1 Inches	Section Modulus, Axis 1-1 Inches	Radius of Gyration, Axis 1-1 Inches	Moment of Inertia, Axis 2-2 Inches	Section Modulus, Axis 2-2 Inches	Radius of Gyration, Axis 2-2 Inches	Distance of Center of Gravity From Outside of Web Inches	Increase of Thickness of Web for Each Pound Increase in Weight. Inches	For Fiber Stress of 16,000 Pounds per Square Inch for Buildings	For Fiber Stress of 12,500 Pounds per Square Inch for Bridges	Uniform Load	Center Load
d		A	t	b	I	S	r	I'	S'	r'	x	t	F	F'	N	N'
3	4.00	1.19	.17	1.41	1.6	1.1	1.17	20	21	.41	44	.098	11,630	9,090	.0004743	.0007589
3	5.00	1.47	26	1.50	1.8	1.2	1.12	25	24	.41	.44		13,140	10,270	0004199	.0006718
3	6.00	1.76	.36	1.60	2.1	1.4	1.08	.31	.27	.42	46		14,710	11,490	.0003751	.0006001
4	5.25	1.55	.18	1.58	3.8	1.9	1.56	.32	.29	.45	.46	.074	20,230	15,800	.0002046	.0003273
4	6.25	1.84	.25	1.65	4.2	2.1	1.51	.38	.32	.45	.46		22,270	17,400	.0001858	.0002973
4	7.25	2.13	.33	1.73	4.6	2.3	1.46	44	.35	.46	46		24,360	19,030	.0001698	.0002717
5	6.50	1.95	.19	1.75	7.4	3.0	1.95	.48	.38	.50	.49	.059	31,640	24,720	.0001046	0001674
5	9.00	2.65	.33	1.89	8.9	3.5	1.83	64	45	.49	.48		37,860	29,570	.0000875	.0001399
5	11.50	3.38	.48	2.04	10.4	4.2	1.75	.82	.54	.49	.51		44,390	34,680	0000746	0001193

6	8 00	2 38	20	1 92	13 0	4.3	2.34	.70	.50	.54	.52	.049	46,210	36,100	0000597	0000855
6	10 50	3 09	32	2 04	15 1	5.0	2 21	.88	.57	53	.50		53.750	42,000	.0000513	0000821
6	13 00	3 82	.44	2 16	17 3	5 8	2 13	1 07	65	.53	52		61 600	48,120	.0000448	0000717
6	15 50	4.56	.56	2 28	19 5	6 5	2.07	1.28	.74	53	55		69.440	54 250	.0000397	.0000636
7	9 75	2 85	.21	2 09	21 1	6 0	2 72	98	.63	.59	.55	.042	64,270	50,210	0000368	0000588
7	12 25	3 60	.32	2 20	24.2	6.9	2.59	1 19	71	.57	.53		73,650	57,540	.0000321	.0000514
7	14 75	4 34	.42	2 30	27 2	7 8	2 50	1.40	.79	.57	.53		82,740	64,690	.0000286	.0000457
7	17 25	5 07	.53	2 41	30 2	8 6	2 44	1 62	.87	56	.55		91,950	71,840	.0000257	.0000411
7	19.75	5 81	.63	2 51	33 2	9 5	2.39	1.85	.96	.56	.58		101,100	78,990	.0000234	.0000374
8	11 25	3 35	.22	2 26	32 3	8 1	3 10	1 33	.79	63	.58	.037	86,140	67,300	0000240	.0000384
8	13 75	4.04	.31	2.35	36 0	9 0	2 98	1.55	.87	.62	.56		95,990	75,000	.0000216	0000345
8	16 25	4 78	.40	2 44	39.9	10 0	2 89	1 78	.95	.61	56		106,450	83,170	0000194	0000311
8	18 75	5 51	.49	2 53	43.8	11 0	2 82	2 01	1.02	.60	57		116,910	91,340	0000177	.0000283
8	21.25	6 25	.58	2 62	47.8	11.9	2.76	2 25	1 11	.60	59		127,370	99,510	.0000162	.0000260
9	13 25	3 89	.23	2 43	47.3	10 5	3 49	1 77	97	67	61	.033	112,170	87,630	0000164	.0000262
9	15 00	4 41	.29	2.49	50 9	11 3	3 40	1 95	1 03	66	59		120,540	94,170	0000153	0000244
9	20 00	5 88	.45	2 65	60 8	13 5	3 21	2 45	1 19	.65	.58		144,070	112,550	.0000128	.0000204
9	25.00	7 35	.61	2 81	70 7	15 7	3.10	2.98	1 36	.64	62		167,590	130,930	0000110	.0000176
10	15 00	4.46	.24	2 60	66 9	13 4	3 87	2.30	1 17	.72	64	.029	142,680	111,470	0000116	0000186
10	20 00	5.88	38	2 74	78 7	15 7	3 66	2 85	1.34	70	61		167,940	131,210	.0000099	0000158
10	25 00	7 35	.53	2 89	91 0	18 2	3 52	3 40	1 50	68	62		194,090	151,630	.0000085	.0000136
10	30 00	8 82	68	3 04	103 2	20 6	3.42	3 99	1 67	.67	.65		220,230	172,060	.0000075	.0000120
10	35.00	10 29	.82	3 18	115 5	23 1	3 35	4 66	1 87	67	69		246,380	192,480	.0000067	.0000107
12	20 50	6 03	.28	2 94	128.1	21.4	4.61	3 91	1 75	.81	.70	025	227,750	177,930	.0000061	.0000097
12	25 00	7 35	39	3 05	144 0	24 0	4 43	4 53	1 91	78	68		256,000	200,000	.0000054	.0000086
12	30 00	8 82	51	3 17	161 6	26 9	4 28	5 21	2 09	.77	68		287,370	224,510	.0000048	.0000077
12	35 00	10 29	64	3 30	179 3	29 9	4 17	5 90	2 27	76	69		318,750	249,020	.0000043	.0000069
12	40 00	11 76	.76	3 42	196 9	32.8	4 09	6 63	2 46	75	72		350,120	273,530	.0000039	.0000063
15	33 00	9 90	.40	3.40	312 6	41 7	5 62	8 23	3 16	91	.79	.020	444,520	347,280	0000025	.0000040
15	35 00	10 29	43	3.43	319 9	42 7	5 57	8.48	3 22	91	79		455,030	355,500	0000024	.0000039
15	40 00	11 76	.52	3 52	347 5	46 3	5 44	9 39	3 43	.89	.78		494,250	386,130	.0000022	.0000036
15	45 00	13 24	.62	3 62	375.1	50 0	5 32	10 29	3 63	.88	.79		533,470	416,770	0000021	.0000033
15	50 00	14.71	.72	3 72	402 7	53 7	5 23	11 22	3 85	.87	80		572,680	447,410	.0000019	.0000031
15	55.00	16 18	.82	3 82	430 2	57 4	5 16	12.19	4 07	.87	.82		611,900	478,050	.0000018	.0000029

TABLE V

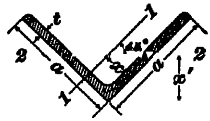
PROPERTIES OF STANDARD ANGLES HAVING EQUAL LEGS



1	2	3	4	5	6	7	8	9	10	11	12
Dimensions Inches	Thickness Inches	Weight per Foot Pounds	Area of Section Square Inches	Distance of Center of Gravity From Back of Flange. Inches	Moment of Inertia, Axis 1-1 Inches	Section Modulus, Axis 1-1 Inches	Radius of Gyration, Axis 1-1 Inches	Distance of Center of Gravity From External Apex. Inches	Least Moment of Inertia, Axis 2-2 Inches	Section Modulus, Axis 2-2 Inches	Least Radius of Gyration, Axis 2-2 Inches
$a \times a$	t		A	x	I	S	r	x'	I'	S'	r'
$\frac{3}{4} \times \frac{3}{4}$	$\frac{1}{8}$	.58	.17	23	.009	.017	.22	.33	004	011	14
$\frac{3}{4} \times \frac{3}{4}$	$\frac{3}{16}$	.84	.25	.25	.012	.024	.22	.36	.005	014	14
1 × 1	$\frac{1}{8}$	.80	.23	30	.022	.031	.30	.42	.009	021	19
1 × 1	$\frac{3}{16}$	1 16	.34	32	.030	.044	.30	.45	.013	028	19
1 × 1	$\frac{1}{4}$	1 49	.44	.34	.037	.056	.29	.48	.016	.034	19
$1\frac{1}{4} \times 1\frac{1}{4}$	$\frac{1}{8}$	1.02	.30	.36	.044	049	38	.51	018	035	24
$1\frac{1}{4} \times 1\frac{1}{4}$	$\frac{3}{16}$	1.47	43	.38	.061	071	.38	.54	025	047	24
$1\frac{1}{4} \times 1\frac{1}{4}$	$\frac{1}{4}$	1 91	56	40	.077	091	.37	.57	.033	057	24
$1\frac{1}{4} \times 1\frac{1}{4}$	$\frac{5}{16}$	2 32	.68	.42	.090	.109	.36	.60	.040	066	.24
$1\frac{1}{2} \times 1\frac{1}{2}$	$\frac{1}{8}$	1.23	.36	.42	08	072	.47	.60	.031	.053	30
$1\frac{1}{2} \times 1\frac{1}{2}$	$\frac{3}{16}$	1 79	.53	.44	.11	.104	.46	.63	.045	072	29
$1\frac{1}{2} \times 1\frac{1}{2}$	$\frac{1}{4}$	2 34	.69	.47	.14	.134	.45	.66	058	088	29
$1\frac{1}{2} \times 1\frac{1}{2}$	$\frac{5}{16}$	2 86	84	.49	.16	.162	.44	.69	070	101	.29
$1\frac{1}{2} \times 1\frac{1}{2}$	$\frac{3}{8}$	3 35	98	.51	.19	188	.44	.72	.082	114	.29
$1\frac{1}{2} \times 1\frac{1}{2}$	$\frac{7}{16}$	3.81	1 12	.53	.21	214	.43	.75	094	.126	29
$1\frac{3}{4} \times 1\frac{3}{4}$	$\frac{3}{16}$	2 11	.62	.51	.18	.14	54	.72	073	.10	34
$1\frac{3}{4} \times 1\frac{3}{4}$	$\frac{1}{4}$	2.76	.81	.53	.23	.19	.53	.75	094	.13	34
$1\frac{3}{4} \times 1\frac{3}{4}$	$\frac{5}{16}$	3 39	1.00	.55	.27	.23	.52	.78	.113	.15	34
$1\frac{3}{4} \times 1\frac{3}{4}$	$\frac{3}{8}$	3 98	1.17	.57	31	.26	51	.81	.133	.16	.34
$1\frac{3}{4} \times 1\frac{3}{4}$	$\frac{7}{16}$	4.56	1 34	.59	35	.30	.51	84	.152	18	.34
$1\frac{3}{4} \times 1\frac{3}{4}$	$\frac{1}{2}$	5 10	1.50	.61	.38	.33	.50	.87	.171	.20	34
2 × 2	$\frac{3}{16}$	2 43	.71	.57	.27	.19	.62	.80	.11	14	39
2 × 2	$\frac{1}{4}$	3 19	.94	.59	.35	.25	.61	.84	14	17	39
2 × 2	$\frac{5}{16}$	3.92	1 15	.61	42	.30	.60	.87	.17	20	.39
2 × 2	$\frac{3}{8}$	4 62	1 36	.64	.48	.35	.59	.90	.20	22	.39
2 × 2	$\frac{7}{16}$	5 30	1 56	.66	.54	.40	.59	.93	23	25	.38
2 × 2	$\frac{1}{2}$	5 95	1.75	.68	.59	.45	.58	.96	.26	27	.38
$2\frac{1}{4} \times 2\frac{1}{4}$	$\frac{3}{16}$	3.1	.90	.69	.55	.30	.78	.98	.22	22	.49
$2\frac{1}{4} \times 2\frac{1}{4}$	$\frac{1}{4}$	4.0	1 19	.72	.70	.39	.77	1 01	.29	28	49
$2\frac{1}{4} \times 2\frac{1}{4}$	$\frac{5}{16}$	5.0	1 46	.74	.80	.48	.76	1 05	35	.33	49
$2\frac{1}{4} \times 2\frac{1}{4}$	$\frac{3}{8}$	5 9	1.73	.76	.98	.57	.75	1.08	.41	.38	.48
$2\frac{1}{4} \times 2\frac{1}{4}$	$\frac{7}{16}$	6.8	2.00	.78	1 11	.65	.75	1.11	46	.42	48
$2\frac{1}{4} \times 2\frac{1}{4}$	$\frac{1}{2}$	7.7	2.25	81	1 23	.72	.74	1.14	.52	46	48
$2\frac{1}{4} \times 2\frac{1}{4}$	$\frac{9}{16}$	8.5	2.50	.83	1.34	.80	.73	1 17	.58	49	48

TABLE V—(Continued)

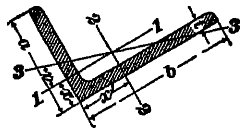
PROPERTIES OF STANDARD ANGLES HAVING EQUAL LEGS



1	2	3	4	5	6	7	8	9	10	11	12
Dimensions Inches	Thickness Inches	Weight per Foot Pounds	Area of Section Square Inches	Distance of Center of Gravity From Back of Flange Inches	Moment of Inertia, Axis 1-1 Inches	Section Modulus, Axis 1-1 Inches	Radius of Gyration, Axis 1-1 Inches	Distance of Center of Gravity From External Apex. Inches	Least Moment of Inertia Axis 2-2 Inches	Section Modulus, Axis 2-2 Inches	Least Radius of Gyration, Axis 2-2 Inches
$a \times a$	t		A	x	I	S	r	x'	I'	S'	r'
3 × 3	$\frac{1}{4}$	4 9	I 44	84	I 24	.58	93	I.19	.50	.42	.59
3 × 3	$\frac{5}{16}$	6 0	I 78	.87	I 51	.71	92	I 22	61	.50	.59
3 × 3	$\frac{3}{8}$	7 2	2.11	89	I 76	.83	91	I 26	.72	.57	.58
3 × 3	$\frac{7}{16}$	8.3	2 43	91	I 99	95	91	I 29	.82	.64	.58
3 × 3	$\frac{1}{2}$	9 4	2.75	93	2 22	I 07	.90	I.32	.92	.70	.58
3 × 3	$\frac{9}{16}$	10 4	3 06	95	2 43	I 19	89	I.35	I 02	.76	.58
3 × 3	$\frac{5}{8}$	11.4	3 36	98	2 62	I 30	88	I.38	I 12	.81	.58
3 × 3	$\frac{11}{16}$	12.4	3 65	I 00	2 81	I.40	88	I.41	I 22	.86	.58
3½ × 3½	$\frac{5}{16}$	7 1	2 09	99	2 45	98	I 08	I 40	.99	.71	.69
3½ × 3½	$\frac{3}{8}$	8 4	2 48	I 01	2 87	I 15	I.07	I 43	I 16	.81	68
3½ × 3½	$\frac{7}{16}$	9 8	2 87	I 04	3 26	I 32	I 07	I 46	I 33	.91	68
3½ × 3½	$\frac{1}{2}$	11 1	3 25	I 06	3 64	I 49	I 06	I 50	I 50	I 00	.68
3½ × 3½	$\frac{9}{16}$	12 3	3 62	I 08	3 99	I 65	I 05	I.53	I 66	I 09	.68
3½ × 3½	$\frac{5}{8}$	13 5	3 98	I 10	4 33	I 81	I 04	I.56	I 82	I 17	.68
3½ × 3½	$\frac{11}{16}$	14 8	4 34	I 12	4 65	I 96	I 04	I 59	I 97	I 24	.67
3½ × 3½	$\frac{3}{4}$	15 9	4 69	I 15	4 96	2 11	I 03	I 62	2 13	I 31	.67
3½ × 3½	$\frac{13}{16}$	17 1	5 03	I 17	5 25	2 25	I 02	I 65	2.28	I 38	.67
3½ × 3½	$\frac{7}{8}$	18 3	5.36	I 19	5 53	2 39	I 02	I.68	2 43	I 45	.67
4 × 4	$\frac{5}{16}$	8.2	2 40	I 12	3 71	I 29	I 24	I 58	I 50	95	.79
4 × 4	$\frac{3}{8}$	9 7	2 86	I 14	4 36	I 52	I 23	I 61	I 77	I 10	.79
4 × 4	$\frac{7}{16}$	11 3	3 31	I 16	4 97	I 75	I 23	I.64	2 02	I 23	78
4 × 4	$\frac{1}{2}$	12 8	3 75	I 18	5 56	I 97	I 22	I 67	2 28	I 36	.78
4 × 4	$\frac{9}{16}$	14 2	4 18	I 21	6 12	2 19	I 21	I 71	2 52	I 48	.78
4 × 4	$\frac{5}{8}$	15 7	4 61	I 23	6 66	2 40	I 20	I.74	2 76	I 59	.77
4 × 4	$\frac{11}{16}$	17 1	5 03	I 25	7 17	2 61	I 19	I.77	3 00	I 70	.77
4 × 4	$\frac{3}{4}$	18 5	5 44	I 27	7 66	2 81	I 19	I 80	3 23	I 80	.77
4 × 4	$\frac{13}{16}$	19 9	5 84	I 29	8 14	3 01	I.18	I 83	3 46	I.89	.77
4 × 4	$\frac{7}{8}$	21 2	6 23	I 31	8 59	3 20	I.17	I 86	3 69	I.99	.77
6 × 6	$\frac{3}{8}$	14 8	4 36	I 64	15 39	3 53	I.88	2.32	6 19	2 67	I 19
6 × 6	$\frac{7}{16}$	17 2	5 06	I 66	17 68	4.07	I.87	2 34	7 13	3 04	I 19
6 × 6	$\frac{1}{2}$	19 6	5 75	I 68	19 91	4 61	I 86	2.38	8.04	3 37	I 18
6 × 6	$\frac{9}{16}$	21 9	6 43	I 71	22 07	5 14	I 85	2.41	8 94	3 70	I 18
6 × 6	$\frac{5}{8}$	24 2	7.11	I 73	24 16	5.66	I 84	2.45	9 81	4 01	I 17
6 × 6	$\frac{11}{16}$	26 4	7 78	I 75	26 19	6.17	I 83	2.48	10 67	4 31	I 17
6 × 6	$\frac{3}{4}$	28 7	8 44	I.78	28 15	6 66	I 83	2.51	11 52	4 59	I 17
6 × 6	$\frac{13}{16}$	30 9	9.09	I.80	30 06	7 15	I 82	2.54	12 35	4 86	I 17
6 × 6	$\frac{7}{8}$	33 1	9 73	I 82	31 92	7 63	I 81	2 57	13 17	5 12	I 16
6 × 6	$\frac{15}{16}$	35 3	10 37	I 84	33 72	8 11	I 80	2 60	13 98	5 37	I.16
6 × 6	I	37 4	11 00	- 86	35 46	8 57	I.80	2.64	14 78	5 61	I.16

TABLE VI

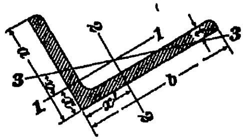
PROPERTIES OF STANDARD ANGLES HAVING UNEQUAL LEGS



1	2	3	4	5	6	7	8	9	10	11	12	13	14
Dimensions Inches	Thickness Inches	Weight per Foot Pounds	Area of Section Square Inches	Distance of Center of Gravity From Back of Longer Flange Inches	Moment Inertia, Axis 1-1 Inches	Section Modulus, Axis 1-1 Inches	Radius of Gyration, Axis 1-1 Inches	Distance of Center of Gravity From Back of Shorter Flange Inches	Moment of Inertia, Axis 2-2 Inches	Section Modulus, Axis 2-2 Inches	Radius of Gyration, Axis 2-2 Inches	Tangent of Angle	Least Radius of Gyration, Axis 2-2 Inches
$b \times a$	t		A	x	I	S	r	x'	I'	S'	r'	a	r''
2½ × 2	3/16	2 7	81	51	29	20	60	.76	51	29	79	632	43
2½ × 2	1/4	3 6	1 06	54	37	.25	59	79	65	38	78	.626	42
2½ × 2	5/16	4 5	1 31	56	45	.31	58	.81	.79	47	78	.620	42
2½ × 2	3/8	5 3	1 55	58	.51	36	58	83	91	55	77	.614	42
2½ × 2	7/16	6 1	1 78	60	58	41	57	85	1 03	62	76	607	.42
2½ × 2	1/2	6 8	2 00	63	64	46	56	88	1 14	70	.75	600	.42
2½ × 2	9/16	7 6	2 21	65	69	51	56	90	1 24	77	75	592	42
3 × 2½	1/4	4 5	1 31	66	.74	40	75	91	1 17	56	95	684	.53
3 × 2½	5/16	5 5	1 62	.68	90	49	74	93	1 42	69	.94	680	.53
3 × 2½	3/8	6 5	1 92	.71	1 04	58	74	.96	1 66	81	93	676	52
3 × 2½	7/16	7 5	2 21	.73	1 18	.66	73	98	1 88	93	92	672	52
3 × 2½	1/2	8 5	2 50	.75	1 30	74	72	1 00	2 08	1 04	91	566	.52
3 × 2½	9/16	9 4	2 78	77	1 42	82	72	1 02	2 28	1 15	91	661	52
3 × 2½	5/8	10 4	3 05	79	1 53	90	71	1 04	2.46	1 26	90	655	52
3½ × 2½	1/4	4 9	1 44	61	78	41	74	1 11	1 80	75	1 12	.506	.54
3½ × 2½	5/16	6 0	1 78	64	94	50	73	1 14	2 19	93	1 11	501	.54
3½ × 2½	3/8	7 2	2 11	66	1 09	59	72	1 16	2 56	1 09	1 10	496	54
3½ × 2½	7/16	8 3	2 43	68	1 23	68	71	1 18	2 91	1 26	1 09	491	.54
3½ × 2½	1/2	9.4	2 75	70	1 36	76	70	1 20	3 24	1 41	1 09	486	.53
3½ × 2½	9/16	10 4	3.06	73	1 49	84	70	1 23	3 55	1 56	1 08	480	.53
3½ × 2½	5/8	11 4	3 36	75	1 61	92	69	1 25	3 85	1 71	1 07	.472	.53
3½ × 2½	11/16	12 4	3 65	77	1 72	99	69	1 27	4 13	1 85	1 06	468	.53
3½ × 2½	3/4	13 4	3 94	.79	1 83	1 07	68	1 29	4 40	1 99	1 06	461	.54
3½ × 3	5/16	6 6	1 93	81	1 58	72	90	1 06	2 33	95	1 10	724	.63
3½ × 3	3/8	7 8	2.30	83	1 85	.85	90	1 08	2 72	1 13	1 09	.721	62
3½ × 3	7/16	9 0	2 65	85	2 09	98	89	1 10	3 10	1 29	1 08	.718	62
3½ × 3	1/2	10 2	3 00	88	2 33	1.10	88	1 13	3.45	1 45	1 07	.714	62
3½ × 3	9/16	11 4	3 34	90	2 55	1 21	87	1 15	3 79	1 61	1 07	.711	62
3½ × 3	5/8	12 5	3 67	92	2 76	1 33	87	1 17	4 11	1 76	1 06	.707	.62
3½ × 3	11/16	13 6	4 00	94	2 96	1 44	86	1 19	4 41	1 91	1 05	.703	62
3½ × 3	3/4	14 7	4 31	96	3 15	1 54	.85	1 21	4 70	2 05	1 04	698	.62
3½ × 3	13/16	15 7	4 62	.98	3 33	1 65	85	1 23	4.98	2 20	1 04	694	.62
3½ × 3	7/8	16 8	4 92	1.00	3 50	1 75	84	1 25	5.24	2 33	1.03	689	63
4 × 3	5/16	7.1	2 09	76	1.65	73	89	1.26	3 38	1 23	1 27	554	65
4 × 3	3/8	8 4	2 48	78	1 92	87	88	1.28	3 96	1 46	1 26	551	64
4 × 3	7/16	9 8	2 87	80	2 18	99	87	1.30	4 52	1 68	1 25	547	64
4 × 3	1/2	11.1	3 25	83	2 42	1 12	86	1.33	5 05	1 89	1 25	543	64
4 × 3	9/16	12 3	3 62	85	2.66	1 23	86	1.35	5 55	2 09	1 24	538	64
4 × 3	5/8	13 5	3 98	87	2 87	1 35	85	1 37	6 03	2,30	1 23	534	64
4 × 3	11/16	14 8	4 34	.89	3 08	1 46	84	1 39	6 49	2 49	1 22	529	64
4 × 3	3/4	15 9	4 69	.92	3 28	1 57	84	1 42	6 93	2 68	1 22	.524	64
4 × 3	13/16	17.1	5 03	.94	3 47	1 68	83	1.44	7 35	2 87	1 21	518	.64
4 × 3	7/8	18.3	5 36	.96	3 66	1 79	.83	1 46	7 75	3 05	1 20	.512	.64

TABLE VI—(Continued)

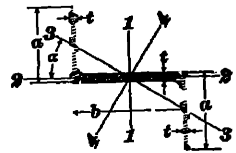
PROPERTIES OF STANDARD ANGLES HAVING UNEQUAL LEGS



1	2	3	4	5	6	7	8	9	10	11	12	13	14
Dimensions Inches	Thickness Inches	Weight per Foot Pounds	Area of Section Square Inches	Distance of Center of Gravity From Back of Longer Flange Inches	Moment of Inertia, Axis 1-1 Inches	Section Modulus, Axis 1-1 Inches	Radius of Gyration, Axis 1-1 Inches	Distance of Center of Gravity From Back of Shorter Flange. Inches	Moment of Inertia, Axis 2-2 Inches	Section Modulus, Axis 2-2 Inches	Radius of Gyration, Axis 2-2 Inches	Tangent of Angle	Least Radius of Gyration, Axis 2-2 Inches
$b \times a$	t		A	x	I	S	r	x'	I'	S'	r'	a	r''
5 × 3	5/16	8 2	2 40	.68	1 75	.75	85	1 68	6 26	1 89	1 61	368	.66
5 × 3	3/8	9 7	2 86	70	2 04	89	84	1 70	7 37	2 24	1 61	364	65
5 × 3	7/16	11 3	3 31	73	2 32	1 02	84	1 73	8 43	2 58	1 60	361	65
5 × 3	1/2	12 8	3 75	75	2 58	1 15	83	1 75	9 45	2 91	1 59	357	65
5 × 3	9/16	14 2	4 18	77	2 83	1.27	82	1 77	10 43	3 23	1 58	353	65
5 × 3	5/8	15 7	4 61	80	3 06	1.39	82	1 80	11 37	3 55	1 57	349	64
5 × 3	11/16	17 1	5 03	82	3 29	1.51	.81	1 82	12 28	3 86	1 56	345	64
5 × 3	3/4	18 5	5 44	84	3 51	1 62	80	1 84	13 15	4 16	1 55	340	64
5 × 3	13/16	19 9	5 84	86	3 71	1 74	.80	1 86	13 98	4 46	1 55	336	64
5 × 3	7/8	21 2	6 23	88	3 91	1 85	.79	1 88	14.78	4 75	1 54	331	64
5 × 3½	5/16	8 7	2 56	.84	2 72	1 02	1 03	1 59	6.60	1 84	1 61	489	77
5 × 3½	3/8	10.4	3 05	.86	3 18	1.21	1.02	1 61	7 78	2 29	1 60	.485	76
5 × 3½	7/16	12 0	3 53	.88	3 63	1 39	1 01	1 63	8 90	2 64	1 59	482	76
5 × 3½	1/2	13 6	4 00	91	4 05	1 56	1 01	1 66	9 99	2 99	1 58	479	75
5 × 3½	9/16	15 2	4 46	93	4 45	1 73	1 00	1 68	11 03	3 32	1 57	476	75
5 × 3½	5/8	16 7	4 92	95	4.83	1 90	99	1 70	12 03	3 65	1 56	472	.75
5 × 3½	11/16	18.3	5 37	97	5 20	2 06	98	1 72	12 99	3 97	1 56	468	75
5 × 3½	3/4	19 8	5 81	1 00	5 55	2 22	98	1 75	13 92	4 28	1 55	464	75
5 × 3½	13/16	21 2	6 25	1 02	5 89	2 37	97	1 77	14 81	4 58	1 54	460	.75
5 × 3½	7/8	22 7	6 67	1 04	6 21	2 52	96	1 79	15 67	4 88	1 53	455	75
5 × 3½	15/16	24 1	7 09	1.06	6 52	2 67	96	1 81	16 49	5 17	1 53	451	.75
6 × 3½	3/8	11.6	3 42	79	3 34	1 23	99	2 04	12 86	3 24	1 94	350	77
6 × 3½	7/16	13 5	3 96	81	3 81	1 41	98	2.06	14 76	3 75	1 93	347	76
6 × 3½	1/2	15 3	4 50	83	4 25	1 59	97	2.08	16 59	4 24	1 92	344	76
6 × 3½	9/16	17 1	5 03	86	4 67	1 77	96	2.11	18.37	4 72	1 91	341	75
6 × 3½	5/8	18 9	5 55	88	5 08	1 94	96	2 13	20 08	5 19	1 90	338	75
6 × 3½	11/16	20 6	6 06	90	5 47	2 11	95	2 15	21.74	5 65	1 89	.334	75
6 × 3½	3/4	22 3	6 56	93	5 84	2 27	94	2 18	23 34	6 10	1 89	331	75
6 × 3½	13/16	24 0	7 06	95	6 20	2 43	94	2.20	24 89	6 55	1 88	.327	75
6 × 3½	7/8	25 7	7 55	97	6 55	2 59	93	2.22	26 39	6 98	1 87	323	.75
6 × 3½	15/16	27 3	8 03	99	6 88	2 74	93	2 24	27 84	7 41	1 86	320	.75
6 × 3½	1	28 9	8 50	1 01	7 21	2 90	92	2.26	29 15	7 80	1 85	.317	.75
6 × 4	3/8	12 3	3 61	94	4 90	1 60	1 17	1 94	13 47	3 32	1 93	446	.88
6 × 4	7/16	14 2	4 18	96	5 60	1 85	1 16	1 96	15 46	3 83	1 92	443	87
6 × 4	1/2	16 2	4 75	99	6 27	2 08	1 15	1.99	17 40	4 33	1 91	440	87
6 × 4	9/16	18 1	5 31	1.01	6 91	2 31	1 14	2 01	19 26	4 83	1 90	438	87
6 × 4	5/8	19 9	5 86	1 03	7 52	2 54	1 13	2 03	21 07	5 31	1 90	434	86
6 × 4	11/16	21 8	6 40	1 06	8 11	2 76	1 13	2.06	22 82	5 78	1 89	431	86
6 × 4	3/4	23 6	6 94	1 08	8 68	2 97	1.12	2 08	24 51	6 25	1 88	428	86
6 × 4	13/16	25 4	7 46	1.10	9 23	3 18	1 11	2 10	26 15	6 70	1 87	425	86
6 × 4	7/8	27 2	7 98	1 12	9 75	3 39	1.11	2 12	27 73	7 15	1 86	421	.86
6 × 4	15/16	28 9	8 50	1 14	10.26	3.59	1 10	2 14	29 26	7 59	1 86	418	.86
6 × 4	1	30 6	9.00	1 17	10 75	3 79	1 09	2 17	30 75	8 02	1 85	414	.86

TABLE VII

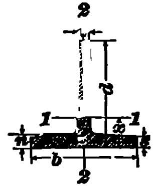
PROPERTIES OF Z BARS



1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
													Coefficient of Strength		Coefficient of Deflection	
Depth of Bar Inches	Length of Legs Inches	Thickness of Web and Legs Inches	Weight per Foot Pounds	Area of Section Square Inches	Moment of Inertia, Axis 1-1 Inches	Section Modulus, Axis 1-1 Inches	Radius of Gyration, Axis 1-1 Inches	Moment of Inertia, Axis 2-2 Inches	Section Modulus, Axis 2-2 Inches	Radius of Gyration, Axis 2-2 Inches	Tangent of Angle α	Least Radius of Gyration, Axis 3-3 Inches	For Fiber Stress of 16,000 Pounds per Square Inch	For Fiber Stress of 12,500 Pounds per Square Inch	Uniform Load	Center Load
b	a	t		A	I	S	r	I'	S'	r'		r''	F	F'	N	N'
3	$2\frac{11}{16}$	$\frac{1}{4}$	6 7	1 97	2 87	1 92	1 21	2 81	1 10	1 19	986	55	20,400	16,000	000270	000432
$3\frac{1}{16}$	$2\frac{3}{4}$	$\frac{5}{16}$	8 4	2.48	3 64	2 38	1 21	3 64	1 40	1 21	1 000	.55	25,400	19,800	.000213	.000341
3	$2\frac{11}{16}$	$\frac{3}{8}$	9 7	2 86	3 85	2 57	1 16	3.92	1.57	1 17	990	.54	27,400	21,400	.000201	.000322
$3\frac{1}{16}$	$2\frac{3}{4}$	$\frac{7}{16}$	11 4	3 36	4 57	2 98	1 17	4 75	1 88	1 19	.975	.55	31,800	24,800	000170	.000272
3	$2\frac{11}{16}$	$\frac{1}{2}$	12 5	3 69	4 59	3 06	1 12	4 85	1 99	1 15	.965	53	32,600	25,500	000169	000271
$3\frac{1}{16}$	$2\frac{3}{4}$	$\frac{9}{16}$	14 2	4 18	5 26	3 43	1 12	5 68	2 30	1.17	.951	.54	36,600	28,600	.000148	.000236

4	3 1/16	1/4	8.2	2.41	6.28	3.14	1.62	4.23	1.44	1.32	.778	.67	33,500	26,200	.000123	.000198
4 1/16	3 1/8	5/16	10.3	3.03	7.94	3.91	1.62	5.46	1.84	1.34	.788	.68	41,700	32,600	.000098	.000156
4 1/8	3 3/16	3/8	12.4	3.66	9.63	4.67	1.62	6.77	2.26	1.36	.798	.69	49,800	38,900	.000081	.000129
4	3 1/16	7/16	13.8	4.05	9.66	4.83	1.54	6.73	2.37	1.29	.794	.66	51,500	40,200	.000080	.000129
4 1/16	3 1/8	1/2	15.8	4.66	11.18	5.50	1.55	7.96	2.77	1.31	.804	.67	58,700	45,900	.000069	.000111
4 1/8	3 3/16	9/16	17.9	5.27	12.74	6.18	1.55	9.26	3.19	1.32	.814	.68	65,900	51,500	.000061	.000097
4	3 1/16	5/8	18.9	5.55	12.11	6.05	1.48	8.73	3.18	1.25	.808	.65	64,600	50,500	.000064	.000103
4 1/16	3 1/8	11/16	20.9	6.14	13.52	6.65	1.48	9.95	3.58	1.27	.818	.67	71,000	55,500	.000057	.000092
4 1/8	3 3/16	3/4	23.0	6.75	14.97	7.26	1.49	11.24	4.00	1.29	.828	.68	77,400	60,500	.000052	.000083
5	3 1/4	5/16	11.6	3.40	13.36	5.34	1.98	6.18	2.00	1.35	.611	.75	57,000	44,500	.000058	.000093
5 1/16	3 5/16	3/8	13.9	4.10	16.18	6.39	1.99	7.65	2.45	1.37	.619	.76	68,200	53,300	.000048	.000077
5 1/8	3 3/8	7/16	16.4	4.81	19.07	7.44	1.99	9.20	2.92	1.38	.628	.76	79,400	62,000	.000041	.000065
5	3 1/4	1/2	17.9	5.25	19.19	7.68	1.91	9.05	3.02	1.31	.616	.74	81,900	64,000	.000040	.000065
5 1/16	3 5/16	9/16	20.2	5.94	21.83	8.62	1.92	10.51	3.47	1.33	.623	.75	92,000	71,900	.000036	.000057
5 1/8	3 3/8	5/8	22.6	6.64	24.53	9.57	1.92	12.06	3.94	1.35	.631	.76	102,100	79,800	.000032	.000051
5	3 1/4	11/16	23.7	6.96	23.68	9.47	1.84	11.37	3.91	1.28	.619	.73	101,000	78,900	.000033	.000052
5 1/16	3 5/16	3/4	26.0	7.64	26.16	10.34	1.85	12.83	4.37	1.30	.626	.74	110,200	86,100	.000030	.000047
5 1/8	3 3/8	13/16	28.3	8.33	28.70	11.20	1.86	14.37	4.84	1.31	.633	.76	119,500	93,300	.000027	.000043
6	3 1/2	3/8	15.6	4.59	25.32	8.44	2.35	9.11	2.75	1.41	.519	.83	90,000	70,300	.000031	.000049
6 1/16	3 9/16	7/16	18.3	5.39	29.80	9.83	2.35	10.94	3.27	1.43	.526	.83	104,900	81,900	.000026	.000042
6 1/8	3 5/8	1/2	21.0	6.19	34.36	11.22	2.36	12.87	3.81	1.44	.532	.84	119,700	93,500	.000023	.000036
6	3 1/2	9/16	22.7	6.68	34.64	11.55	2.28	12.59	3.91	1.37	.520	.81	123,200	96,200	.000022	.000036
6 1/16	3 9/16	5/8	25.4	7.46	38.87	12.82	2.28	14.41	4.44	1.39	.526	.82	136,800	106,800	.000020	.000032
6 1/8	3 5/8	11/16	28.1	8.25	43.18	14.10	2.29	16.34	4.98	1.41	.532	.84	150,400	117,500	.000018	.000029
6	3 1/2	3/4	29.3	8.63	42.12	14.04	2.21	15.44	4.94	1.34	.519	.81	149,800	117,000	.000018	.000029
6 1/16	3 9/16	13/16	31.9	9.39	46.13	15.22	2.22	17.27	5.47	1.36	.525	.82	162,300	126,800	.000017	.000027
6 1/8	3 5/8	7/8	34.6	10.17	50.22	16.40	2.22	19.18	6.02	1.37	.530	.83	174,900	136,700	.000015	.000025
7 1/2	3	3/8	16.3	4.78	38.19	10.18	2.83	5.59	1.99	1.08	.29	.72	108,600	84,800	.000020	.000033
8	3	3/8	16.9	4.97	44.64	11.16	3.00	5.60	1.99	1.06	.27	.72	119,000	93,000	.000017	.000028
10	3	3/8	19.4	5.72	76.87	15.37	3.67	5.60	1.99	.99	.19	.71	163,900	128,100	.000010	.000016

TABLE VIII—PROPERTIES OF T BARS



EQUAL LEGS

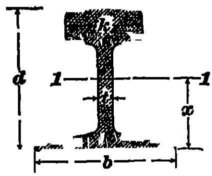
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Dimensions													Coefficient of Strength	
Width of Flange Inches	Depth of Bar Inches	Thickness of Flange Inches	Thickness of Stem Inches	Weight per Foot Pounds	Area of Section Square Inches	Distance of Center of Gravity From Outside of Flange Inches	Moment of Inertia, Axis 1-1 Inches	Section Modulus, Axis 1-1 Inches	Radius of Gyration, Axis 1-1 Inches	Moment of Inertia, Axis 2-2 Inches	Section Modulus, Axis 2-2 Inches	Radius of Gyration, Axis 2-2 Inches	For Fiber Stress of 16,000 Pounds per Square Inch	For Fiber Stress of 12,500 Pounds per Square Inch
b	d	s to n'	t to t'		A	x	I	S	r	I'	S'	r'	F	F'
1	1	1/8 to 5/32	1/8 to 5/32	89	26	29	02	03	30	01	02	21	350	270
1 1/8	1 1/8	3/16 to 7/32	5/32 to 7/32	1 39	.41	.33	04	.05	32	02	04	25	560	440
1 3/16	1 3/16	3/16 to 1/4	5/32 to 7/32	1 53	45	34	05	06	33	03	05	26	630	490
1 1/4	1 1/4	3/16 to 1/4	5/32 to 1/4	1 61	47	36	06	07	35	03	05	27	700	550
1 3/8	1 3/8	3/16 to 1/4	5/32 to 1/4	1 85	.54	39	08	08	.39	05	07	.29	890	690
2	2	1/4 to 5/16	1/4 to 5/16	3 7	1.05	.59	37	26	59	18	18	42	2,770	2,160
2	2	5/16 to 3/8	5/16 to 3/8	4 3	1 26	61	.43	31	59	23	.23	42	3,330	2,600
2 1/4	2 1/4	1/4 to 5/16	1/4 to 5/16	4 1	1 19	68	51	32	65	.24	21	45	3,440	2,690
2 1/4	2 1/4	5/16 to 3/8	5/16 to 3/8	4.9	1 45	67	64	40	66	32	29	47	4,290	3,360
2 1/2	2 1/2	5/16 to 3/8	5/16 to 3/8	5.5	1 60	.73	87	.49	74	44	35	52	5,250	4,100
3	3	5/16 to 3/8	5/16 to 3/8	6 6	1 95	86	1 58	74	90	75	50	62	7,860	6,140
3	3	3/8 to 7/16	3/8 to 7/16	7 8	2 27	88	1 82	.86	.90	92	.61	.64	9,180	7,180
3 1/2	3 1/2	3/8 to 7/16	3/8 to 7/16	9 3	2 74	.99	3 10	1 23	1 08	1.42	81	.73	13,140	10,260

UNEQUAL LEGS

1 1/4	1 1/16	3/16 to 1/4	5/32 to 7/32	1.49	.44	29	04	05	.29	03	01	28	500	390
2 1/2	1 1/4	3/16 to 9/32	3/16 to 5/16	2 9	84	30	08	09	31	28	22	58	930	730
3	2 1/2	3/8 to 7/16	3/8 to 7/16	7 2	2.07	.71	1 08	60	64	.90	60	.66	6,400	5,000
3 1/2	4	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

TABLE IX

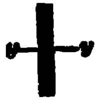
PROPERTIES AND PRINCIPAL DIMENSIONS OF STANDARD T RAILS



1	2	3	4	5	6	7	8	9
							Axis 1-1	
Weight per Yard Pounds	Area Square Inches	b Inches	d Inches	k Inches	t Inches	x Inches	Moment of Inertia I	Section Modulus S
8	.78	$1\frac{1}{2}$	$1\frac{1}{2}$	$\frac{13}{16}$	$\frac{5}{32}$	.75	.23	.31
12	1.18	$1\frac{7}{8}$	$1\frac{7}{8}$	$1\frac{1}{16}$	$\frac{3}{16}$	.92	.55	.58
16	1.57	$2\frac{1}{4}$	$2\frac{1}{4}$	$1\frac{3}{16}$	$\frac{7}{32}$	1.10	1.13	.99
20	2.00	$2\frac{3}{8}$	$2\frac{3}{8}$	$1\frac{3}{8}$	$\frac{1}{4}$	1.2	1.5	1.2
25	2.5	$2\frac{3}{4}$	$2\frac{3}{4}$	$1\frac{1}{2}$	$\frac{19}{64}$	1.4	2.4	1.7
30	2.9	$3\frac{5}{16}$	3	$1\frac{11}{16}$	$\frac{11}{32}$	1.5	3.7	2.4
35	3.4	$3\frac{3}{8}$	$3\frac{1}{16}$	$1\frac{27}{32}$	$\frac{13}{32}$	1.5	4.3	2.8
40	3.9	$3\frac{1}{2}$	$3\frac{1}{2}$	$1\frac{27}{32}$	$\frac{23}{64}$	1.7	6.0	3.4
45	4.4	$3\frac{3}{4}$	$3\frac{3}{4}$	$1\frac{59}{64}$	$\frac{3}{8}$	1.8	7.6	3.9
50	4.9	$3\frac{7}{8}$	$3\frac{7}{8}$	$2\frac{1}{8}$	$\frac{7}{16}$	1.9	10.1	5.1
55	5.4	$4\frac{1}{16}$	$4\frac{1}{16}$	$2\frac{1}{4}$	$\frac{15}{32}$	2.0	12.2	5.9
60	5.9	$4\frac{1}{4}$	$4\frac{1}{4}$	$2\frac{3}{8}$	$\frac{31}{64}$	2.1	14.7	6.7
65	6.4	$4\frac{7}{16}$	$4\frac{7}{16}$	$2\frac{13}{32}$	$\frac{1}{2}$	2.1	17.0	7.4
70	6.9	$4\frac{5}{8}$	$4\frac{5}{8}$	$2\frac{7}{16}$	$\frac{33}{64}$	2.2	20.0	8.4
75	7.4	$4\frac{13}{16}$	$4\frac{13}{16}$	$2\frac{15}{32}$	$\frac{17}{32}$	2.3	23.0	9.1
80	7.8	5	5	$2\frac{1}{2}$	$\frac{35}{64}$	2.4	26.7	10.1
85	8.3	$5\frac{3}{16}$	$5\frac{3}{16}$	$2\frac{9}{16}$	$\frac{9}{16}$	2.5	30.5	11.2
90	8.8	$5\frac{3}{8}$	$5\frac{3}{8}$	$2\frac{5}{8}$	$\frac{9}{16}$	2.6	35.2	12.6
100	9.8	$5\frac{3}{4}$	$5\frac{3}{4}$	$2\frac{3}{4}$	$\frac{9}{16}$	2.8	44.4	15.0
150	14.7	6	6	$4\frac{1}{4}$	1	3.0	69.3	22.9

TABLE X

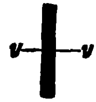
MOMENT OF INERTIA OF RECTANGULAR SECTIONS



Depth in Inches	Width of Rectangle in Inches						
	1/4	5/16	3/8	7/16	1/2	9/16	5/8
2	.17	.21	.25	.29	.33	.38	.42
3	.56	.70	.84	.98	1.13	1.27	1.41
4	1.33	1.67	2 00	2.33	2.67	3 00	3.33
5	2.60	3.26	3 91	4.56	5.21	5 86	6.51
6	4 50	5.63	6 75	7 88	9.00	10 13	11 25
7	7.15	8.93	10 72	12.51	14.29	16 08	17 86
8	10 67	13.33	16 00	18.67	21.33	24 00	26 67
9	15.19	18.98	22 78	26 58	30.38	34.17	37.97
10	20.83	26.04	31 25	36 46	41.67	46.87	52.08
11	27.73	34.66	41.59	48 53	55.46	62 39	69.32
12	36 00	45.00	54.00	63 00	72.00	81 00	90 00
13	45.77	57.21	68.66	80 10	91.54	102 98	114 43
14	57.17	71 46	85 75	100 04	114.33	128 63	142 92
15	70 31	87.89	105.47	123 05	140.63	158 20	175 78
16	85 33	106 67	128 00	149 33	170.67	192 00	213 33
17	102 35	127 94	153 53	179.12	204 71	230.30	255 89
18	121.50	151 88	182.25	212.63	243.00	273 38	303 75
19	142.90	178 62	214.34	250.07	285.79	321 52	357 24
20	166.67	208 33	250.00	291.67	333 33	375 00	416 67
21	192 94	241.17	289.41	337.64	385 88	434 11	482 34
22	221 83	277.29	332 75	388.21	443 67	499 13	554 58
23	253 48	316.85	380 22	443.59	506 96	570 33	633.70
24	288.00	360 00	432.00	504 00	576 00	648.00	720 00
25	325 52	406 90	488 28	569.66	651 04	732.42	813 80
26	366 17	457 71	549.25	640.79	732 33	823 88	915 42
27	410.06	512 58	615 09	717.61	820 13	922 64	1,025 16
28	457.33	571 67	686 00	800 33	914 67	1,029 00	1,143 33
29	508 10	635.13	762 16	889.18	1,016.21	1,143 23	1,270.26
30	562 50	703.13	843 75	984.38	1,125.00	1,265 63	1,406 25
32	682 67	853.33	1,024.00	1,194.67	1,365.33	1,536 00	1,706 67
34	818.83	1,023.54	1,228.25	1,432 96	1,637.67	1,842 38	2.047 08
36	972.00	1,215 00	1,458 00	1,701.00	1,944 00	2,187 00	2,430 00
38	1,143 17	1,428 96	1,714 75	2,000.54	2,286.33	2,572.13	2,857 92
40	1,333 33	1,666 67	2,000.00	2,333.33	2,666.67	3,000 00	3,333.33
42	1,543.50	1,929 38	2,315 25	2,701.13	3,087 00	3,472 88	3,858.75
44	1,774.67	2,218.33	2,662 00	3,105 67	3,549 33	3,993 00	4,436.67
46	2,027 83	2,534.79	3,041 75	3,548 71	4,055 67	4,562.63	5,069 58
48	2,304 00	2,880 00	3,456 00	4,032.00	4,608 00	5,184.00	5,760 00
50	2,604 17	3,255 21	3,906 25	4,557.29	5,208 33	5,859.38	6,510 42
52	2,929.33	3,661.67	4,394.00	5,126 33	5,858 67	6,591.00	7,323 33
54	3,280.50	4,100 63	4,920.75	5,740 88	6,561 00	7,381.13	8,201 25
56	3,658 67	4,573 33	5,488.00	6,402 67	7,317 33	8,232.00	9,146 67
58	4,064.83	5,081.04	6,097 25	7,113.46	8,129 67	9,145.87	10,162.08
60	4,500.00	5,625.00	6,750.00	7,875.00	9,000 00	10,125.00	11,250.00

TABLE X—(Continued)

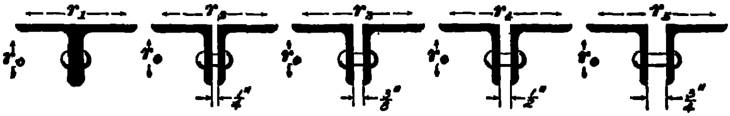
MOMENT OF INERTIA OF RECTANGULAR SECTIONS



Depth in Inches	Width of Rectangle in Inches					
	$\frac{11}{16}$	$\frac{3}{4}$	$\frac{13}{16}$	$\frac{7}{8}$	$\frac{15}{16}$	1
2	.46	.50	.54	.58	.63	67
3	1.55	1 69	1.83	1 97	2 11	2 25
4	3.67	4 00	4.33	4 67	5 00	5 33
5	7.16	7 81	8.46	9 11	9.77	10 42
6	12.38	13 50	14.63	15 75	16.88	18 00
7	19.65	21.44	23.22	25 01	26.80	28.58
8	29.33	32.00	34.67	37 33	40 00	42.67
9	41.77	45.56	49.36	53 16	56.95	60.75
10	57.29	62.50	67.71	72 92	78 13	83.33
11	76.26	83 19	90.12	97 05	103 98	110 92
12	99.00	108 00	117 00	126 00	135 00	144 00
13	125.87	137 31	148.75	160 20	171 64	183 08
14	157.21	171 50	185 79	200 08	214 38	228 67
15	193 36	210 94	228 52	246 09	263 67	281 25
16	234 67	256 00	277 33	298 67	320 00	341 33
17	281.47	307 06	332 65	358 24	383 83	409 42
18	334 13	364 50	394 88	425 25	455 63	486 00
19	392.96	428 69	464 41	500 14	535 86	571 58
20	458.33	500 00	541 67	583 33	625 00	666 67
21	530 58	578 81	627 05	675 28	723 52	771.75
22	610 04	665.50	720 96	776 42	831 87	887.33
23	·697 07	760 44	823 81	887 18	950 55	1,013.92
24	792 00	864.00	936 00	1,008 00	1,080 00	1,152 00
25	895 18	976 56	1,057 94	1,139 32	1,220 70	1,302 08
26	1,006 96	1,098 50	1,190 04	1,281 58	1,373 13	1,464 67
27	1,127.67	1,230 19	1,332 70	1,435 22	1,537.73	1,640 25
28	1,257 67	1,372.00	1,486 33	1,600 67	1,715 00	1,829 33
29	1,397 29	1,524.31	1,651 34	1,778 36	1,905 39	2,032 42
30	1,546 88	1,687.50	1,828 13	1,968 75	2,109 38	2,250 00
32	1,877 33	2,048.00	2,218 67	2,389 33	2,560 00	2,730 67
34	2,251.79	2,456.50	2,661.21	2,865 92	3,070 63	3,275.33
36	2,673 00	2,916.00	3,159 00	3,402 00	3,645 00	3,888.00
38	3,143 71	3,429 50	3,715 29	4,001 08	4,286 88	4,572.67
40	3,666.67	4,000.00	4,333 33	4,666.67	5,000 00	5,333.33
42	4,244.63	4,630.50	5,016 38	5,402.25	5,788 13	6,174.00
44	4,880 33	5,324.00	5,767 67	6,211 33	6,655.00	7,098.67
46	5,576 54	6,083.50	6,590 46	7,097 42	7,604 38	8,111 33
48	6,336 00	6,912.00	7,488 00	8,064 00	8,640 00	9,216 00
50	7,161 46	7,812.50	8,463 54	9,114.58	9,765 63	10,416 67
52	8,055 67	8,788.00	9,520 33	10,252 67	10,985 00	11,717.33
54	9,021.38	9,841.50	10,661 63	11,481 75	12,301 88	13,122.00
56	10,061 33	10,976 00	11,890 67	12,805 33	13,720 00	14,634 67
58	11,178 29	12,194.50	13,210 71	14 226 92	15,243 12	16,259.33
60	12,375 00	13,500.00	14,625 00	15,750.00	16,875.00	18,000.00

TABLE XI

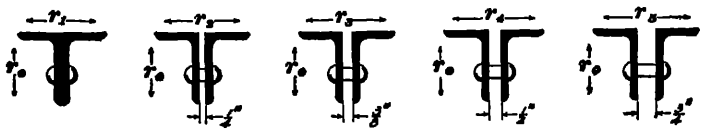
RADII OF GYRATION FOR TWO ANGLES, HAVING EQUAL LEGS, PLACED BACK TO BACK



Dimensions Inches	Thickness Inches	Area of Two Angles Square Inches	Radii of Gyration					
			r_0	r_1	r_2	r_3	r_4	r_5
¾ × ¾	⅛	.34	.22	.32	.42	.48	.53	.65
¾ × ¾	3/16	.49	.22	.33	.44	.49	.55	.67
1 × 1	⅛	.47	.30	.42	.52	.57	.62	.74
1 × 1	¼	.88	.29	.45	.55	.60	.66	.77
1¼ × 1¼	⅛	.60	.38	.52	.62	.67	.72	.83
1¼ × 1¼	5/16	1.37	.36	.56	.66	.71	.77	.88
1½ × 1½	3/16	1.05	.46	.64	.73	.78	.83	.94
1½ × 1½	⅜	1.97	.44	.67	.77	.82	.88	.99
1¾ × 1¾	3/16	1.24	.54	.74	.83	.88	.93	1.03
1¾ × 1¾	7/16	2.68	.51	.78	.88	.93	.98	1.09
2 × 2	3/16	1.43	.62	.84	.93	.98	1.03	1.13
2 × 2	5/16	2.30	.60	.86	.95	1.00	1.05	1.16
2 × 2	7/16	3.12	.59	.88	.98	1.03	1.08	1.19
2¼ × 2¼	3/16	1.62	.70	.94	1.03	1.08	1.12	1.22
2¼ × 2¼	⅜	3.09	.67	.97	1.06	1.11	1.16	1.27
2½ × 2½	¼	2.38	.77	1.05	1.14	1.19	1.24	1.34
2½ × 2½	⅜	3.47	.75	1.07	1.16	1.21	1.26	1.36
2½ × 2½	½	4.50	.74	1.09	1.19	1.24	1.29	1.39
2¾ × 2¾	3/16	1.99	.86	1.14	1.23	1.28	1.32	1.42
2¾ × 2¾	5/16	3.24	.84	1.16	1.25	1.30	1.35	1.45
2¾ × 2¾	7/16	4.43	.83	1.18	1.28	1.32	1.37	1.47
3 × 3	¼	2.88	.93	1.26	1.34	1.39	1.43	1.53
3 × 3	7/16	4.87	.91	1.28	1.37	1.42	1.47	1.57
3 × 3	⅝	6.72	.88	1.32	1.41	1.46	1.51	1.61
3½ × 3½	⅜	4.97	1.07	1.48	1.56	1.61	1.66	1.75
3½ × 3½	⅝	7.97	1.04	1.52	1.61	1.66	1.71	1.81
3½ × 3½	13/16	10.05	1.02	1.55	1.65	1.70	1.75	1.85
4 × 4	5/16	4.80	1.24	1.67	1.76	1.80	1.85	1.94
4 × 4	9/16	8.37	1.21	1.71	1.80	1.85	1.89	1.99
4 × 4	13/16	11.68	1.18	1.75	1.85	1.89	1.94	2.04
4½ × 4½	5/16	5.43	1.40	1.87	1.96	2.00	2.05	2.14
4½ × 4½	½	8.50	1.38	1.90	1.99	2.04	2.08	2.18
4½ × 4½	⅝	10.47	1.36	1.92	2.01	2.06	2.10	2.20
5 × 5	⅜	7.22	1.56	2.09	2.17	2.22	2.26	2.35
5 × 5	½	9.50	1.54	2.10	2.19	2.24	2.28	2.38
5 × 5	⅝	11.72	1.52	2.12	2.21	2.26	2.30	2.40
6 × 6	7/16	10.12	1.87	2.50	2.58	2.63	2.67	2.76
6 × 6	⅝	14.22	1.84	2.53	2.62	2.66	2.71	2.80
6 × 6	⅞	19.47	1.81	2.57	2.66	2.70	2.75	2.85

TABLE XII

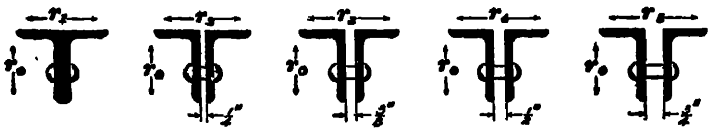
RADII OF GYRATION FOR TWO ANGLES, HAVING UNEQUAL LEGS, PLACED BACK TO BACK



Dimensions Inches	Thickness Inches	Area of Two Angles Square Inches	Radii of Gyration					
			r_0	r_1	r_2	r_3	r_4	r_5
2 × 1 3/8	3/16	1.20	.63	54	.62	.67	72	.83
2 × 1 3/8	3/8	2 25	.61	.56	.66	.71	.76	.88
2 × 1 1/2	3/16	1.24	.63	59	.68	.73	.78	88
2 × 1 1/2	3/8	2 34	.61	62	.72	.77	.82	.93
2 1/2 × 1 1/4	3/16	1 34	.80	44	.52	.58	.63	.74
2 1/2 × 1 1/4	3/8	2 53	.78	47	.57	.62	.68	.79
2 1/2 × 1 1/2	3/16	1 43	.80	.55	.64	.69	.74	.84
2 1/2 × 1 1/2	3/8	2.72	.78	58	.68	.73	.78	.89
2 1/2 × 1 3/4	3/16	1.52	.80	.67	.75	.80	85	.95
2 1/2 × 1 3/4	1/4	2 00	.79	.68	77	.81	.86	.97
2 1/2 × 2	3/16	1 62	.79	.79	.88	92	.97	1 07
2 1/2 × 2	3/8	3.09	.77	.82	.91	.96	1.01	1.12
2 1/2 × 2	1/2	4 00	.75	.84	.94	.99	1.04	1.15
2 3/4 × 1 1/2	3/16	1 52	.89	.53	.62	.67	.72	.82
2 3/4 × 1 1/2	5/16	2 46	.87	.55	.65	.70	.75	.86
2 3/4 × 1 1/2	7/16	3 34	.85	.58	.68	.73	.78	89
3 × 2	3/16	1 80	.97	.75	.83	.88	.93	1 03
3 × 2	5/16	2 93	.95	.76	.85	.90	95	1 05
3 × 2	7/16	3 99	.93	.79	.88	.93	98	1 09
3 × 2 1/2	1/4	2.63	.95	1.00	1 09	1 13	1 18	1 28
3 × 2 1/2	3/8	3.84	.93	1 02	1 11	1 16	1 21	1.31
3 × 2 1/2	9/16	5.55	.91	1 05	1 15	1 20	1.25	1.35
3 1/2 × 2 1/2	1/4	2.88	1.12	96	1 04	1 09	1.13	1 23
3 1/2 × 2 1/2	1/2	5.50	1 09	1 00	1 09	1.14	1 19	1 29
3 1/2 × 2 1/2	11/16	7.30	1 06	1 03	1.13	1.18	1.23	1 33
3 1/2 × 3	5/16	3.87	1.10	1 21	1 30	1 35	1.39	1.49
3 1/2 × 3	9/16	6 68	1.07	1 25	1.34	1 39	1 44	1.54
3 1/2 × 3	13/16	9 24	1.04	1 30	1.40	1 45	1 50	1.60

TABLE XII—(Continued)

RADII OF GYRATION FOR TWO ANGLES, HAVING UNEQUAL LEGS, PLACED BACK TO BACK



Dimensions Inches	Thickness Inches	Area of Two Angles Square Inches	Radii of Gyration r_0	r_1	r_2	r_3	r_4	r_5
4 × 3	5/16	4.18	1.27	1.17	1.25	1.30	1.34	1.44
4 × 3	9/16	7.24	1.24	1.21	1.30	1.34	1.39	1.49
4 × 3	13/16	10.05	1.21	1.25	1.35	1.40	1.45	1.55
4 × 3½	5/16	4.49	1.26	1.42	1.50	1.55	1.59	1.69
4 × 3½	1/2	7.00	1.23	1.44	1.53	1.58	1.63	1.72
4 × 3½	5/8	8.59	1.22	1.46	1.55	1.60	1.65	1.75
4½ × 3	3/8	5.34	1.44	1.14	1.22	1.27	1.31	1.41
4½ × 3	1/2	7.00	1.42	1.15	1.24	1.29	1.34	1.44
4½ × 3	5/8	8.59	1.40	1.18	1.27	1.31	1.36	1.46
5 × 3	5/16	4.80	1.61	1.09	1.17	1.22	1.26	1.36
5 × 3	9/16	8.37	1.58	1.13	1.22	1.26	1.31	1.41
5 × 3	13/16	11.68	1.55	1.17	1.27	1.32	1.37	1.47
5 × 3½	3/8	6.09	1.60	1.34	1.42	1.46	1.51	1.60
5 × 3½	5/8	9.84	1.56	1.37	1.46	1.51	1.56	1.66
5 × 3½	7/8	13.34	1.53	1.42	1.51	1.56	1.61	1.71
5 × 4	3/8	6.47	1.59	1.58	1.66	1.71	1.75	1.85
5 × 4	1/2	8.50	1.57	1.60	1.68	1.73	1.78	1.87
5 × 4	5/8	10.47	1.55	1.62	1.71	1.75	1.80	1.90
6 × 3½	3/8	6.84	1.94	1.26	1.34	1.39	1.43	1.53
6 × 3½	5/8	11.09	1.90	1.30	1.39	1.43	1.48	1.58
6 × 3½	7/8	15.09	1.87	1.34	1.44	1.49	1.53	1.64
6 × 4	3/8	7.22	1.93	1.50	1.58	1.62	1.67	1.76
6 × 4	5/8	11.72	1.90	1.53	1.62	1.67	1.71	1.81
6 × 4	7/8	15.97	1.86	1.58	1.67	1.71	1.76	1.86
7 × 3½	7/16	8.80	2.26	1.16	1.29	1.33	1.38	1.47
7 × 3½	1/2	10.00	2.25	1.22	1.30	1.35	1.39	1.48
7 × 3½	5/8	12.34	2.24	1.24	1.32	1.37	1.42	1.51
7 × 3½	13/16	15.74	2.21	1.27	1.36	1.41	1.46	1.56
7 × 3½	1	19.00	2.19	1.31	1.40	1.45	1.50	1.60

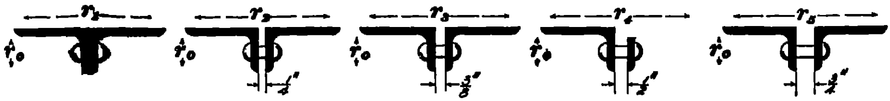
TABLE XIII

RADII OF GYRATION FOR TWO ANGLES, HAVING UNEQUAL LEGS, PLACED BACK TO BACK

Dimensions Inches	Thickness Inches	Area of Two Angles Square Inches	Radii of Gyration r_0	r_1	r_2	r_3	r_4	r_5
2 × 1⅜	$\frac{3}{16}$	1.20	.41	.92	1.01	1.06	1.11	1.22
2 × 1⅜	⅜	2.25	.38	.95	1.05	1.10	1.15	1.26
2 × 1½	$\frac{3}{16}$	1.24	.44	.90	.99	1.05	1.09	1.20
2 × 1½	⅜	2.34	.42	.93	1.09	1.14	1.19	1.29
2¼ × 1¼	$\frac{3}{16}$	1.34	.33	1.21	1.31	1.36	1.41	1.51
2¼ × 1¼	⅜	2.53	.32	1.25	1.35	1.40	1.45	1.56
2½ × 1½	$\frac{3}{16}$	1.43	.42	1.17	1.26	1.31	1.36	1.47
2½ × 1½	⅜	2.72	.40	1.20	1.30	1.35	1.40	1.51
2½ × 1¾	$\frac{3}{16}$	1.52	.51	1.13	1.23	1.27	1.32	1.43
2½ × 1¾	¼	2.00	.50	1.14	1.24	1.29	1.34	1.44
2½ × 2	$\frac{3}{16}$	1.62	.60	1.10	1.19	1.24	1.29	1.39
2½ × 2	⅜	3.09	.58	1.13	1.23	1.28	1.33	1.43
2½ × 2	½	4.00	.56	1.15	1.25	1.30	1.35	1.46
2¾ × 1½	$\frac{3}{16}$	1.52	.41	1.31	1.40	1.45	1.50	1.60
2¾ × 1½	$\frac{5}{16}$	2.46	.40	1.33	1.43	1.48	1.53	1.63
2¾ × 1½	$\frac{7}{16}$	3.34	.39	1.36	1.45	1.51	1.56	1.66
3 × 2	$\frac{3}{16}$	1.80	.58	1.37	1.46	1.51	1.56	1.66
3 × 2	$\frac{5}{16}$	2.93	.57	1.39	1.48	1.53	1.58	1.68
3 × 2	$\frac{7}{16}$	3.99	.55	1.41	1.51	1.56	1.61	1.71
3 × 2½	¼	2.63	.75	1.31	1.40	1.45	1.50	1.60
3 × 2½	⅜	3.84	.74	1.33	1.42	1.47	1.52	1.63
3 × 2½	$\frac{9}{16}$	5.55	.72	1.37	1.46	1.51	1.56	1.66
3½ × 2½	¼	2.88	.74	1.58	1.67	1.72	1.76	1.86
3½ × 2½	½	5.50	.70	1.62	1.72	1.77	1.81	1.92
3½ × 2½	$\frac{11}{16}$	7.30	.69	1.66	1.75	1.80	1.86	1.96
3½ × 3	$\frac{5}{16}$	3.87	.90	1.52	1.61	1.66	1.71	1.80
3½ × 3	$\frac{9}{16}$	6.68	.87	1.57	1.66	1.71	1.76	1.86
3½ × 3	$\frac{13}{16}$	9.24	.85	1.61	1.71	1.76	1.81	1.91

TABLE XIII—(Continued)

RADII OF GYRATION FOR TWO ANGLES, HAVING UNEQUAL LEGS, PLACED BACK TO BACK



Dimensions Inches	Thickness Inches	Area of Two Angles Square Inches	Radii of Gyration					
			r_0	r_1	r_2	r_3	r_4	r_5
4×3	5/16	4 18	.89	1 79	1.88	1 93	1.97	2.07
4×3	9/16	7.24	.86	1 83	1.93	1 97	2 02	2 12
4×3	13/16	10.05	83	1.88	1.97	2 02	2.08	2 18
4×3½	5/16	4.49	1 07	1 73	1.81	1.86	1 91	2.00
4×3½	½	7.00	1 04	1 76	1 85	1.89	1.94	2 04
4×3½	⅝	8 59	1 02	1.78	1.87	1.92	1.97	2.07
4½×3	⅜	5 34	86	2.07	2.16	2.21	2 26	2 35
4½×3	½	7.00	85	2.09	2 18	2 23	2 28	2 38
4½×3	⅝	8.59	83	2 11	2.21	2.26	2 31	2 41
5×3	5/16	4 80	85	2 33	2 42	2.47	2 52	2 61
5×3	9/16	8 37	82	2 37	2 47	2.52	2 57	2 67
5×3	13/16	11.68	80	2 42	2 52	2.57	2 62	2.72
5×3½	⅜	6.09	1 02	2.27	2.36	2.41	2.45	2 55
5×3½	⅝	9 84	99	2.31	2.40	2.45	2 50	2 60
×3½	⅞	13 34	96	2.36	2 45	2.50	2 55	2 65
5×4	⅜	6 47	1 20	2 20	2 29	2.34	2.38	2 48
5×4	½	8 50	1 18	2 22	2.31	2 36	2 41	2 50
5×4	⅝	10 47	1 17	2 24	2.33	2 38	2 43	2 53
6×3½	⅜	6.84	99	2 81	2 90	2.95	3 00	3 09
6×3½	⅝	11.09	.96	2 86	2 95	3 00	3.05	3.15
6×3½	⅞	15.09	93	2 90	3 00	3 05	3 10	3.20
6×4	⅜	7 22	1.17	2.74	2.83	2 87	2 92	3 02
6×4	⅝	11 72	1.13	2.78	2.87	2.92	2.97	3 06
6×4	⅞	15.97	1.11	2.82	2 92	2 97	3 02	3 12
7×3½	7/16	8 80	.95	3 37	3 47	3 52	3 56	3 66
7×3½	½	10.00	.94	3 39	3 48	3 53	3 58	3.67
7×3½	⅝	12 34	.93	3 40	3 50	3.55	3.60	3 70
7×3½	13/16	15 64	.91	3 45	3.54	3.59	3 64	3 74
7×3½	1	19.00	.89	3 48	3.58	3 63	3 68	3 78

MATERIALS OF STRUCTURAL ENGINEERING

(PART 1)

FOUNDATION SOILS

EXAMINATION OF SOILS

1. Methods of Investigation.—Before any permanent structure is founded upon the earth, an accurate knowledge of the character of the soil at and below the foundation line should be obtained by examination. If the building is to be placed in a well-settled locality, this information may possibly be obtained from the history of the adjacent excavations; although, unless the geological formation of the ground is regular and uniform, this should not be relied on. If the building is to be placed in an unsettled locality, or if, in a settled locality, it is to be of greater weight than the adjoining buildings, maps of the geological formation, if available, should be consulted and may furnish valuable information as to the probable bearing capacity of the soils. If the building is an important one and doubt exists as to the capacity of the soil to sustain its weight, borings should be made for some distance below the bottom of the footings in order to determine the character of the underlying strata. These borings can be made by methods similar to those used in the driving of an ordinary pipe well, from which samples of the material passed through can be obtained as the pipe is driven down.

Tests may also be made by driving a gas pipe, say 2 inches or more in diameter, with a maul or hammer, and withdrawing the pipe from time to time to obtain samples of the material passed through. If the pipe passes through gravel or hard material, difficulty will be experienced in extracting the material after the pipe is withdrawn. This is at best a crude method and should not be used if accurate results are desired.

A better method of making this test is to sink a pipe and use an auger to bring up the material. A common wood auger with levers 2 or 3 feet long, turned by two men, will bring up samples that may be sufficient to determine the nature of the soil, but such samples cannot be taken as indicating the compactness of the soil, as the driving of the pipe compresses the material inside of it and the auger subsequently disturbs it. The same thing is true of the samples taken from a driven well, as the material inside the pipe is first compressed and then taken out with an auger, or, if the material is hard, broken up with a bit or chisel, and then removed with a sand pump.

In making tests with pipes, too great care cannot be exercised in arriving at conclusions regarding the character of the underlying material.

The soil may also be tested by digging an ordinary well 4 to 6 feet in diameter, curbing it with wood as the excavation proceeds. This method will permit accurate conclusions to be formed as to the character of the material in its natural position. If the ground is wet, it will be necessary to make the curbing water-tight and to remove the water.

If the site of the proposed structure is in a locality of apparently recent geological formation, such as the bottom lands in the valleys, more care must be exercised in obtaining the required information than is necessary in localities having an older geological history. In land formed of alluvial deposits a wide variation may be found between two contiguous sites. The difference may have been caused by changes in the river currents and channels due to floods or other natural causes, which may cause gravel to be

deposited in considerable layers in one spot and soft material in an adjoining spot. In alluvial soils, deep borings should always be made, as a shallow excavation or test well may disclose gravel that is of variable thickness and may be underlaid with softer and yielding material. If the gravel is thick enough, it may be advisable not to go below it, but in any event the actual knowledge of the conditions should be ascertained before decision as to the character of the foundations is made.

2. **Determination of Bearing Capacity.**—After a full knowledge of the nature of the soil has been obtained, if the material is compressible and spread foundations are to be used, a test of the actual bearing capacity of the soil should be made by loading a platform of certain area and measuring accurately the settlements under increasing loads.

In testing the bearing capacity of the soil on which the New York State Capitol at Albany was erected, a measured load was applied to a square foot and also to a square yard. For the first test a timber mast 12 inches square, held in a vertical position by guys, was fitted with a cross-frame to hold the weights. A hole 3 feet deep was dug in the blue clay at the bottom of the foundation. The hole was 18 inches square at the top and 14 inches square at the bottom. Small stakes were driven in the ground on lines radiating from the center of the hole. The tops of the stakes were brought exactly to the same level so that any change in the surface of the ground adjacent to the hole could readily be detected by means of a straightedge. The foot of the mast was placed in the hole and the weights applied. No change in the surface of the adjacent ground was observed until the load reached 5.9 tons per square foot, when an uplift of the surrounding earth was observed in the form of a ring with an irregular rounded surface. Similar experiments were made by applying the load to a square yard, with essentially the same result. The loads were allowed to remain some time and the settlements observed.

Before building the Congressional Library at Washington, similar experiments were made with a frame, which rested

upon four corner posts, each a foot square. The frame could therefore be moved from place to place upon wheels and a number of tests were made on different parts of the site.

3. Experimental Precautions.—In making experimental tests, it is necessary to take extraordinary precautions to avoid obtaining erroneous results, and more particularly is extra care necessary if the bearing area tested is a small one, as slight errors either way may vitiate the correctness of the finding.

In placing posts on the ground, such as 12″ × 12″ timbers, great care is necessary to see that they are evenly and gently placed upon the surface to be tested; and if only a square foot is tested, allowance must be made for the greater resistance to displacement which the surrounding ground offers to a small area, as compared with a larger area. It is likewise well to consider that the employment of a square testing area gives four cutting edges, a condition likely to produce a greater and more rapid settlement than the parallel edges of a footing course.

CHARACTERISTICS OF FOUNDATION SOILS

4. The capacity of the earth to support superimposed loads without lateral displacement or crushing depends on the character of the soil, which may vary from solid rock through all intermediate stages to a soft or semifluid condition, as mud, silt, or marshy matter. Rock furnishes the most stable foundation. Gravel, which is composed of waterworn pieces of rock, is the next best. Hard dry sand, wet sand if confined, dry clay, wet clay, silt, and marshy matter decrease in bearing capacity approximately in the order named.

5. Rock.—The ultimate crushing strength of **rock** can only be determined from tests made on cubes of it in the testing machine, or, if the variety is well known, from the history of the stone. The ultimate strength of the rock varies from about 15 tons, the crushing strength of cemented gravel in the case of decayed rock and some of the softest

varieties, up to about 1,800 tons per square foot for the hardest rocks. In making tests of cubes of rock for the ultimate strength, in order to secure accurate results, the beds must be truly parallel and reduced to a true plane.

The safe bearing value of rock is considered to be not less than about one-tenth the ultimate crushing strength of the cubes. Under ordinary conditions it may safely be said that all rock, except that of the softest and most friable character, will safely sustain any load that may be placed upon it by a building of ordinary character, the area of the foundations being determined by the safe stresses that may be placed on the material forming the foundation courses of the superstructure. In ordinary practice, the safe bearing value of sound rock is taken from 14 to 20 tons per square foot.

6. Gravel.—Broken rock from which the earthy substance has been worn away by the action of water, or *detritus of rock*, as it is called, composes **gravel.** If gravel is found in a compact mass and in layers or strata of considerable thickness, it forms a most excellent foundation, compression within the safe limits of loading being so small that it can be neglected as a factor in design. In founding a building upon gravel, care must be taken to see that the stratum is of sufficient thickness and that the material underneath it is not such that it would be displaced by the load coming upon the strata above it. If a bed of gravel overlays a dry material, such as clay or sandy clay, it can be regarded as an excellent foundation; but if the clay is wet, a test of the ultimate bearing capacity should be made. The safe bearing loads upon gravel are usually taken from 6 to 10 tons per square foot on hard cemented gravel and from 4 to 6 tons per square foot on loose and sandy gravel, although the lower limit is considered the safer one to use under ordinary circumstances.

Strata, or sometimes merely pockets, of gravel are frequently found between layers of alluvial mud in land that has been formed through the action of water. In building

upon such soil, care must be taken that the gravel is not merely a local deposit, and soundings should be made over the whole site before the foundation is designed.

7. Sand.—Sandy soils vary from fine sand to coarse gravel. Dry **sand** is incompressible, and if beyond the danger of scour or of being carried away by running water is one of the best foundations obtainable. Where foundations are built upon sand, it is necessary to make sure provision that water will not remove it from underneath the footing courses. Dry sand when confined may be considered as having the same safe load as hard cemented gravel; but no rule can be laid down for the safe load of wet sand.

8. Clay.—Soils that are composed of **clay** vary greatly in hardness and bearing capacity, and range from slate or shale to soft, wet, or semiliquid clay. The bearing capacity of clays may roughly be said to vary inversely in proportion to the water contained in them. It is therefore evident that their bearing capacity may be improved by draining them and thereby removing the water. Hard clay, such as slate or shale, will support the same load as rock of equal hardness, while soft, wet clay will squeeze out laterally when any pressure, such as ordinarily occurs in building, is placed upon it. Wet clay is at best a dangerous material for foundations, because it has a tendency to squeeze out around the edges of the footings, or, if it is not soft enough for this, settlement of the building takes place by the load pressing the water out of the clay and thereby reducing its volume.

In making an excavation below the water-line, clay may be found quite hard when first uncovered, but as it has a great capacity for absorbing water, it may swell up and increase in volume and become unfit for a foundation that would permit of no settlement.

If coarse sand or gravel is found mixed with the clay its bearing capacity is increased in the same proportion as these materials are mixed with it. It must be remembered, however, that clay is an unctuous material, that is, of a fatty or

greasy nature, and acts as a lubricant between the coarser particles, and therefore sand and gravel mixed with a very slight proportion of clay, if the material is wet, has a much smaller bearing capacity than the sand and gravel without the clay, but if the material is dry the bearing capacity is very much greater than if it is wet.

The city of Chicago is underlaid very largely with a stratum of clay of variable thickness, and the Building Laws there make the following restrictions: If the soil is a layer of pure clay at least 15 feet thick, without a mixture of any foreign substance, excepting gravel, it shall not be loaded more than at the rate of 3,500 pounds per square foot. If the soil is dry and thoroughly compressed, it may be loaded not to exceed 4,500 pounds per square foot. If the soil is a mixture of clay and sand, it shall not be loaded more than at the rate of 3,000 pounds per square foot. With the loads authorized by the Building Department of Chicago, the settlement of a building, due to the compression of the soil, amounts to from 3 inches to 5 inches.

From the experimental tests made on the soil at the State Capitol at Albany, it was found that the ultimate bearing capacity of the clay soil was less than 6 tons per square foot, and the building was designed to impose a load of 2 tons per square foot. This soil contained from 60 to 90 per cent. of alumina clay, the remainder being fine silicious sand. It was also found to contain from 27 to 43 per cent., usually about 40 per cent., of water, and samples of it weighed from 81 to 101 pounds per cubic foot.

In the soil underlying the Congressional Library at Washington, the ultimate bearing capacity of the soil, which was yellow clay mixed with sand, was 13½ tons per square foot, and the building was designed to impose a pressure of 2½ tons per square foot on this material.

Hard, stiff clay, when dry, may be estimated to have a safe bearing capacity of from 4 to 6 tons, but if it is allowed to become wet, its resistance is greatly decreased and it is probably not good for more than 1½ to 2 tons; even then the compression and consequent settlement may be such as will

make it necessary to design all the footings so that the settlement on each will be the same, and a consideration of the safe capacity of such soil to properly bear the weight of the structure put upon it leads directly to the question of the design of foundations for which settlement has to be provided.

Where subsequent settlement of the foundations might cause cracks in the superstructure, the footing of a heavier part of the building, such as a tower, for instance, is temporarily loaded, the load being removed as the superstructure is built. In the Chicago Auditorium, a large tower rises 94 feet above the main building, and as a precaution against the heavy concentrated weight of the tower cracking the adjoining wall, a direct load, approximately equal to the weight of the finished tower, was placed upon the tower footings, and gradually removed as the walls were carried up.

In Chicago, when piles are not resorted to and the building is founded upon the compressible soil, the wall piers are built detached and are carefully calculated for the dead load that comes upon them. The live load is either omitted entirely or a very small proportion of it is used.

The great difficulty in designing a building to meet such conditions of supporting ground is that of accurately proportioning the foundations where the interior columns carry but little dead load, as, for instance, the interior columns in a store or other building having an open interior. In the case of office buildings, the dead load being a considerable part of the total load, the problem is easy of solution, but in store buildings it is a problem requiring great judgment to reach a satisfactory conclusion.

In designing foundations for which a settlement must be allowed, great care must be taken to have the center of resistance under the center of gravity of the load, as, with a yielding material, the overloading of one side or one edge may cause an undue compression of that side or edge and thereby tilt the footings with disastrous consequences to the superstructure. For compressible soil, it is necessary to spread the foundation over a large area, so as to have a small

load per square foot of surface. This is usually accomplished by means of I beams bearing on or embedded in a concrete slab or plate, but in buildings of no great importance this spreading of the footing is often done by means of timbers. When timbers are used, however, it is absolutely necessary that they be below the permanent water-line, as otherwise they will decay; it is also necessary that the bending stresses on the timbers be kept within very small limits, as otherwise the deflection of the timber under load will cause settlement.

9. **Mud, Silt, Marsh, or Semiliquid Soils.**—With soils of this character, it is necessary either to drive piles to a hard substratum, if it can be found, or to sink caissons to the hard material below, the method to be pursued usually depending on the importance of the building and the value of the land. With ordinary buildings, the pile foundation is undoubtedly the simplest and the cheapest method, but in New York city, in the lower part of the island, it is found necessary to go down to the bed rock, and in the most recent important structures there pneumatic caissons have been sunk to bed rock and afterwards filled with concrete, so arranged as to form a continuous retaining wall around the building. The material inside of this retaining wall is excavated and the space utilized for various purposes, such as basements, subbasements, and office or store purposes, the light, heat, and air being furnished by mechanical means.

Rankin gives the following formula for ascertaining the unit bearing capacity P_u of soft soil:

$$P_u = w\,h\left(\frac{1 + \sin a}{1 - \sin a}\right)^2$$

in which w = weight of soil, in pounds per cubic foot;
h = depth of marsh, in feet;
a = angle of repose of the soil.

For example, if a equals 5°, the supporting power of the soil is 1.42 $w\,h$ per unit of area; if a equals 10°, it is 2.02 $w\,h$; and if a equals 15°, it is 2.88 $w\,h$. The weight of mud, silt, and quicksand varies from about 100 to 130 pounds per cubic foot.

The city of New Orleans is underlaid with alluvial soil, that is, a soil deposited by water, and experiments there made on quicksand would indicate that with a load of $\frac{1}{2}$ ton to 1 ton per square foot, the settlement would not be excessive.

In designing the foundation for any building, the particular character of the building, its cost, and permanence must determine the character of the footings. If the foundation is of soft material, the possibility of adjacent excavations wrecking the building must be carefully considered. If it is necessary to establish the foundation of a building on soft, yielding material, either too soft for a grillage or spread foundation, piles must usually be driven. If, also, the character of the building is such that it cannot be designed to insure equal settlement of all its pieces or parts, and a firm bearing soil cannot be found within convenient distance, piles long enough to sustain the load coming upon them should be driven into the ground. The piles can be used only in soils that are permanently saturated with water, as, if exposed to alternate wetting and drying, or if all the piling is not thoroughly immersed, fungus soon destroys the wood, and the building resting on the piles must fail.

10. Quicksand.—The term **quicksand** is applied to any silicious material so saturated with water that it will flow more or less easily when its natural condition of equilibrium is destroyed by excavating pits, trenches, shafts, or tunnels. Quicksand may be found on the surface, underlaid by firm material, or it may be found in a stratum of greater or less thickness, confined by firm strata both above and below. If the quicksand is on the surface, and underlaid by a hard material, it is no more difficult to deal with than water, but when found between strata of harder material at considerable depth, great difficulty will be ordinarily experienced in working in it, for the reason that as the depth below the surface increases, the pressure increases and the flow of the liquid or semiliquid materials allows the stratum on top of it to settle, bringing great pressure on the sides of the caissons, sheet piling, or cribbing, either crushing it or throwing it out of

line. The material falling on top of the quicksand increases the amount of material to be excavated and usually adds very greatly to the cost of the foundation of the structure.

Owing to the fact that quicksand flows and runs so readily, the greatest precaution must be exercised in working in this soil adjacent to the foundations of a building or structure. Instances have been known where the undue removal of sand and water, while boring a hole for a direct plunger elevator, imperiled the adjacent foundations by removing some of the stratum from beneath the footings with the detritus from the hole. It is likewise a necessary precaution to have all sewers and drain lines running near footings and on a lower level, and that are built in soils partaking of the nature of quicksand, so carefully constructed and inspected that there can be no possibility of bleeding the foundation stratum.

11. If quicksand is encountered in building construction, the only successful method of dealing with it is to use sheet piling in the excavation, keeping it in position by horizontal bars and bracing. The piling must be of sufficient thickness to withstand the bending strains due to the pressure of the quicksand. These piles should be tongued and grooved or connected with loose slip tongues. If the excavation can be carried on at the same time with the driving, it is possible to drive the piling with a heavy maul, or, if the material is soft enough, it may be driven with a maul before the excavation is made. If the material is of a clayey nature, and the excavation can not be carried along as the piling is driven, it may be impossible to drive it any distance with the maul. In such cases, a portable steam pile driver, which fits on the head of the pile, built on the same lines as a steam rock drill, may be used to advantage. By means of this driver, the piles may be driven to a considerable depth before the excavation is made, and as the material is thrown out of the trench or pit, shores can be put in place and the trenches carried down for some distance without any displacement of the adjacent material unless it is so thin that it will slip down under the piles and come up into the trench, in which case, if the

amount of material coming into the trench is considerable, the whole temporary structure must collapse because of the displacement of the material behind it. In such cases, several methods of founding may be followed. One is the freezing process, in which the earth adjacent to the excavation is frozen hard by means of brine from a portable refrigerating plant, which is circulated through pipes that are sunk into the ground around the sides of the proposed excavation. After the ground is frozen, the excavation can be carried on as with other hard material.

Another method is by sinking either pneumatic or open caissons through the material. If the material is very soft and fluid, the pneumatic caisson must be employed. Open caissons of hollow brick or concrete, or iron cylinders, or timber-lined shafts can be sunk to great depths through soft material and ultimately filled with concrete and masonry. In sinking these open caissons, the material is excavated from the inside and weights are built or placed upon the caissons, causing them to sink. If brick or concrete cylinders are used, the bottom must rest on a timber or iron curb with a cutting edge.

Pneumatic caissons usually consist of a strong grillage of timber, laid diagonally and crosswise, strongly bolted together, beneath which a strong cutting edge of timber, sheathed with iron, is provided. The height of this cutting edge is sufficient to permit the work to be carried on beneath the under side of the caisson. An air pressure sufficient to prevent the material from running in around the cutting edge is maintained in this open space. As the material is excavated from beneath it, the caisson sinks. The excavated material is taken out in buckets through the air lock. As the caisson sinks, the masonry is built upon it until the cutting edge finally reaches the hard material, after which the space between the cutting edge and the under side of the timber platform is filled with concrete.

Another method of dealing with quicksand is to solidify it by liquid cement by means of pipes sunk into the ground. Liquid hydraulic cement is forced down through one pipe

and suction is maintained in the other; by this means the cement is drawn from the supply pipe over to the exhaust pipe and the intervening material is saturated with it. By changing the position of the pipes often enough, a stratum or area of quicksand saturated with cement can be produced, which, upon hardening, forms an artificial stone.

12. Culm.—It is frequently necessary in the anthracite regions to build at least temporary structures upon culm or refuse from mines. This material is composed of fine particles of coal and slate, being exceedingly springy or elastic when in beds of considerable thickness and is a dangerous material upon which to build. This is especially true when it is not thoroughly confined or the building is so situated that its foundations are not well within the slope of repose, or slope which the material naturally assumes when dumped or piled loosely. Another great danger in building upon culm and similar materials exists in the liability of the material to be washed away by heavy rains, fire-streams, or leaky mains. Under no condition should culm be subjected to a greater pressure than $\frac{1}{2}$ to $\frac{3}{4}$ ton per square foot.

13. Foundations Over Mines, Etc.—Large buildings have been destroyed by the failure of the principal piers built over long-disused cisterns or vaults, which in the course of time have been forgotten and filled over. A principal pier in one instance did not break through the covering of the cistern until the building was far along in its construction, so that the failure of the pier precipitated the destruction of almost the entire building.

In the hard-coal section of this country, cities of importance are frequently undermined by galleries or veins which have been worked, and these are often near the surface. When heavy buildings are erected in such localities, the position of the building with reference to these mine workings must be investigated; then, if the roof of the working does not seem to offer adequate strength, some means of support must be provided. The working, if abandoned, may be filled, or a better method would be to extend piers through to the solid floor.

One great danger exists in such localities from the fact that surface water is likely to run along the hard-pan or shaly roof of the working, and thence down through fissures in some abandoned working. In such an instance a tunnel will frequently be formed by the action of the subterranean stream, and where a heavy foundation is placed above, damaging settlement is likely to occur which may possibly cause the failure of the building.

BEARING VALUE OF FOUNDATION SOILS

14. There is some difference of opinion regarding the safe bearing value of foundation soils, due probably to the difficulty of arriving at any experimental results that will have a general application. Conservative engineering practice, however, dictates that the greatest unit pressure on the different foundation soils shall not exceed the values given in the following table:

TABLE I

SAFE BEARING VALUES OF DIFFERENT FOUNDATION SOILS

Materials	Tons per Square Foot
Granite rock formation	30
Limestone, compact beds	25
Sandstone, compact beds	20
Shale formation, or soft friable rock	8 to 10
Gravel and sand, compact	6 to 10
Gravel, dry and coarse, packed and confined	6
Gravel and sand, mixed with dry clay	4 to 6
Clay, absolutely dry and in thick beds	4
Clay, moderately dry and in thick beds	3
Clay, soft (similar to Chicago clay)	1 to 1½
Sand, compact, well-cemented, and confined	4
Sand, clean and dry, in natural beds and confined	2
Earth, solid, dry, and in natural beds	4

BUILDING LAWS REGARDING FOUNDATION SOILS

15. The observance of the revised building laws of the several cities is considered good engineering practice, for they are usually the results of careful investigations and records of long experience. The following, quoted from the New York Building Law, is interesting and gives bearing values that are well within the safe limits.

Where no test of the sustaining power of the soil is made, different soils, excluding mud, at the bottom of the footings shall be deemed to safely sustain the following loads to the superficial foot, namely:

Soft clay, 1 ton per square foot.

Ordinary clay and sand together, in layers, wet and springy, 2 tons per square foot.

Loam, clay, or fine sand, firm and dry, 3 tons per square foot.

Very firm, coarse sand, stiff gravel, or hard clay, 4 tons per square foot, or as otherwise determined by the Commissioner of Buildings having jurisdiction.

Where a test is made of the sustaining power of the soil, the Commissioner of Buildings shall be notified so that he may be present in person or by representative. The record of the test shall be filed in the Department of Buildings.

When a doubt arises as to the safe sustaining power of the earth upon which a building is to be erected, the Department of Buildings may order borings to be made, or direct the sustaining power of the soil to be tested by and at the expense of the owner of the proposed building.

MATERIALS OF STRUCTURAL ENGINEERING

(PART 2)

MASONRY

INTRODUCTION

1. The materials of masonry construction include stone, brick, terra cotta, and the cementing materials—cement, lime, and sand—used in the manufacture of the mortars. That an intelligent working knowledge of these materials may be had, it is necessary to know something of the elements that compose them.

Though an extensive knowledge of chemistry and geology would be valuable to the structural engineer, his practice involves only an elementary knowledge of these subjects. This acquaintance is necessary, however, in order that he may know the chemical and physical nature of the materials with which he builds. Therefore, the first portion of this Section is devoted to a brief description of the principal elements of nature that combine to constitute stones and minerals, and following this is given a condensed geological description that will furnish an insight into the physical formation of the rocks from which building stones are obtained.

2. Elements and Compounds.—Every substance or mass of matter is either an *element*, a *compound*, or a *mixture*.

Any substance that can be decomposed or divided into separate substances is called a **compound**. For example, if an electric current is passed through water, the water slowly

disappears and two gases are formed. These gases are entirely unlike, and neither resembles the water from which it was produced. Likewise, lime can be divided into two other substances, calcium and oxygen.

There are substances, however, like iron, gold, sulphur, and arsenic, that have never been decomposed into other substances. These are called **elements.**

In referring to an element, it is customary to simply use the *symbol*, which is usually the first letter of the name. Thus, *H* stands for hydrogen, *C* for carbon, etc.

CHEMICAL ELEMENTS

3. About seventy elements have been found in the earth, but of these, fourteen in various combinations form nearly the entire crust of the earth. These elements, named in the approximate order of their importance, are oxygen, silicon, aluminum, iron, calcium, magnesium, potassium, sodium, carbon, hydrogen, phosphorus, sulphur, chlorine, and manganese, to which may be added nitrogen, for though it does not enter into the rocks to a great extent, it forms about 79 per cent. of the air.

As all these elements are ready to form combinations with others, they are seldom found uncombined and there is a constant tendency to change, the old combinations being broken up and new combinations being formed. One of the most active of the elements in this respect is oxygen.

4. Chemical Combination.—When two or more elements are brought into contact under favorable circumstances, they will combine and form a substance unlike either of the elements. For example, hydrogen, an exceedingly light gas, will burn in the air with a light blue flame. This burning is a form of chemical combination, in which the hydrogen unites with another gas, the oxygen of the air, and forms water, a substance with which we are all familiar.

Chemical combination produces heat; chemical separation, on the other hand, absorbs heat. Thus, if carbon and oxygen are brought together at high temperature, they will

combine and form carbon dioxide; hydrogen and oxygen combine to form water; hydrogen, nitrogen, and oxygen, when combined in certain proportions, form nitric acid; a given volume of nitrogen and three times that volume of hydrogen combine and form ammonia, a gas that differs greatly from both nitrogen and hydrogen. In each of these combinations a certain amount of heat is produced; to separate the elements again an amount of heat will be absorbed exactly equal to that produced by their combination.

5. It is supposed that the *molecules* of most elements, such as hydrogen or oxygen, are composed of two *atoms*. A **molecule** is the physical unit, or the smallest portion which, so long as the substance is chemically unchanged, keeps together without complete separation of its parts, while an **atom** is the chemical unit, or the smallest mass of an element that can exist. It is further supposed, by chemists, that equal volumes of all gases, whether simple or compound, contain the same number of molecules; thus, a cubic foot of hydrogen, a cubic foot of air, a cubic foot of steam, contain the same number of molecules.

Suppose, now, that a cubic foot of hydrogen gas is allowed to come into contact with a cubic foot of chlorine gas; if the mixture is exposed to heat or light, the gases combine. The process of combination is explained as follows: There is a certain attraction or affinity between the hydrogen and the chlorine atoms. Under the influence of heat or light, this attraction becomes so strong that the two atoms composing the molecule of hydrogen are torn apart. Likewise, the atoms composing a molecule of chlorine separate. Each atom of chlorine seizes upon an atom of hydrogen and forms a molecule of an entirely new gas, viz., hydrochloric acid gas. Since each atom of chlorine takes one atom of hydrogen, it is plain that the number of molecules of each gas must be the same. In other words, a cubic foot of chlorine requires a cubic foot of hydrogen to combine with it; these gases cannot be made to combine in any other proportion. For example, if 3 cubic feet of

chlorine were placed in contact with 2 cubic feet of hydrogen, 4 cubic feet of hydrochloric acid gas would be formed, and the extra cubic foot would still remain chlorine. The symbol for hydrochloric gas is HCl.

Suppose, now, that hydrogen and oxygen are placed in contact and heated; they will combine and form steam (or water). But it will be found that each atom of oxygen seizes two atoms of hydrogen to form a molecule of water, and, therefore, the volume of hydrogen must be double the volume of the oxygen with which it combines. This is shown by the symbol for water, H_2O; that is, two parts or atoms of hydrogen to one of oxygen. Similarly, the symbol for ammonia is NH_3; that is, three parts of hydrogen to one of nitrogen. Again, hydrogen and carbon form a compound; each atom of carbon seizes four atoms of hydrogen and forms a molecule of marsh gas; the symbol for marsh gas is therefore CH_4.

The symbol of any compound indicates how the atoms of the elements combine to form the compound. Thus, the symbol H_2O shows that two atoms of hydrogen and one of oxygen unite to form a molecule of water. The symbol H_2SO_4 (sulphuric acid) shows that a molecule of the sulphuric acid contains two atoms of hydrogen, one of sulphur, and four of oxygen.

6. Combination by Weight.—A cubic foot of hydrogen combines with just 1 cubic foot of chlorine. On weighing each gas, it is found that the cubic foot of chlorine weighs 35.5 times as much as the cubic foot of hydrogen. A cubic foot of oxygen weighs 16 times as much as a cubic foot of hydrogen.

As like volumes of gases contain equal numbers of molecules, and oxygen and hydrogen molecules contain two atoms each, like volumes of the two gases must contain equal numbers of atoms. Now, since the former weighs 16 times as much as the latter, it follows that an atom of oxygen weighs 16 times as much as an atom of hydrogen. Similarly, an atom of chlorine weighs 35.5 times as much as an atom of

hydrogen. This ratio between the weight of an atom of any element and the weight of an atom of hydrogen is called the **atomic weight** of the element. The molecules of many elements have the same number of atoms as hydrogen, and the atomic weight of such elements may be found by dividing the weight of a given volume of the element, when in a gaseous state, by the weight of a cubic foot of hydrogen. The atomic weight of such gases is, therefore, the same as their specific gravity, hydrogen being the base of comparison instead of water or air. With liquids, solids, and gases whose molecules do not have the same number of atoms as hydrogen, the above statement is not true.

7. The atomic weights of some of the elements considered in this Section are as follows:

Hydrogen, H	1.0
Oxygen, O	16.0
Nitrogen, N	14.0
Carbon, C	12.0
Sulphur, S	32.0
Chlorine, Cl	35.5

By the aid of these atomic weights, the composition of any substance, by weight, can be found when its symbol is known. For example, take water, H_2O; multiply the number of atoms of each element by the atomic weight of the atom. Thus,

$$\begin{aligned} 2 \times 1 &= 2 \text{ parts, by weight, of hydrogen} \\ 1 \times 16 &= \underline{16} \text{ parts, by weight, of oxygen} \\ & 18 \text{ parts, by weight, of water} \end{aligned}$$

Water, then, is composed of $\frac{2}{18} = 11.11$ per cent. of hydrogen, and $\frac{16}{18} = 88.89$ per cent. of oxygen.

As another example, take carbon dioxide, CO_2. We have

$$\begin{aligned} 1 \text{ atom of } C \times \text{atomic weight, } 12 &= 12 \text{ parts, by weight, of } C \\ 2 \text{ atoms of } O \times \text{atomic weight, } 16 &= \underline{32} \text{ parts, by weight, of } O \\ & 44 \text{ parts, by weight, of } CO_2 \end{aligned}$$

Hence, CO_2 contains $\frac{12}{44} = 27.27$ per cent. of carbon and $\frac{32}{44} = 72.73$ per cent. of oxygen. From these examples it is

plain that the atomic weight of water is 18, and of carbon dioxide 44.

8. **Mixtures.**—Two or more elementary substances may be mixed together and yet not combine to form a new substance; they are then said to form a **mixture.** The mixture has the properties of the elements composing it. The most familiar example of a mixture is ordinary air, which is composed of 23 parts, by weight, of oxygen and 77 parts, by weight, of nitrogen. The two gases are not combined chemically; they are simply mixed.

9. **Specific Gravity.**—This value is the ratio of the weight of a given substance to that of a standard substance. For solids and liquids, the standard is pure water at a temperature of 62° F.; while for gas, the standard is usually hydrogen gas, though sometimes air. Frequently, however, the specific gravity of gases is measured with water as a standard.

The weight of a cubic foot of pure water at 62° F. is 62.355 pounds; so that in order to find the weight per cubic foot of any substance, the specific gravity of the substance is multiplied by 62.355; while to find the specific gravity of a substance, its weight per cubic foot is multiplied by the reciprocal of 62.355 or .016037. To ascertain the weight per cubic inch of a substance, its specific gravity is multiplied by .036085. With each element, briefly treated in the following description, are given the chemical symbols and the specific gravity. The values given for the gaseous elements are based on hydrogen as the standard, while those for solids have pure water at 62° F. for the basis of the value.

GASEOUS ELEMENTS

10. **Oxygen,** *O*, specific gravity 16, is a transparent, colorless, odorless gas. The word *oxygen* is derived from two Greek words that literally mean sour generator, or acid former, as it was formerly believed that oxygen was a constituent of all acids. The chemist Davy, however, showed that hydrogen and not oxygen was necessary for an acid.

Oxygen is found in both a combined and a free state. In the air it is mixed, but not combined, with nitrogen in the proportions of about 23 per cent. of oxygen and 77 per cent. of nitrogen. It is an important constituent of all animal and vegetable substances, and is contained in all mineral substances. In granite, slate, clay, limestone, and other rocks, except coal, the quantity of oxygen is nearly 50 per cent. It combines with all other elements except fluorine, which is a gaseous element never found uncombined. Water dissolves about 4 per cent. of oxygen; about 47 per cent. of the earth's crust is composed of it, while it constitutes about 86 per cent. of the water. Decay and the ordinary forms of decomposition are simply the result of oxidation.

11. **Hydrogen,** *H*, specific gravity 1, is the lightest of all known substances. As one of the two elements constituting water, it is found everywhere present in the crust of the earth. In combination with water, many minerals form the group of hydrous minerals. It exists in combination everywhere, as a constituent of water, of all plants and animals, and in numerous minerals, abundantly in coal, petroleum, bitumen, and in a lesser degree in rocks. The amount of water chemically combined in the rocks or mechanically held in the crevices is estimated to many times exceed the amount of water in the oceans.

Hydrogen, in combination with carbon, forms the *hydrocarbons* that are found in the rocks that contain the remains of animal and plant life, such as the rocks from which petroleum and natural gas are obtained.

Hydrogen, combined with the minerals, forms but .21 per cent. of the crust of the earth. The amount of it present in the air and water, however, increases the importance of this element.

12. **Chlorine,** *C*, specific gravity 35.5, is a nearly transparent gas, of a greenish-yellow color. Combined with magnesium, potassium, or sodium, it forms the chloride of which common salt is the most familiar example. It is never found in a free state. Chlorine is one of the strongest

oxidizing agents. It has a great affinity for hydrogen and most metals, of which it forms chlorides. Its most abundant compounds are chlorides, of which sodium chloride, $NaCl$, and magnesium chloride, $MgCl_2$, are the most common and widely distributed. Sodium and magnesium chlorides are found in sea-water, and all soils of the earth contain some sodium chloride.

Chlorine forms about .01 per cent. of the crust of the earth, but about 2 per cent. of the oceans.

SOLID ELEMENTS

13. Silicon, Si, specific gravity 2.49, is never found in a free state but is found chiefly in the form of the oxide, SiO_2, commonly called *silica* or *silicon dioxide*, and also in combination with oxygen and several of the common metals, particularly with sodium, potassium, aluminum, and calcium, in the form of *silicates*, which are combinations of silicon with oxygen and other elements, as, for instance, feldspar, which is a complex silicate of aluminum and potassium. Silicon in one or the other of these forms is found in nearly all the rocks of the earth. In sandstone, it constitutes the greater part of the rock, while in coal it is only present in small quantities.

Next to oxygen, silicon is the most abundant element. Some extensive mountain ranges consist almost entirely of silicon dioxide, SiO_2, in the forms known as *quartz* and *quartzite*. Other ranges are made up of silicates that are compounds formed by the combination of silicon dioxide and bases; that is, the greater number of rocks of the earth's surface are combinations of complex silicates, limestone, $CaCO_3$, being the only important exception. The clay of valleys, river beds, etc. also contains silicon in large quantities, while the sand found so abundantly on the seashore is mostly silicon dioxide, or pulverized quartz.

While it is difficult to decompose the oxide in such a way as to obtain the element, under proper conditions the silicon can be obtained in the form of crystals that are of a gray color and are harder than glass, although they are only a chemical curiosity. Crystalline silica is found as quartz,

rock crystal, and sand in variously colored varieties, and the amorphous (not crystalline) forms of flint, chalcedony, opal, and others. The specific gravity of quartz is about 2.6. It is very hard but not so hard as ruby, corundum, Al_2O_3, and diamond. All forms of silica are more or less soluble in alkali by prolonged heating, but crystalline quartz is the least attacked. Silicon will not burn or dissolve in any acids except a mixture of nitric and hydrofluoric acids.

About 27 per cent. of the crust of the earth is composed of silicon, and therefore about 74 per cent. of the rocks are composed of oxygen and silicon.

14. Aluminum, *Al*, specific gravity 2.6, is a light, silver-white metal that is never found uncombined, its compounds are mostly silicates. It is the most abundant and widely distributed of all metals. Combined with oxygen it forms the oxide of aluminum, or *alumina*, Al_2O_3. Aluminum is commonly found in combination with both oxygen and silicon and forms the group of minerals known as the silicates of alumina. Alumina is found in all rock formations, except limestone, and in nearly 200 minerals. Corundum, Al_2O_3, is the richest of its ores. Emery is a mixture of the oxides of aluminum and iron.

The compounds of aluminum are difficult of reduction. The ruby and sapphire are Al_2O_3, colored by impurities. About 8 per cent. of the crust of the earth is composed of aluminum and about 80 per cent. of the crust of the earth is composed of oxygen, silicon, and aluminum. The greater part of the clay soil and rocks is composed of these three elements.

15. Iron (*ferrum*), *Fe*, specific gravity 7.8, is rarely found in an uncombined state, but is usually found as an oxide. It is widely disseminated throughout the earth's crust, of which it constitutes about 5 per cent., and its presence is shown in the red and yellow colors in the soil and rocks.

Iron has a great affinity for oxygen and, combined with it, forms what is commonly known as *rust*, which is the hydrated

oxide of iron. It commonly exists in the soil in the form of the highest oxide and gives it the red and yellow color, but in the presence of an excess of organic matter, the ferric oxide is reduced to ferrous carbonate, and its red or yellow color is destroyed. The clay of swamps is always bluish, but if this clay is burned to brick the organic matter is destroyed and the iron peroxidized colors the brick red.

16. Calcium, *Ca*, specific gravity 1.578, is a brass-yellow lustrous metal that is never found in an uncombined state. In combination it is very widely distributed and is found in enormous quantities. It is found principally as *carbonate*, $CaCO_3$, in the form of limestone, marble, and chalk; as *sulphate*, $CaSO_4$, in the form of gypsum; and as *phosphate*, Ca_3PO_2. Calcium combined with carbon dioxide, CO_2, and with another molecule of oxygen forms *calcite*, $CaCO_3$, which is present in some marbles and most limestones. Calcium is slightly soluble in water, and in solution forms the source from which fish, shell fish, and reef-building corals derive the materials from which to build their shells, cells, etc. Calcium in combination with sulphur and oxygen forms *gypsum*, $CaSO_4+2H_2O$.

Calcium enters into the composition of many of the complex silicates, and therefore forms a constituent of many of the rocks. In the form of carbonate, such as chalk and the various limestones, it forms an enormous rock mass in different parts of the earth. In some districts the sulphate or gypsum is found in considerable quantities. The chief varieties of native calcium carbonate, or carbonate of lime, are the amorphous forms, chalk, and the various kinds of limestone, the crystalline forms of marble, and the well-crystallized forms of calcite, or Iceland spar, and aragonite. Nearly 4 per cent. of the crust of the earth is composed of calcium.

17. Magnesium, *Mg*, specific gravity 1.7, is a less abundant element than calcium, but is almost invariably associated with it. It is a silver-white metal with a high luster, and is never found in a free state. Magnesium carbonate

is found as *magnesite*, $MgCO_3$, but usually occurs with calcium carbonate in dolomite and magnesian limestones. The sulphate and carbonate may be found in many natural waters. As silicate, magnesium enters into the composition of numerous minerals, as soapstone, serpentine, and meerschaum, while asbestos, hornblende, and many other silicates, contain this element. Magnesium forms about 2½ per cent. of the crust of the earth.

18. Potassium (*kalium*), *K*, specific gravity .875, is a white metal with a bright, metallic luster. It is so light as to float on water. It is a constituent of many minerals, particularly feldspar, which is a complex silicate of aluminum and potassium. It is never found in a free state, but is present as a silicate in the older Plutonic rocks, which are rocks formed from molten material at a great depth and pressure and including the granites, and in a small quantity in most soils. It is also present in sea-water. Potassium forms about 2½ per cent. of the crust of the earth.

19. Sodium (*natrium*), *Na*, specific gravity .973, is abundant, very widely distributed, and found in large quantities, principally as *sodium chloride*. It is found in a number of silicates and enters into the composition of an enormous number of rocks, being present in traces almost everywhere. It is a silver-white metal with a bright metallic luster, with a very light pink tinge, and is so light as to float on water. Sodium forms about 2½ per cent. of the crust of the earth.

20. Carbon, *C*, specific gravity 3.52, is commonly found in combination with oxygen. In an uncombined state it is found in the form of diamond and graphite. It also forms part of the tissue of all animals and plants, and when these tissues decay or burn the carbon unites with oxygen to form *carbon dioxide*, CO_2, which in combination with other elements forms the carbonates of which calcite (calcium carbonate) is an example.

Carbon is the central element of organic nature; every living thing contains it as an essential constituent. The number of compounds that it forms is almost infinite, and they

present such peculiarities that they are commonly treated under the head of "organic chemistry."

Carbon is present in a number of rocks, having been placed there by the agency of organic life, as in coal, where it has been taken from the air, and in limestone, where it has been taken from the water. .22 per cent. of carbon is estimated to be in the crust of the earth. The *diamond* is pure crystallized carbon.

21. Phosphorus, *P*, specific gravity 1.84 to 2.01, is a yellowish, translucent, crystalline, waxy solid that is never found in a free state because of its affinity for oxygen and other elements. Combined with oxygen it forms the phosphates, of which phosphate of lime is the most common. Its compounds are quite abundant and widely diffused, the principal one being calcium phosphate, which occurs in certain minerals as *phosphorite* and *apatite.*

Phosphorus, in minute quantities, enters into the bones and tissues of many animals and the tissues of many plants. It constitutes but about .1 per cent. of the crust of the earth.

22. Sulphur, *S*, specific gravity 7.086, is found particularly in the neighborhood of volcanoes. It occurs also as sulphide, in combination with many metals, as in *iron pyrites*, FeS_2, *copper pyrites*, $FeCuS_2$, etc. As sulphate, it occurs in combination with metals and oxygen, as *calcium sulphate*, or *gypsum*, $CaSO_4 + 2H_2O$.

Sulphur is found in the tissues of many animals and plants, and is found in the rocks in small quantities. It forms but about .03 per cent. of the crust of the earth.

23. Manganese, *Mn*, specific gravity 7.14 to 7.20, is said to be the hardest of metals. It is brittle, almost infusible, and looks like cast iron. It is easily oxidized in the air and dissolves in diluted acids. It forms alloys and imparts valuable properties to steel, perhaps by combining with such impurities as sulphur and phosphorus, but this is uncertain. Manganese is widely disseminated and its presence is often shown by black and purple stains on the surface of the rocks. It forms less than 1 per cent. of the crust of the earth.

COMMON MINERALS

24. The elements, when they unite, follow definite laws and produce definite results, uniting themselves in regular proportions, forming molecules, which, if free to act under favorable conditions, often assume regular geometrical forms or crystals. These combinations produce *minerals*, which (with the exception of mercury) may be defined as homogeneous solids of definite chemical composition, found in nature, but not of apparent organic origin.

A **mineral** may be composed of a single element, such as gold, silver, copper, lead, etc., or it may be made up of two or more elements chemically combined, as is usually the case. These elements may be only slightly bound together, as in the case of sulphide of mercury, which may be disunited by the action of heat, or they may be so firmly united and their chemical affinity so strong that, like silica, they may resist almost any effort to separate them. Therefore, some minerals are enduring and others are unstable, and while the chemical composition is definite, it is subject to change whenever the conditions are favorable. These changes are the cause of the decay and crumbling of rocks.

Minerals are frequently formed under conditions that do not permit sufficient room for the perfect development of the crystals; therefore, the perfect crystal is rare. When the crystal is not perfect, though the internal structure shows a crystal formation, the mineral is termed *crystalline*.

Some minerals, like diamonds and quartz, are exceedingly hard, while others, like graphite or black lead, are soft. Some are brittle and others are plastic. Most of the minerals are crystalline, though sometimes built up in irregular shapes, to which is applied the term *amorphous*, which means not crystalline.

The minerals that make up the greater part of the earth's crust, named approximately in the order of their importance, are:

1. Quartz.
2. The group of feldspars.
3. The group including calcite, dolomite, and siderite.
4. The group of micas.
5. The amphibole group.
6. The pyroxene group.
7. The group of iron minerals.
8. Gypsum.
9. Salt.
10. Ice and its liquid form, water.

25. Quartz.—Quartz (*silica*, SiO_2) is the most abundant mineral in the crust of the earth. In color it varies from a transparent rock crystal to a jet-black glassy mass. It is often colorless, though sometimes topaz-yellow, amethystine, rose, smoky, and other tints occur, also quartz of various shades of red, yellow, green, blue, or brown are found. In some varieties, the colors are in bands, stripes, or clouds, of all degrees of transparency to opaque, of a vitreous luster, with brilliant crystals, sometimes dull and often waxy.

The common mineral impurities of quartz are chlorite, rutile, asbestos, actinolite, tourmaline, hematite, and limonite. Hematite, or the red oxide of iron, is usually the red coloring matter; limonite, another oxide of iron, is the yellow coloring matter. Chlorite and actinolite give the green color, and an oxide of silicate of nickel an apple-green tint. Manganese gives an amethystine color, and carbonaceous matter, such as color marsh waters, the smoky-brown shades. Quartz crystals often contain liquid in cavities, either water, petroleum, or liquid carbon dioxide. Clear quartz is sometimes speckled with scales of mica or rendered opaline by means of asbestos. Flint or chert are often colored by mixture with the materials of the enclosing rock.

Quartz is sometimes amorphous, but usually crystalline, and is often found in perfect crystals; the varieties of the latter include agate, amethyst, and jasper. Quartz is light in weight, brittle, and so hard that it cannot be scratched with a knife. Its chemical composition is strongly fixed and silica, or quartz, once formed, remains as silica throughout all the changes to which rocks are ordinarily subjected. By reason of its hardness and chemical strength, quartz may

give a great durability to rock. There is no common mineral that resists destruction so well.

Quartz is found in various forms and colors, but may be distinguished by the lack of true cleavage, by hardness, infusibility before the blowpipe, and insolubility with the common acids.

It is found nearly everywhere on the earth's surface, but is particularly abundant in granite rocks, sandstone, and most of the soils. Wherever found, it is fresh, pure silica, and in this it differs from many of the common minerals, which are so liable to decay.

The varieties of quartz may be classified as *vitreous*, distinguished by their glassy fracture; *chalcedonic*, having a subvitreous or waxy luster and generally translucent; *jaspery cryptocrystalline*, having little or no luster, and opaque. The vitreous varieties comprise rock crystal, amethyst, rose quartz, false topaz, smoky quartz, prase, aventurine quartz, and ferruginous quartz. The chalcedonic varieties comprise chalcedony, chrysoprase, carnelian, sard, agate, onyx, cat's-eye, flint, rhomb spar, chert, and plasma. The jaspery varieties comprise jasper, bloodstone, Lydian stone, touchstone, basanite, besides other varieties arising from the structure.

26. Feldspar Group.—The name *feldspar* is given to all silicates of alumina with some other elements, such as magnesium, potassium, and calcium. Each species differs slightly from the other in chemical composition and mineral form. The two most common divisions are **orthoclase** and **plagioclase**. The former name signifies that the mineral has the lines of cleavage at right angles to each other, while the latter designation means that there are two prominent lines of cleavage oblique in direction.

Feldspar is variable in color, light in weight, and almost as hard as quartz. While quartz may be distinguished by its lack of cleavage, feldspar has distinct cleavage planes. Feldspar is always crystalline, though good crystals are not common. It is not as soluble as the nearly insoluble quartz,

but when exposed to the weather it begins to change and crumble and in time changes from a clear, hard, glassy mineral to a dull, opaque substance that can be scratched with a knife, and finally becomes a powdery white clay known as *kaolin*, from which fine chinaware is made. In changing to kaolin, some of the original sodium, calcium, or potassium enters into combination with other elements, producing a soluble salt that can be removed by solution in water; therefore, when exposed to the weather, rocks that contain feldspar decay and crumble.

27. Calcite Group.—The mineral in the calcite group is carbonate of lime, $CaCO_3$, and though commonly white, may be any color. It is usually crystalline but frequently amorphous. It is easily scratched with a knife, light in weight, and cleaves readily in two or three directions. Its principal varieties are Iceland spar, dog-tooth spar, satin spar, and limestone (a general name for a massive calcite as well as for massive dolomite), as well as granular or crystalline limestone, which includes all the fine marbles, compact limestone, chalk, hydraulic limestone, oolite, pisolite, argentine, etc.

Calcite is the main constituent of limestone and is one of the most abundant of the common minerals. It is present in nearly all waters on or in the earth, and is being constantly formed by the destruction of minerals that contain calcium. Its presence in water makes it possible for many animals and some plants to take it from solution and build it into their skeletons or substance, from the remains of which great beds of limestone are deposited in the sea.

Calcite, not being chemically strong, is not a durable substance, and hence rocks made of it do not resist the weather well.

28. Dolomite, or Magnesian Limestone.—Dolomite is calcium magnesium carbonate, $CaMgCO_3$, and its usual colors are white, or white tinged with yellow, red, green, brown, and sometimes black. Iron and manganese are often present, replacing part of the magnesium or calcium.

Iron-bearing varieties become brown on exposure, and magnesia-bearing varieties, black.

The principal kinds of dolomite are the white, crystalline, granular, and the white, massive varieties, of which the former resembles granular limestone and the latter is extensively used as a marble. The latter species include pearl spar, rhomb spar, and brown spar. Dolomite resembles calcite in that it burns to quicklime but is somewhat more resistant. It forms great beds of magnesian or dolomitic limestone, which is closely associated with calcite.

29. Siderite, or Spathic Iron.—Spathic iron, iron carbonate, or chalybite, FeO_2C or $FeO + CO_2$, often contains some manganese oxide or magnesia and lime replacing some of the *iron protoxide*, FeO. The iron, on exposure to air, becomes *hydrous sesquioxide*, that is, an iron oxide having three parts oxygen to two parts of iron, and gives the siderite a brown or brownish-yellow color. The crystallized or foliated variety is called *spathic* or *sparry iron* because the mineral has the aspect of a spar or crystallized mineral. The cleavage of siderite resembles calcite, calcinite, and dolomite.

The argillaceous variety occurs in nodular or lumpy forms and is called clay ironstone, and is abundant in coal measures.

Siderite occurs in rocks of various ages and often accompanies other ores. Large deposits exist in gneiss and mica schist, clay, slate, also in some limestone and in the coal formations, principally in the form of clay ironstone.

30. Mica Group.—Mica is a general name applied to a large number of minerals that are complex silicates of aluminum and some other metals, as potassium, lithium, and magnesium. There are numerous species of mica, depending on differences in chemical composition. The one characteristic feature of them all, however, is the cleavage, which is such that the mineral splits readily into thin elastic plates.

Micas vary in color from light brown to deep black, and are very soft. They decay readily, forming soluble and insoluble products, the soluble part passing off in the water, the

insoluble parts usually remaining as clayey remnants. Some micas are not so easily decayed, however, and are frequently found in the soil, on the beach, and in beds that have been made from the decay of other rocks.

Mica is common in lava, granites, and many other rocks. The common mica is called *muscovite*, which varies in color from white to green, yellow, and brownish shades, rarely rose red or reddish violet. Mica possesses a pearly luster and is transparent or translucent. It is a constituent of granite, gneiss, and mica schist.

31. **Amphibole and Pyroxene Groups.**—The minerals of these groups possess distinct chemical and crystalline characteristics, which can be distinguished in the crystal, but which resemble each other so closely that they cannot readily be distinguished by the naked eye. *Hornblende* is the common representative of the amphibole group and *augite*, of the pyroxene group.

These minerals are found in many of the lavas and granitic rocks and are commonly dark-colored with jet-black grains. They are complex silicates, and iron is often present. They easily decay, forming reddish or yellowish stains of iron rust as the iron becomes an oxide. Much of the iron coloring matter of the soil is formed by their disintegration.

32. **Ores of Iron.**—In addition to siderite, which is a carbonate of iron, several oxides of iron and the sulphide of iron are quite common. Of the oxides, the most common is *magnetite*, Fe_3O_4, a black mineral, frequently crystalline, and found in many of the volcanic rocks, in ore beds, and elsewhere. When magnetite rusts, it forms *hematite*, Fe_2O_3, which is the commonest of the iron ores, and forms the red coloring matter of the rocks. When hematite is further oxidized, it becomes *limonite*, $2Fe_2O_33H_2O$. The sulphide of iron or pyrites, FeS_2, is found in many of the rocks in the form of cubical or other crystals, and is of a brownish-yellow color and commonly known as *fool's gold.*

33. **Gypsum (Hydrous Calcium Sulphate).**—Gypsum, $CaSO_4 + 2H_2O$, is sulphate of lime, and is formed by the

decomposition of lime-bearing minerals or by alteration of the carbonate of lime by the sulphate. It is soluble in water. Gypsum, in appearance, resembles calcite somewhat and has a cleavage nearly as perfect as that of some of the micas, but the cleavage plates are not elastic. While present all through the crust of the earth gypsum is not abundant. It is rarely found in large quantities, and when it is thus found it is mined for plaster of Paris. Its principal varieties are selenite, radiated and fibrous gypsum, and alabaster.

34. Salt, or Halite.—Salt is *chloride of sodium*, *NaCl*. Chlorine, which is one of the strongest of the oxidizing agents, has such strong affinities that it never occurs in a free state. Its most abundant compound is sodium chloride, which is very soluble. The salt in the ocean has been originally derived from the rocks reaching it in solution.

35. Ice.—Ice is a substance that plays a very important part in the changes that occur in the earth's crust. Water in freezing and producing ice expands with great force; this expansion is one of the most destructive agents in nature, and one with which the engineer has frequently to combat.

FORMATION AND CHARACTERISTICS OF ROCKS

36. Definition.—The term **rock** is commonly defined as a hard mass of mineral matter, composed of one or more kinds of minerals, having, as a rule, no definite external form and liable to vary considerably in chemical composition. According to geology, however, rock includes all the consolidated materials forming the crust of the earth, such as sand, gravel, and clay, as well as the fragmental or detrital beds that have been derived from it.

37. Classification.—Rocks are divided into two principal kinds, according to their structure and origin, namely, *stratified* and *unstratified*. **Stratified rocks** are more or less consolidated sediments and are therefore aqueous in origin and earthy in structure. **Unstratified rocks** have

been more or less fused and are therefore igneous in origin and either crystalline or glassy in structure.

The rocks of the earth's crust owe their formation to five different causes:

1. The solidification of molten rock, as, for instance, the lavas.

2. The chemical precipitation from water, as illustrated by the beds of salt.

3. The action of animals or plants, as exemplified in the case of coral or coal strata.

4. The mechanical destruction of other rocks, as in the sand and clay beds.

5. The alteration or metamorphism of one of these classes of rocks, as exemplified in the case of marble.

Rocks that owe their formation to the first cause are termed *igneous*. Rocks that owe their formation to the second, third, and fourth causes are termed *sedimentary* or *stratified*; the fifth class is termed *metamorphic*.

UNSTRATIFIED, OR IGNEOUS, ROCKS

38. Unstratified, or igneous, rocks can be distinguished by the absence of true stratification or the lamination by sorting of material, by the absence of fossils, by crystalline or glassy texture in place of an earthy texture, and by their mode of occurrence, all of which characteristics are due to their mode of origin. The igneous rocks have consolidated from a state of fusion or semifusion instead of being deposited as sediments. Their original fused condition is shown by the crystalline or glassy texture, and by their occurrence in fissures, which shows that they were injected in a molten state; also by their effects, due to their heated condition, on the stratified rocks with which they come in contact.

Igneous rocks occur in three principal positions: underlying stratified rock and appearing on the surface in large masses, particularly in the mountain regions; in vertical sheets intersecting the stratified rocks or other igneous rocks;

in streams or sheets overlying the stratified rocks or sometimes between the strata, sometimes as veins connected with the underlying masses of igneous rocks. They occupy but a small portion of the surface of the earth, estimated to be about one-tenth of the land surface, but underneath the stratified rocks they are supposed to form the great mass of the earth.

Igneous rocks are classified into two groups: the *Plutonics*, or the *granitics*, and the *volcanics*, or the true *eruptives*. The **Plutonics** are coarse-grained and occur only in great masses either underlying the stratified rocks or appearing on the surface over wide areas, especially in the axes of mountain ranges. Usually, the **granitics** have not been erupted at all, although they often form the reservoirs from which the eruptive rocks are derived. The **volcanic, or eruptive,** rocks are fine-grained, sometimes glassy, and are found in sheets injected among the strata or as streams and sheets outpoured on the surface. The term *trappean* has been applied to the eruptive rocks that are injected among the stratified rocks.

These rocks may be briefly summarized as the *granitic*, which occur beneath, the *trappean*, which are injected among, and the *volcanic*, which are outpoured on, the stratified rocks.

39. Granitic, or Plutonic, Rocks.—The granitic groups are found in great masses and never in sheets or streams. They are very coarse-grained in texture and have a mottled or speckled appearance, due to the crystals of which they are formed being of considerable sizes and of various colors aggregated. These crystals mainly consist of quartz, feldspar, mica, and hornblende. The quartz crystals are bluish, glassy, transparent spots. The feldspar crystals are opaque, whitish, greenish, or rose-colored crystals with fluted surfaces. The hornblende crystals are usually black spots. The mica may be distinguished by its thin, scaly structure, the color of which is pearly or black. The term *granitic* has been applied to this group because granite is its best type; in fact, all these rocks are commonly

called *granite* by those unfamiliar with mineralogy. Thus, true granite consists of quartz, feldspar, and mica, with hornblende and talc sometimes appearing as impurities. Syenite is feldspar and hornblende frequently associated with quartz, mica, etc.

Igneous rocks, whether Plutonic or volcanic, are divisible into two subgroups: the *acidic rocks* and the *basic rocks*. In the **acidic rocks,** quartz and potash feldspar (orthoclase) predominate; in the **basic rocks,** hornblende or augite and soda lime feldspar (plagioclase) predominate. The acidic rocks are light-colored and not so dense as the basic group; the basic rocks are darker and heavier than the acidic group. These two groups, while sometimes sharply defined, frequently grade into each other. Granite is the best type of the acidics, and diorite, particularly gabbro and diabase, of the basics.

40. Trappean, or Intrusive, Rocks.—This group of rocks is intermediate between the Plutonics and the volcanics. They occur in sheets intruded among the strata, particularly of the older rocks. They are finer grained than the Plutonics and more crystalline than the volcanics, presumably because they cooled more rapidly than the Plutonics and less rapidly than the volcanics.

The trappean rocks are also divisible into acidics and basics. *Felsite* and *porphyry* are among the acidics, and *diorite* and *diabase* are among the basics. **Diorite** and **diabase,** when occurring among the intrusive rocks, are finer grained than the massive varieties. **Felsite** is a fine-grained, light-grayish rock, consisting essentially of orthoclase and quartz. **Porphyry** consists of a fine-grained, feldspar, ground mass, with large crystals of feldspar disseminated through it. Any rock is said to be porphyritic, however, if it consists of a fine-grained ground mass with large crystals of any kind disseminated through it, as porphyritic diorite and porphyritic granite.

41. Volcanic, or Eruptive, Rocks.—The volcanic, or eruptive, rocks are distinguished from the Plutonic and

trappean by their texture and mode of occurrence. In texture they are characterized by a crystalline structure, though some of the fine-grained varieties are microcrystalline and are in a more or less uncrystalline or glassy base or cement, showing that the fused mass has cooled too quickly to permit of complete crystallization. Frequently these rocks are in a wholly glassy condition.

These rocks are also divisible into the acidic and the basic. **Trachyte** may be taken as a type of the acidics. It is a light-colored rock and rough to the touch, consisting essentially of orthoclase with more or less quartz. When the quartz grains are conspicuous it becomes *rhyolite*. **Phonolite** is a dense variety, of light grayish color, that splits into slabs on weathering and has a metallic ring under the hammer. **Obsidian** and **pumice** are glassy scoriaceous (that is, of cinder or slaglike foundation) varieties of trachyte. **Basalt** is a type of the basics. It is a very dark, almost black, heavy rock, with an almost invisible fine-grained texture, and consists of plagioclase with augite, olivine, and magnetite. **Dolorite** has a similar composition, but has a more distinctly crystalline texture and is therefore dark grayish in color.

42. The volcanic or eruptive rocks have been formed in two ways: through the craters of volcanoes, in which the fused mass comes up through the opening and flows off in streams or is thrown out as cinders and ashes; or, it comes up through great fissures in the crust of the earth, frequently hundreds of miles long, and spreads as extensive sheets. These two methods of formation may be called *crater eruption* and *fissure eruption*. The greater part of the rocks on the surface of the earth is due to the latter method.

The fissure eruptions may be subdivided into three classes: *dikes*, *overflow sheets*, and *intercalary beds*.

Dikes are vertical sheets, filling great fissures in other igneous or stratified rocks and are the most common of all methods of occurrence of eruptives and intrusive rocks.

Overflow sheets are masses of lava that have come up, in liquid form, through great fissures and spread out on the

surface as extensive sheets. These sheets are often outpoured one on the other until they reach a total thickness of 2,000 to 3,000 feet. Some of these lava floods are of great thickness and extent. The whole of Northern California, Northwestern Nevada, and a great part of Oregon, Washington, and Idaho, and part of Montana and British Columbia, are covered with these lava floods, which are supposed to have come up in fissures in the Cascade and Blue Mountains, and spread as sheets covering the whole intervening space. The most extensive overflow sheets are usually basalt. The basic lavas, like basalt, were very liquid and spread out in thin sheets, while the acidic lavas were stiffly viscous and were squeezed out in dome shape.

Intercalary beds are sheets of rock found between strata. As such they may have been poured out on the bed of the sea or lake, and afterwards covered with sediment, or they may have broken through the strata for a certain distance and then have spread between the separated strata.

SEDIMENTARY AND METAMORPHIC ROCKS

43. Sedimentary.—Some rocks disintegrate through mechanical or chemical agencies more readily than others. For instance, in granite, the quartz will waste very slowly and then only as it is dissolved, for it will not be altered chemically; the feldspar and hornblende, being more complex, are less durable and soon commence to change, finally becoming the clay from which some of the elements go off in solution; the quartz grains, left without cementing material, fall out and the granite crumbles. From the original minerals of this rock, three different products result: soluble salts, fine clayey fragments, and larger grains of pure quartz. What is true of granite is also true, in a certain degree, of all rocks.

44. According to the way in which these mineral products are gathered into layers or strata, they may be classified into three groups of sedimentary rocks: the *fragmental*, or *clastic;* the *chemical precipitates;* the *organic.* The chemical

precipitates and some of the fragmental and organic rocks are of aqueous formation. Nearly all are stratified and some of each group are truly sedimentary.

45. Fragmental, or clastic, rocks are composed of distinct fragments of other rocks, gathered into layers by means of wind, ice, water, or volcanic eruption. Sandstones are formed of particles of sand that have been gathered into beds by one of these agencies and consolidated. Strata in which the material is transported by wind must necessarily be composed of fine grains or dust; ice, water, and volcanoes can transport particles of any size. The size of the particles transported by the wind or water will be determined by the velocity with which they are moved; therefore, by the varying action of these transporting agencies, the fragments are assorted into layers, according to their size, the coarsest being moved only by the stronger currents, while the finer particles settle in quiet air or water, giving rise to banding or stratification. This is also true of rocks transported by means of volcanic action, the larger pieces falling first and the finer particles or ash being carried great distances.

46. The pebbly rocks formed at the base of a cliff may be transported by water and rounded, forming pebbly or granite beds which, when consolidated into hard rock, become known as a **conglomerate.** The matrix is usually sand that contains a cement which binds the fragments together and consolidates the rock. Among the various kinds of conglomerate rock are limestone conglomerates, shale conglomerates, quartz-pebbly conglomerates, granite-pebbly conglomerates, etc. There are also volcanic conglomerates, composed of larger fragments of volcanic pumice and ash.

47. The **sandstones** may be coarse in texture, according to the size of the conglomerate, or very fine in grain, almost like clay, as exemplified in bluestone. Sandstones are usually composed of grains of quartz. There are also shale sands, magnetite sands, and garnetiferous sands.

Sandstones have been given different names, depending on the cement that consolidates the sand grains. *Argillaceous*

sandstone is applied to rock that has a clayey cement. If the grain is fine it is sometimes called an *arenaceous clay*. In *calcareous sandstone*, the cementing material is lime. If the cementing material is of iron, the rock is *ferruginous*, as in the brown and red sandstone. In *siliceous sandstone*, the cementing material is silica. In some sandstones, there are many small angular fragments giving the rock the character of a grit, which is used for grindstones. Sandstone rocks that split easily in every direction are called *freestones*. Sometimes, owing to the presence of many mica flakes, sandstone cleaves readily in only one direction and it is then said to be *shaly* or *micaceous sandstone*.

48. The **clay rocks** are so constructed that they frequently split into layers. Such rock is called a *shale*, the cleavage depending on many minute flat particles of mineral, often mica. Near the coral islands, the grinding action of the waves on the beach often wears the coral into fine clay, which, in settling to the bottom, forms a limy mud, afterwards becoming transformed into limestone. There are many other kinds of clay rocks, such as the *kaolin clay*, formed by the decay of feldspar; and *fireclay*, which has lost its alkalies by having them extracted by the plants that grew upon it, leaving the clay so free of alkali that it resists the action of fire. There are also sandy or *arenaceous clays*, containing considerable fine sand, and *carbonaceous clays* that contain fragments of plants.

49. **Chemically precipitated rocks** are interesting but not important. Chemical precipitation, however, has much to do with the destruction of rocks.

Water passing through rocks often takes a mineral in solution from one place and places it in another; this is one of the ways in which rocks are cemented. The water often produces chemical reactions and sometimes completely changes entire beds of rock, as, for instance, certain limestones have been changed to magnesian limestone or dolomite, while others have been changed to iron beds by the precipitation of siderite or some other salt of iron from

some solution of iron in water. Other limestones have been changed to gypsum.

Pure rain water exerts very little power as a solvent in the dissolving of most minerals, but from decaying vegetation and from the air it derives impurities, including carbonic-acid gas, or it may encounter alkaline substances that are easily dissolved. These impurities transform the water, in some cases, into a weak acid, and in other cases, into a weak alkali, in which condition it may attack the minerals directly.

50. **Organic rocks** comprise calcareous, silicious, and phosphate rocks, and plant deposits.

1. *Calcareous Rocks.*—The greater part of the limestone beds of the earth have been formed through the agency of animals. In the oceans, there are reefs built of coral fragments, which are made of carbonate of lime that the coral animals have abstracted from the ocean water. On a great part of the ocean floor, an ooze or limestone mud is now forming deposits like the chalk, which originated in a similar manner. In past geological ages, a species of animal, now very rare in the ocean, built limestone beds that are known as *crinoidal limestone.* Limestones are composed of carbonate of lime, sometimes accompanied by numerous impurities that color them. When clay is present in large quantities, the rock is called an argillaceous limestone, which grades into a calcareous clay rock. They usually contain many fossil fragments of shells or corals.

2. *Silicious Rocks.*—These rocks, of animal origin, are not common. **Infusorial earth** is composed of the skeletons of microscopic animals and is found in shallow lakes and beneath some of the swamps. **Diatomaceous earth** is a silicious rock, geologically, containing large numbers of silicious cells of a plant belonging to the group of diatoms.

3. *Phosphate Rocks.*—In some places in the United States, particularly near Charleston, S. C., and Florida, and in certain sections of Tennessee, the bones of marine and land animals of great size have accumulated into bone beds.

These rocks are phosphate of lime. They are known as **phosphate rocks** and are used to form the phosphoric acid of fertilizers. These phosphate rocks have no value for structural purposes.

4. *Plant Deposits.*—Vegetation takes carbon from the air and mineral substances from the water. With these substances they form their structure, which, on decaying, are returned in large part to the air of the earth. In swamps, where decay is retarded, the plant remains may accumulate in beds of peat which may later on become transformed into coal or mineral fuel, which is mostly composed of carbon. Peat, lignite or brown coal, bituminous coal, anthracite, and graphite are successive transformations of organic matter.

51. There are other forms of rock that are directly or indirectly of organic origin, while nearly all the sedimentary strata have some sedimentary animal or plant remains. Flint and chert are dense, hard layers of nodules of silica, some of which have been formed by silicious animals, although most of them appear to be of chemical origin.

52. Oolite is usually a limestone, although occasionally it is an iron or even a silicious rock. It is made up of minute grains resembling bunches of fish eggs, whence its name. Each of the rounded layers is made up of concentric layers like an onion. Oolite may be formed in three ways: by the action of lower forms of plant life, which build up the grains; by chemical deposit in water at the surface, as in the geyser origin of the Yellowstone; and by a chemical change, which causes a rock of a different origin to assume the conditions of an oolite. Oolite grains are now accumulating on many shores, as those of Florida and the Great Salt Lake.

Sedimentary rocks, particularly those that owe their origin to mechanical and organic agencies, are of the greatest importance to man, as they furnish him with most of his building stones. At present sedimentary rocks are forming, all over the globe, on land and in the sea. More than one-half of the earth's crust that is visible is made of sedimentary strata.

53. Metamorphic Rocks.—These rocks are an intermediate series between the stratified and unstratified rocks. They are stratified, banded, or foliated rocks, but crystalline in texture, like the igneous rocks, and are usually destitute of fossils. This banding, however, is quite different from stratification, for it is an arrangement of crystalline minerals, while in sedimentary rocks it is usually a banding of fragments arranged according to size or color, etc. The metamorphic strata result from complex changes of other rocks, in which the elements are often made to combine in a new manner. Given the same assemblage of elements, the effects of metamorphism will produce the same result; thus, a schist may be formed from either a shale or a lava.

Metamorphic rocks are supposed to have been formed from sediments, like stratified rocks, but to have been subsequently changed by (1) heat, (2) water, (3) alkali, (4) pressure, (5) crushing. To produce metamorphism by heat alone, that is, dry heat, requires a temperature of 2,500° to 3,000° F., but in the presence of water an incipient change begins at 400° F., while complete hydrothermal fusion takes place at 800°. If any alkaline carbonate be present in the water, these effects occur at a still lower temperature. Pressure is necessary because without it it is impossible to have even such moderate heat in the presence of water.

Metamorphic rocks may grade into stratified rocks on the one hand and into igneous rocks on the other. They cover a large area; the older rocks especially, are found along the axes of great mountain chains. The whole of Labrador, the larger portion of Canada, the whole eastern slope of the Appalachian system, and also the axes of the Colorado and Sierra ranges consist of them. In Canada, they are supposed to be 40,000 to 50,000 feet thick. They are very much crumbled.

54. Metamorphism is nearly always associated with great thickness and crumbling. Some rocks, such as the igneous, were originally solid; many, such as the fragmental and organic, were unconsolidated. Metamorphism solidifies these

rocks and in some places changes them entirely. As a result of these changes, the strata are so altered that their original condition cannot be told at a glance. Sandstone becomes a dense quartz rock, called *quartzite*, in which the sand grains may be no longer visible to the eye. A peat bog may be changed to anthracite or even to graphite. A dense, apparently structureless, limestone, may become transformed to a white or variegated marble composed of many crystals of calcite, or a clay stratum may be metamorphosed into a slate, in which the dense rock becomes harder, although at the same time an ability to split easily in one direction is introduced. This is called slaty cleavage and is one of the features of metamorphism. The cleavage is developed because of many plates of micaceous material formed in the rock as a result of heat or the mashing together of the whole rock mass in a direction at right angles to the cleavage plane. In this change one of the conditions is great pressure, and as the minerals develop they grow in a plane at right angles to the direction of the pressure because this is the plane of the least resistance, and because of the many cleavage planes of the newly formed micaceous material, slate splits easily along these lines.

55. Metamorphism of slate rock continued further would develop other minerals and would soon be altered in character and become a **schist,** in which the various minerals are arranged in bands having a cleavage no less marked than that of slate. A schist will split into layers much less easily and uniformly than the slate. Schist is known by various names, depending on the minerals present, such as mica schist, hornblende schist, chlorite schist, etc. All these rocks are characterized by the banding and the ease of splitting, which is due to the schistose structure arising from the banding of the minerals along these planes. As in the case of slate, these bands are at right angles to the direction of the pressure that was present during the metamorphism.

A final metamorphic change is one in which the original condition is hidden. The last stage before actual melting

takes place produces **gneiss**, which resembles granite, except that it has its minerals more or less perfectly banded, while granite is massive and without layers. In granite and gneiss, the minerals are frequently the same in kind and in general formation.

56. Where metamorphism has taken place in sedimentary rocks, the changes are sometimes so pronounced that the stratification is destroyed and the original nature of the rocks is so changed that it is impossible to tell whether they were originally sedimentary or igneous. Metamorphic rocks are found in the earth's crust in three positions: first, near some mass of intruded igneous rock; second, in the core of mountains deep below the original surface; third, among the most ancient rocks, particularly the Archæan, which are the lowest and the oldest of all the rocks. In the latter place the metamorphic rocks occur in great masses.

It was formerly believed that metamorphic rocks were part of the original crust of the earth and represented the most ancient strata, but this belief has been proved erroneous and it can now be shown that the rocks that have been very much altered belong to later ages and are transformations of sediments belonging to the same general age as deposits of shales, limestone, and sandstone in adjoining regions.

JOINTS AND FISSURES IN ROCKS

57. All rocks—igneous, sedimentary, and metamorphic—are divided into separable blocks of different sizes and shapes by cracks in various directions; these cracks are called **joints.** In stratified rocks one of the division planes is between the strata and the other two nearly at right angles to this. The different kinds of rocks have blocks of characteristic shapes and sizes. In sandstone, for instance, the blocks are usually very large and roughly prismatic; in limestone, they are usually very regularly cubic; in shale, oblong rhomboidal; in slate, small and sharply rhombic; in granite, sometimes large and roughly cubic, sometimes scaling in concentric shells producing domes; in eruptives, of many shapes, such

as roughly cubic, roughly spherical columnar, etc. These joints are supposed to have been formed by the shrinkage of the rocks; in stratified rocks, in consolidating from sediments; in igneous or metamorphic rocks, in cooling from a state of fusion or semifusion. In stratified rocks, the joints are usually confined to the stratum, though some of the larger joints (*master joints*) run through several strata.

Fissures are undoubtedly formed by the movements of the earth's crust. Joints are cracks in the individual strata, while fissures are fractures of the earth's crust extending through many formations and continuing for great distances. Owing to the great horizontal pressure, due to the shrinkage of the crust of the earth, the crust is sometimes thrown into ridges and hollows, thereby producing enormous fractures parallel to the axis of bending; thus mountain ranges are produced parallel to mountain ranges, while sometimes a system is at right angles to the main system.

Great fissures occur in systems, usually parallel to the axis of elevation or length, and frequently extend for hundreds of miles, and are miles deep. When filled, at the moment of formation, with fused matter from below, they form *dikes*, and all great dikes and igneous overflows have been through such fissures. If these fissures are not filled at once with fused matter, but are afterwards slowly filled with mineral matter, they form *fissure veins*.

PHYSICAL PROPERTIES OF BUILDING STONE

58. The structural manner in which the constituent parts of the rocks are grouped together bears a greater relation to the value or quality of the rock than the character of the minerals composing it; or in other words, the **physical characteristics** may be, and frequently are, more important than the chemical qualities.

59. Density.—The weight, strength, and absorptive properties of stone are dependent on the **density**. Thus, among rocks having the same mineral composition but

differing as to structure, generally the strongest will be the densest, and the heaviest will be the least absorptive.

60. Hardness.—The manner in which the mineral constituents of a rock are cemented to each other and the individual hardness of such mineral constituents determine the **hardness** of the rock as a structure. The minerals composing a rock may be hard but the rock itself as a structure will be soft if the particles do not strongly adhere to one another. Thus, some of the softest sandstones are composed of quartz, which is a hard mineral, but the grains are so weakly cemented together that the stone as a whole is soft.

61. Structure.—The structure of a rock depends on the form, size, and arrangement of its component minerals. All rocks may be approximately classified as *crystalline*, *vitreous* or *glassy*, and *fragmental*. Granite and crystalline limestone may be taken as types of the crystalline group; obsidian and pitchstone may be taken as types of the vitreous group; while the sandstones are types of the fragmental group.

62. Though all rocks have some common structural characteristics, certain peculiarities are found only in single types of rock. If the structure can be recognized by the unaided eye, the rock is said to have a *macroscopic structure*, and such rocks may be said to be granular, massive, polished, stratified, porphyritic, and concretionary.

The term *granular*, as its name implies, is applied to rocks built up of distinct grains of crystalline, or fragmental and water-worn character.

The term *massive*, or *unstratified*, is applied to rocks that are not arranged in any definite form in layers, or strata, but have the constituent parts mingled together, as in diabase and granite.

The term *polished*, or *schistose*, is applied to rocks that have their constituents arranged in definite planes nearly parallel to each other.

The term *stratified* is applied to rocks composed of parallel layers or beds, as is frequently seen in limestone and

sandstone. When the strata are thin and fine, the rock structure is said to be scaly or laminated.

The term *porphyritic* is applied to rocks that consist of a ground mass of fine or compact and evenly crystallized material, with larger crystals of feldspar scattered through it. A granite fragment has a porphyritic structure, but it is difficult to distinguish owing to the similarity of color existing between the crystals and the ground mass. In such rocks as the felsites, it is quite noticeable. In the porphyries of Eastern Massachusetts, the ground mass is of a black or no color and very compact and dense, while the large white crystal feldspars are in marked contrast.

The porphyritic structure is so noticeable that any rocks possessing this characteristic in a marked degree are commonly termed *porphyries* without regard to the mineral composition. The word *porphyry* is now commonly applied as an adjective, because any rock may possess this structure, whatever may be its origin or composition.

The term *concretionary* is applied to rocks composed of concretions or rounded particles built up by the collections of mineral matter around a center forming a rounded mass of concentric layers like the coating of an onion. When the concretions are small, like the roe of a fish, the structure is called *oolitic*, or if large like a pea, the structure is called *pisolitic*. The Bedford, Ind., limestones are examples of the oolitic type. The concretionary structure is rarely found in crystalline rocks.

63. Aggregation of Particles.—The hardness of the rock depends largely on the aggregation of the particles; therefore, the working qualities of the rock are fixed by the character of this aggregation. If the grains are loosely coherent in a rock composed of hard minerals it may work readily, while a rock consisting of softer materials may be worked with difficulty because the particles tenaciously adhere to each other.

The durability of a stone is, to a great extent, a matter of texture. If the grains adhere closely, the stone will be less

absorbent and less durable than one in which the adhesion is not so great, as in the friable and loose-textured rocks.

The kind of fracture shown by a rock is determined by the fineness or the coarseness of the grain and the relation of the particles to themselves or their state of aggregation Such rocks as flint, obsidian, and some varieties of limestone have a compact fine grain and show, on fracture, a concave or convex shell-like face of conchoidal form, and are difficult to dress. Other stones show, on fracture, a jagged surface or split along certain planes, all dependent on the aggregation of the particles.

64. Rift and Grain.—Rocks that do not possess rift and grain cannot be worked into rectangular form without great difficulty, unless they are of a very soft nature, but with these qualities the hardest rocks can be readily worked; for instance, the South Dakota quartzite, which is one of the hardest rocks known, can be as easily broken into pieces for paving as a soft sandstone or a granite.

The **rift** of a rock is a line of cleavage parallel to the bed and is visible in such rocks as mica schist, gneiss, and other sedimentary rocks. It is along these lines that the rock can be readily split. Rift, however, is commonly found in massive rocks, although it is not so easily discerned as in the examples cited. The **grain** of a rock is always at right angles to the plane of the rift or bed.

65. Color.—The chemical properties of a rock, as a rule, determine its **color**. The color of granites, however, is affected by the action of the light on the feldspars, which when, clear and glassy, absorb the light, making the rock apparently darker than when the feldspars are white and opaque and reflect the light.

Iron, the principal coloring matter in rocks, may be found in chemical composition with other minerals or in such simpler compounds as the sulphides and carbonates, or as an oxide distributed throughout the mass of rock. The brownish or reddish hues are due to the free oxides of iron while the bluish or grayish hues are caused by the carbonates

or the sulphides. The absence of iron in any of its forms is usually indicated by the white, or nearly white, color of the rock. The permanency of the color of the rock depends on the form in which the iron is found. Oxidation is likely to result if it is in the form of a sulphide, carbonate, or other protoxide compound. Therefore, stone containing these forms of iron is apt to fade and turn yellowish and stain on exposure. The sesquioxide, being in the last stages of oxidation, can undergo no further change from oxidation and is therefore a permanent color; hence, the decidedly red color may be considered permanent. The blue and the black colors of marbles and limestones are largely caused by the presence of carbonaceous matter, usually of vegetable origin.

SILICIOUS STONES

66. The granitic group of igneous rocks is richest in silica and therefore its members are known as **silicious stones.** Of this class, granite, syenite, gneiss, greenstone, and trap and the harder varieties of sandstone are most commonly used for structural purposes.

67. These stones compose the **primary rocks,** which are those rocks supposed to have been formed from the slow cooling of the incandescent earth. The granites are also unstratified, eruptive rocks, and underlie the stratified rocks. They are composed of an aggregation or assemblage of crystals of feldspar, quartz, and mica, the principal impurities being hornblende and talc: Quartz (pure silica, SiO_2) has a hardness of 7; feldspar (silica and aluminum, together with potash), a hardness of 6; hornblende, a hardness of from 5 to 6; the small scales of mica, a hardness of 3.

The colors of granite are white, grayish white, yellowish, reddish, rose, flesh color, or deep red, but rarely green. It is distinguished by its even and brilliant fracture, its pearly luster, and its outline, which is seldom regular, but in which may be recognized rectangles and parallelograms.

Granite varies in quality according to the proportions of its components and their method of aggregation. Stone

of the greatest durability and hardness contains a greater proportion of quartz and a less proportion of feldspar and mica. Hornblende renders the stone tough and heavy. Feldspar renders it lighter in color, easier to cut, and more susceptible to decomposition by the solution of potash contained in it. Mica renders it friable.

The granites are among the most valuable of the building stones and are extensively used in important works. They can be readily quarried and by reason of the lack of grain in the stone, blocks can be obtained of any size. On account of its great hardness, granite is difficult to work and therefore very costly to use if the stone is required to be cut. It weighs about 166 pounds per cubic foot.

Granite is found in the eastern part of the United States, in Canada, in many parts of the Alleghany and Rocky Mountains, and, as a rule, wherever the later rock formations and the underlying beds have been left exposed. It is generally classified into gray and red. Gray granite is found throughout New England, the border states, and in Virginia. Red granite is composed of red orthoclase (aluminum potassium silicates), bluish quartz, and a little hornblende, with very little mica. It is hard and takes a fine polish. It is found on the Bay of Fundy, in the islands on the St. Lawrence River, Virginia, Lake Superior, Maine, and many points in the Rocky Mountains.

68. Syenite.—This stone derives its name from Syene in Egypt. It consists of feldspar and hornblende, frequently associated with mica and quartz; is of a granular texture closely resembling granite; and is hard and tough, somewhat coarse-grained, and will not take a polish. It is one of the most durable of the granitic rocks when its feldspar constituent is not too readily decomposed by the removal of its potash when open to the weather. For this reason it should be carefully tested before it is used.

69. Gneiss and Mica Slate.—These are similar to granite in composition but differ from it in being stratified. Granite, syenite, and gneiss resemble one another so closely

that they are all frequently called granite by those not familiar with their characteristics.

Gneiss is not as valuable a stone as granite on account of its stratification, which will not permit it to be split in any direction. It is, however, a good building material and often answers as well as granite.

70. Greenstone, Trap, and Basalt.—These stones are igneous, unstratified rocks, consisting of hornblende and feldspar. The term *trap* has been suggested as a generic name for these rocks. The greenstone is not as coarse-grained as granite, and in the trap and basalt the granular structure is not apparent. The greenstone and trap break into blocks and the basalt into columns of prismatic form. They are found in veins and dikes and injected among the stratified rock of all ages.

These rocks vary in color, from nearly white in some varieties of greenstone, to nearly black, as in basalt, the difference of color being determined by variation in the proportions of hornblende, which gives a dark color, and feldspar, which gives a light color. The green is due to chromium. These stones, while making very durable building material, cannot be obtained in large blocks and are difficult to cut. Trap rock forms one of the best aggregates for use in making concrete.

SEDIMENTARY STONES

71. Sandstones.—Such material as sandstone consists of fragmentary rocks, composed mostly of grains of silica (or quartz), cemented together by a deposition of silica, carbonate of lime, oxide of lime, and aluminous matter. Sandstone is a stratified rock and belongs to the later geological periods.

If the cementing material is silica, the rock is very durable, but difficult to work. Iron oxide in the cementing material, consisting of carbonate of lime, and clayey matter, gives the stone a reddish or brownish color. Lime renders the stone particularly liable to disintegration when exposed

to an atmosphere containing gases, or when used for foundations where the soil is impregnated with acid water. The presence of clay or the oxide of iron is also deleterious

Sandstones are variable in character, some being nearly as valuable as granite, while others are practically useless for permanent construction. The best stone is characterized by small grains with a small proportion of cementing material, and when broken has a bright, clear, sharp fracture. It is usually found in thick beds and shows slight evidences of stratification. Water can readily penetrate between the layers of this stone; therefore, in foundations it should be laid on its natural bed so that the penetration of moisture and possible disintegration by freezing may be prevented as far as it may be possible.

Sandstone of good quality possesses strength and durability and can be readily cut and dressed, which qualities make it one of the most frequently used of our common building stones. When the grains are extremely small, it is termed a "freestone" because of the ease with which it can be quarried, cut, and dressed.

Sandstones vary much in color: The Ohio and Nova Scotia varieties are yellowish and cream color and sometimes nearly white; the Missouri sandstone is of a yellowish drab color and possesses durability; the Portland, Conn., Newark, N. J., Marquette, Mich., and Bass Island, in Lake Superior, sandstones are of a dark brownish-red color, which is due to the presence of iron, and are termed *brownstones*. The Potsdam, N. Y., red sandstone is durable, hard, highly silicious, and of a reddish color. The Hummelstown, Pa., sandstone has a brownish color.

72. Soapstone.—This stone is the silicate of magnesia and is found in many places in the United States. It possesses valuable qualities where a stone capable of resisting high temperature is required.

73. Argillaceous Stone.—Stones of this nature are generally weak and soft and are not durable when exposed to the weather. They are therefore of little value as building

stones. Clay slate is a sedimentary argillaceous rock, fine-grained, compact, and of a laminated structure. Its colors are usually dark purple, blue, and light green. The best varieties of clay slate are used for roofing and flagging.

CALCAREOUS STONES

74. Calcareous stones are composed largely of lime; therefore, limestones and marbles are the most familiar examples. Limestone is a carbonate of lime and effervesces when attacked by acids that are stronger than the carbonic acid in its composition, the weaker acid being rejected and new lime salts being formed. The carbonic acid can also be expelled by heat, in which case the product is caustic lime, commonly termed *quicklime*, which is uncombined lime. All varieties of calcareous stone are found in the United States. Extensive deposits are found in a line parallel with the Atlantic Coast; another deposit underlies the Middle States. The marbles are mostly confined to the mountainous districts, while the common limestones are frequently found in immense strata that have been deposited on the bed of an ancient ocean.

75. Limestones.—These common building stones are of various qualities, some being hard and strong while others are soft and friable. There are two sorts, the granular and the compact, from either of which excellent stones may be obtained. As they are usually easily worked, they are among the lowest-priced dressed stones in the building.

76. Marble.—Metamorphosed limestone gives masonry material known as **marble**, which is easily dressed to a smooth surface and polished. The granular varieties are generally superior to the compact for building purposes. The impure carbonates of lime are sometimes of great value as marble. The magnesian limestones, or the dolomites, are usually of excellent quality.

White marble is found in the Laurentian rocks, Canada, but much of that used in the Northern Atlantic States is obtained from the Green Mountains, which extend through

Vermont, Western Massachusetts, Western Connecticut, and Southeastern New York. Quarries exist at Granden, Rutland, Danby, Dorset, and Manchester, in Vermont; at Lanesborough, Lee, Stockbridge, Great Barrington, and Sheffield, in Massachusetts; at Canaan, in Connecticut; and at Pleasantville and Tuckahoe, in New York. The snowflake marble is obtained from the Pleasantville quarries, and a fine grade of statuary marble from Rutland, Vt. From this place toward the south, the marbles become coarser and harder and more suitable for building purposes. Dolomitic marbles are found in the southeastern part of New York and in Delaware. White dolomite marble is found in Maryland.

The colored marbles used in building construction are of several varieties and are found in Vermont, Connecticut, New York, Pennsylvania, and Tennessee. **Brecciated marbles,** that is, those in which the conglomerate fragments are angular instead of water-worn, are found in Vermont on the shores of Lake Champlain, and a dove-colored marble with greenish veins is found at Rutland. Black marbles are found at Shoreham, Conn., and Williamsport, Pa. Black Trenton limestone is found at Glen Falls, N. Y. The Warwick marble, found in Orange County, N. Y., is beautifully colored with carmine, with white veins. The Knoxville marble is of a reddish-brown color with lines of blue. Tennessee marble is brown and white mottled. The foreign marbles are largely imported from Italy, Spain, and Belgium. The Bardiglio of Italy is of a gray color shaded with black; the Siena of Spain is a pale yellow color; the Lisbon of Portugal a pale reddish color; and the Belgian, of Belgium, is black. Verde antique is composed of bands of serpentine and white marble.

77. Chalk.—Soft limestone in which the minute shells composing it have not been entirely destroyed by the pressure to which it has been subjected in early geological times is called **chalk.** It is not suitable for constructive purposes but is very useful in making lime and cement.

78. Quicklime.—This material is obtained by *calcination* from various limestones and is the basis of common mortar; the act, or operation, of calcination is the expelling, by heat, of some white substance by which the stone or the cementing material is broken down and reduced to a friable state. Limestones are necessary in the manufacture of iron, as they afford an alkaline base that unites chemically with the silica, alumina, and other impurities of the ores and allows the metal to separate into a state of approximate purity.

79. Gypsum, alabaster, or **plaster of Paris** is a sulphate of lime containing the water of crystallization. The term "plaster of Paris" is due to the fact that large deposits of this stone underlie the city of Paris. This natural sulphate of lime, when raised to a high temperature, loses its water of crystallization and is then ground into a fine powder. This becomes the plaster of Paris of commerce, which is used for molds, ornaments, and casts, as well as in wall plaster and staff. Gypsum is found in many parts of the United States, great quantities being found in the state of New York.

MANUFACTURED STONES

80. The artificial stones include *brick*, *firebrick*, *concrete*, and *terra cotta*. While there are several ornamental stones manufactured, they have little structural significance, and only brick, firebrick, and concrete, which are entirely materials of construction, will be discussed in this Section. Terra cotta is much used in modern construction to obtain decorative and architectural effect, but its chief structural use is in the construction of fireproof floors, partitions, and coverings.

81. Bricks.—As is generally known, **bricks** are made of burnt clay and are used extensively in all classes of building operations of permanency. They can be cheaply made, readily handled, and formed into structures of almost any desired form. Brick, if properly made, burned hard, and

laid up with Portland cement mortar, is one of the most durable building materials in use.

The clay of which the common brick is made consists principally of silicate of alumina, but usually also contains lime, magnesia, and oxide of iron; the oxide of iron gives the brick strength and hardness; silicate of lime renders the clay easily fusible and causes the brick to become distorted in the burning. Uncombined silica is beneficial, if there is not too much of it, as it preserves the shape of the brick at high temperature. If it is in excess, however, it renders the brick weak and brittle, because it destroys cohesion; 20 to 25 per cent. of silica makes a good proportion.

Bricks are made by hand and machine, the machine-made ones usually being the denser. They are burnt for about 2 weeks, first at a moderate heat, until all the moisture has been expelled, and then at a slowly increasing temperature until, at the end of 24 hours, the "arch bricks" attain a white heat, when the temperature is slightly lowered, but a constant high temperature is kept up until the end of the allotted time, after which the openings in the kiln are closed, the fire is drawn, and the kiln is allowed to cool very slowly. There are various kinds of kilns, some temporary and some permanent; the permanent kilns are the best and most economical.

Three kinds of bricks are usually taken from the kilns: those forming the top and sides of the arches in which the fire is placed are overburned and partly vitrified; the lower bricks in the arch are usually overburned on one end and underburned on the other and are called **arch bricks**; they are hard, brittle, and have little strength. Bricks from the interior of the pile are usually of the best quality in the kiln and are termed **body bricks**, sometimes **hard** or **cherry bricks**. Bricks from the exterior of the kiln are usually underburned and are called **soft** or **salmon bricks**. Salmon bricks are too soft for use in important places, as they are deficient in strength and will not resist the weather.

Bricks of good quality should be of regular shape with parallel surfaces, plane faces, and sharp square edges, of

uniform texture, and should be burned hard. They should be thoroughly sound, free from cracks, and should ring clearly when struck a sharp blow. They will fail under a compressive stress of about 10,000 pounds per square inch; soft bricks will not resist more than about one-tenth of this stress. Pressed bricks will bear about twice as much as good hard bricks. Hard, well-burned brick should not absorb more than 6 per cent. of its weight in water. Brickwork in masses will crush under a very much smaller load than a single brick, presumably because of the combined stresses due to bending, etc. The Watertown Arsenal tests of brick piers are worth careful study.

82. **Firebrick.**—Firebrick is usually made of a very pure clay with clean sand, or sometimes of pure silica cemented with a small proportion of clay. The clay should be silicate of alumina. Oxide of iron in the clay is very injurious, and if it reaches 6 per cent. the brick is not suitable for the purpose. Specifications for firebrick should require that the oxide of iron shall be less than this amount, and that the aggregate of lime, soda, potash, and magnesia shall be less than 3 per cent. The sulphide of iron or pyrites has a harmful effect on the fireclay, and brick containing it should not be accepted. An excess of silica in the brick makes it refractory in extremely high temperatures. Where the brick has to resist the action of metallic oxides, which would have a tendency to unite with silica, alumina should be in excess.

83. **Concrete.**—This name is given to any mixture of gravel, slag, cinders, or broken stone with cement and sand. The best concrete for heavy structural purposes has for its base broken stone or slag, while gravel and cinders are used for such work as filling in fireproof construction and for the foundation of cement pavements. An excellent concrete is made of cement and sand with a mixture of broken stone and gravel.

The broken stone, slag, cinders, or gravel forming the base of the concrete is known as the *aggregate*, while the

cementing material, composed of the cement and sand, is known as the *matrix*. The combination of the aggregate and the matrix forms, when the cement has attained its final set, an artificial conglomerate rock that is superior in strength and durability to many of the natural rocks or stones used for building purposes. The purpose of the aggregate is to provide strength and solidity to the mass and to make up the bulk as cheaply as possible, so that the amount of the cement mortar, which is the expensive material, may be reduced to a minimum. The matrix holds the aggregate together by its adhesion, and in a complete concrete must fill all *voids*, as the spaces that would exist between the parts of the aggregate are called.

In mixing concrete, water should be judiciously used, for the mixture should be pulverulent, or nearly so, rather than wet. An excess of water tends to weaken the concrete and prevents the proper distribution of the matrix throughout the mass of the aggregate.

Concrete should be put in place in layers of from 6 to 10 inches in depth and should be tamped until the moisture begins to appear on the top of the layer. In placing concrete, layer upon layer, the upper surface of the bed should be free from dirt and dust and moderately wet.

84. Stone and Gravel Concrete.—When concrete is composed of broken stone and gravel as a base, the stone, or gravel, and sand must be sharp and thoroughly clean in order that the concrete may attain its maximum strength. Sand containing loam, dirt, or dust, which will soil the hands when rubbed, should not be used in the manufacture of concrete where dependence must be placed on it for structural stability.

The proportion of parts for the ingredients composing stone and gravel concrete depend entirely on the purpose or use for which the material is required. In many instances, a *poor concrete*, that is, one in which a small quantity of cement is used in proportion to the stone, gravel, and sand, will fulfil the requirements and promote economy, while it is

frequently imperative that a *rich concrete* be used, or one in which considerable cement is employed in the mixture.

For structural work requiring strength and durability, engineers favor a mixture consisting of one part of Portland cement, two parts of sand, and five parts of broken stone that will pass through a 2- or 2½-inch ring. An excellent concrete is also made by using 5½ cubic feet of cement, 7 cubic feet of sand, and 27 cubic feet of broken stone. These proportions are just sufficient to make 1 cubic yard of concrete.

When gravel is used in making the concrete it assists in filling the voids and consequently decreases the amount of cement mortar required. Good proportions for concrete of this character consist of one part of Portland cement, two parts of sand, three parts of gravel, and four parts of broken stone. This formula, which is easily remembered, has been used by the engineer corps of the United States army in the construction of foundations. Rosendale or natural-rock cements may be substituted for Portland, if desired. Concrete composed of one part of natural-rock or Rosendale cement to two parts of sand and four parts of chips or broken stone is specified by many architects and was used in the construction of the East River Bridge. The mass of the foundation for the Statue of Liberty in New York harbor was made of two parts of Rosendale cement, two parts of sand, and seven parts of broken trap stone.

The modulus of elasticity of good stone concrete is about 700,000 pounds, while of neat cement it is approximately 3,000,000 pounds. The unit adhesive strength of stone concrete to rough iron or steel is about 600 pounds, while the adhesion of cement to brick or stone, in pounds per square inch, is approximately 15.

85. Cinder Concrete.—This material has fire-resisting qualities superior to stone concrete, owing to the fact that its mass is more porous and that the air spaces confined within are non-conductors and also because the cinders are more refractory than the broken stone commonly used

in stone concrete. Besides, it adapts itself more quickly to sudden changes in temperature and therefore is not as liable to crack, split, or disintegrate when rapidly cooled by streams of water used in the extinguishing of a fire.

Cinder concrete is light in weight, compared with stone concrete, and is used extensively for fireproof-floor construction both as a filling material and as a structural element of the construction, though the laying of any material in which the matrix is Portland cement should not be conducted when the thermometer is below 28° or 30° F. Cinder concrete, in case of necessity, can be laid at a much lower temperature, especially if it is being used in poor construction where the centers are so open as to allow the surplus water to drain away before freezing can take place in the body of the concrete.

86. Experience teaches that the rapid and complete destruction of iron or steel when in contact with damp ashes or cinders is certain. Even cast-iron pipes, which are less liable to corrosion than steel, running through cinder banks, oxidize rapidly and in a short time are unfit for use. It is only natural, therefore, that on this account cinders as aggregate for concrete should be looked on with some disfavor by those who contemplate using that material in conjunction with steel or iron.

The active agents producing corrosion are water, carbonic acid, and sulphuric acid. The water and carbonic acid occur in the atmosphere, and if allowed to come in contact with the bare metal will cause corrosion. Where the metal is embedded in the concrete, however, there is little chance for this to occur, and no more opportunity is offered by the use of cinder concrete than in that in which the principal ingredient is stone, the cement in any case being the protective material.

That cement will protect iron and steel from corrosion indefinitely is well known from observation and experiments; and if it protects the metal against the action of moisture and carbonic acid, it is reasonable to suppose that, if the cinders are embedded in a sufficient matrix of cement mortar,

there will be no action on the metal due to the sulphuric acid, or to sulphur as sulphide, in the cinders. Sufficient matrix in order to completely protect bare iron or steel, can only be had in complete cinder concrete, which is obtained by the proportions of 1 part of cement, 2.75 parts of sand, and 4.75 parts of cinders. Such a concrete is not so good a fire- and water-resisting material as that made of 1 part of cement, 2 parts of sand, and 5 parts of cinders, and is not as practicable, for it is heavier, and is not in accordance with the building ordinances, as usually framed in the principal cities.

Though the usual concrete in the proportion of 1, 2, and 5, will not entirely protect bare iron or steel, and some oxidation will take place more or less rapidly, depending on the absence or presence of moisture, it is hardly likely that the corrosion will be serious with the iron or steel used in the ordinarily dry atmosphere between the floors of a building. The concrete, however, contains considerable moisture when it is first laid, and an initial corrosion is likely to take place before the concrete has dried out. In order to avoid this initial corrosion, which is serious, for when it once starts it is liable to continue owing to chemical action and reaction, the iron or steel should be painted with some good paint, preferably one free from oils that are likely to be affected by acids and alkalies. Some paint of the asphaltum variety would probably be best.

Since meager concrete in the proportion of 1, 2, and 5 fails to entirely protect bare iron or steel, it is good practice, where possible, to cover the metal with a thin coat or layer of cement mortar of 1 part of Portland cement and 2.75 parts of sand, and then follow with the concrete. If this is done, the concrete will not come in contact with the metal, and the steel or iron will be fully protected from corrosion. which, though slight and probably locally confined, is likely to be instituted from the several causes previously cited.

FIRE-STONES

87. Fire-stones are stones capable of resisting the action of great heat without fusing, exfoliating, or cracking. Lime and magnesia, except in the form of silicates, are prejudicial to the quality of fire-stones; potash, also, is very injurious because it increases the fusibility of the stone, which, on melting, causes the formation of a fusible glass. Quartz and mica alone or in combination make the most refractory stones. Mica, slate, and gneiss make an excellent combination. Gneiss is particularly refractory when it contains a considerable portion of arenaceous quartz; that is, quartz in which the particles partake of the nature of sand.

Limestones do not stand well in the presence of high temperatures, as they sometimes explode, owing to the rapid expulsion of the carbonic-acid gas.

Granitic and other primary rocks usually contain some water, which, in the presence of fire, causes them to crack and sometimes explode.

Sandstones, if somewhat porous, uncrystallized, and free from feldspar, are the most refractory of the common building stones.

Firebrick is perhaps the most fire-resisting material now known, while common hard-burned brick is more refractory than any of the building stones.

Concrete made of Portland cement and stone is a fire-resisting medium of only fair value.

DURABILITY OF BUILDING STONE

88. In the structural use of building stones, it is seldom that the full safe strength of the stone is required to resist the stresses imposed, and consequently the range of choice is not limited by this consideration so much as by the factor of *durability*. While in architectural work color is of great importance, for on it the architect depends to a large extent for the success of his design, it is exceptional in purely structural work for the color of the stone to be a deciding factor.

The durability of a building stone depends not only on the physical and chemical formation of the stone, but to a considerable extent on the climate in which the structure is to be built and also the method employed in quarrying the material. Where a selection is to be made of two stones equally durable and structurally fit for their purpose, the economic consideration influences the choice. The cost of structural building stones is regulated by the difficulties of quarrying, the refractory nature of the stone in finishing, and the distance it must be transported.

89. Physical Structure.—The most durable building stones are generally of a compact and uniform texture and show a clean fracture free from earthy or soluble mineral matter. Stones showing lamination or layers are not likely to prove as durable as those of a more homogeneous structure, especially when laid with the laminations perpendicular to the bed of the wall, or on *edge*.

Non-porosity is not always a quality synonymous with durability, for many stones that absorb moisture also permit of its rapid evaporation; such stones are likely to prove more durable than those that absorb less moisture and part with it more reluctantly.

Stones showing a streaked appearance and lack of uniformity in color are usually composed of several minerals of various degrees of hardness, and in some instances one of them may be slightly soluble. Such stones are not likely to weather well, for the softer or more soluble mineral will be corroded and washed away, leaving the harder substance to protrude. When the less durable substance is in small pockets or spots, the stone will, on long exposure, be pitted; while if the softer mineral is in streaks or veins the material will be grooved, fissured, or channeled.

Small fossils or shells embedded in the substance of a building stone have usually a deleterious influence on its durability and weathering qualities. Such fossils and shells are calcareous in nature and generally soft and partially soluble under atmospheric influences.

90. Climate and Environment.—Building stones of the most durable character are required in climates where the diurnal changes in temperature are great and where there is much moisture in the atmosphere. The structure of a stone consists of minute particles that are surrounded by a matrix that forms the cementing material of the mass or are closely attached to each other by cohesion. In either case, changes in temperature cause these particles to expand and contract with considerable force, thus loosening particles from the matrix or from each other, causing deterioration and the ultimate destruction of the rock. The freezing of water in the pores of the stone or in the crevices and the spaces between the laminations in stratified stones, is the primary cause of the rapid destruction of some building stones. Water in freezing expands about one-tenth of its bulk and is said to exert a pressure of about 150 tons per square foot, which is sufficient, under favorable conditions, to split and rend the strongest rocks. The freezing of moisture within the pores of the stone is very deleterious to the stability of its structure, especially if there is not contained in the rock reserve pore space sufficient to accommodate the increased bulk of the water when frozen.

When water freezes in the crevices or spaces between the laminations, its action is that of an adjustable wedge tending to split the rock and widening the crevice more and more with each repetition of the freezing process. By this means, stones of laminated or stratified structure are particularly liable to disfigurement, when laid on edge by *exfoliation* or the scaling of the surface. In the large cities, it is not uncommon to observe balusters and carved details partially destroyed from this cause. The damages from the freezing of water in the spaces and crevices between the laminations is not as great when the stone is laid on its bed as when laid on edge, because there is not the same opportunity for the space lying in a horizontal plane to collect the moisture and also because the pressure on the stone from the superimposed masonry nullifies, to some extent, the wedging or bursting action of the freezing water.

The severest atmosphere on building stone is one that frequently and for long periods contains great quantities of suspended moisture in the shape of fogs and is also subjected, by environment, to much smoke and gas from the bituminous coals of manufactories. Such atmospheres are likely to contain carbonic-acid gas and sulphurous fumes, which have an appreciable effect on limestones and marbles. The actions from these sources affecting the durability of building stones are especially marked where the atmosphere is extremely moist.

91. Effect of Quarrying and Finishing.—Before stones are used in an important structure they must be thoroughly seasoned. When detached from the rock, stone is generally saturated with quarry water. It should, therefore, be exposed for some months, preferably under cover, to allow this water to evaporate. If the stone is not seasoned before it is placed in the wall of the structure, it is likely to remain peculiarly damp, and the excess of moisture, in freezing, will influence the durability of the material.

The use of heavy explosives for detaching dimensioned stone is detrimental to the quality of durability, from the fact that the severe concussion is likely to jar the particles and partially destroy their cementation and cohesion, producing incipient cracks and flaws that make the face of the stone more permeable to moisture and thus facilitating the deterioration of the stone by freezing and chemical action. For the same reasons stones sawed to size are more durable than those hammered and broken; and stones taken from the quarry by channeling or cutting are preferable to those procured by wedging.

92. Effect of Fire.—The fierce conflagrations that occur in the large cities frequently subject the stone walls of structures to intense heat. While stone is an excellent nonconductor, it is not as a rule as durable, when subjected to intense heat, as brick.

The severest test to which a stone can be subjected in a fire is for it to be heated intensely and then cooled by the

sudden application of water from the fire-hose. This rapid change of temperature causes the exterior heated layer of the stone to contract more rapidly than the mass, and from many stones under this condition, large pieces will crack and break off; the process being several times repeated results in the entire destruction of the stone.

The silicious sandstones are the least destructible by fire, while the granites and conglomerates are probably the most affected by intense heat and the sudden cooling incident to the application of water. Limestones are very refractory in temperatures less than 1,000°, and at this temperature are not liable to deterioration by sudden cooling, though above this temperature, they may be reduced to quicklime, which crumbles and falls away after a few weeks' exposure to the air.

STRENGTH OF STONES AND MASONRY

93. The resistance of stones to stress varies greatly, and the strength of masonry depends not only on the material of which it is composed but on the manner in which these materials are handled; that is, on the workmanship. Stones that are the densest usually possess the greatest resistance, and masonry composed of squared stones with close joints is the strongest.

Many tables, based on the results of tests, give the strength values of building stones, but they differ widely, the discrepancy being due to the following causes:

1. Samples are taken from different quarries or from different parts of the same quarry.
2. The pieces of stone used for testing are not uniformly seasoned.
3. Test pieces are of different sizes.
4. They are not uniformly dressed or finished.
5. Variations exist in the method of placing the test specimens in the machine.

Frequently stones quarried from different parts of the same bed will show 20 or 30 per cent. difference in their crushing resistance, and stones that have been quarried some

time and exposed will show a different resistance from those lately detached. The larger the test piece, the greater will be the unit stress developed, for small cubes do not develop as great a unit resistance as large ones, and within certain limits the unit stress that test cubes of the same material will sustain varies directly as the cube of the sides.

The method of finishing the test pieces and the accuracy and fineness with which the sides are dressed have much to do with the results of the test. Specimens that have been sawed to shape test higher than those that have been finished with a tool or chisel. Microscopic examination of the surface finished with a chisel reveals numerous minute cracks, caused by the excessive jars, that tend to reduce the crushing strength by starting fractures. The fineness of the surface finish also affects the result, from the fact that when the bearing surfaces are rough, transverse stresses that tend to disrupt the specimen are created.

94. It is well determined that from lack of homogeneousness or uniformity of texture, building stones and masonry of the same material have variable strength values. This uncertainty regarding the exact strength of masonry materials, together with their usually rapid deterioration, necessitates the use of a high factor of safety, so that in all work of this class minimum safety factors ranging from 10 to 20 are employed. When, therefore, the average strength values of commercial masonry materials are known and a high factor of safety is used, the basis on which the design is made is assuredly safe.

95. The average strength values for masonry materials, which are sufficiently conservative for good engineering practice, are given in the following tables:

TABLE I

STRENGTH OF BUILDING STONES AND MASONRY

Materials	Weight per Cubic Foot Pounds	Compressive Strength Pounds per Square Inch	Tensile Strength Pounds per Square Inch	Transverse Strength Pounds per Square Inch
Granite, Colorado	166	15,000		
Connecticut	166	14,000		1,500
Massachusetts	165	16,000		
Maine	165	15,000		
Minnesota	166	25,000		
New York	166	16,000	600	1,800
New Hampshire	166	12,000		
Sandstone, bluestone	160	15,000	1,400	2,700
Connecticut, Middletown	148	7,000	590	1,000
Massachusetts, Longmeadow, brown	142	10,000	450	
Massachusetts, Longmeadow, red	149	12,000	450	
New York, Hudson River		12,000		
New York, Little Falls, brown		10,000		
Ohio	139	8,000	100	479
Pennsylvania, Hummelstown, brown		12,000		
Limestone, New York, Kingston	168	12,000		
Limestone, New York, Garrison Station	164	18,000	Average	Average
Indiana, Bedford, oolitic	146	8,000	1,000	1,500
Michigan, Marquette	146	8,000		
Pennsylvania, Conshohocken		15,000		
Marble, Pennsylvania, Montgomery County		11,000		
Massachusetts, Lee, dolomite		22,800	Average	Average
New York, Pleasantville, dolomite		22,000	700	1,200
Italian	168	12,000		
Vermont	167	10,000		
Slate	160–180	10,000	10,000	5,000
Rubble, in lime mortar	150	500		

TABLE II

STRENGTH OF BRICKS, BRICKWORK, AND RUBBLE

Materials	Weight per Cubic Foot Pounds	Compressive Strength Pounds per Square Inch	Tensile Strength Pounds per Square Inch	Transverse Strength Pounds per Square Inch
Bricks, soft, inferior	100	1,000	40	
good, common	120	10,000	200	600
best, hard	125	12,000	400	800
paving	130	5,000		
Philadelphia, pressed	150	6,000	200	600
Brickwork, common, in lime mortar	120	1,000	50	
stretchers, in cement and lime	125	1,500	100	
best, hard in cement mortar	130	2,000	300	
Terra cotta	110	5,000		
Terra-cotta work	112	2,000		
Rubble	150	500		

TABLE III

STRENGTH OF CEMENTS, MORTARS, AND CONCRETE

Materials		Weight per Cubic Foot Pounds	Compressive Strength Pounds per Square Inch	Tensile Strength Pounds per Square Inch	Transverse Strength Pounds per Square Inch
Neat cement,	Portland, 1 month old		2,000	300	400
	Portland, 1 year old		3,000	550	800
	natural rock, 1 month old		1,200	150	200
	natural rock, 1 year old		2,000	400	400
	lime	59	600		200
	plaster of Paris	79	600	70	
Concrete,	Portland cement, 1 month old		1,000	200	100
	Portland cement, 1 year old	120–140	2,000	400	150
	natural cement, 1 month old	120–140	500	100	75
	natural cement, 1 year old				
Mortar,	lime	98	400	50	125
	natural cement and lime	100	600	75	200
	natural cement	102	1,000	145	102
	Portland cement	109	2,000	300	700
	plastering	86	400		

TABLE IV

ALLOWABLE UNIT STRESSES FOR MASONRY MATERIALS

Description of Material		Compressive Strength Pounds per Square Inch	Transverse Strength Pounds per Square Inch
Capstones,	bluestone	700	300
Templets,	granite	700	180
Monoliths,	limestone	500	150
	marble	400	120
	sandstone, other than bluestone	350	100
	slate	700	400
Squared-Stone	bluestone	350	
Masonry,	granite	350	
	limestone	250	
	sandstone, other than bluestone	175	
Rubble,	laid in Portland cement mortar	150	20
	laid in natural cement mortar	120	
	laid in lime-and-cement mortar	100	
	laid in lime mortar	80	
Brickwork,	laid in Portland cement; cement 1, sand 3	250	50
	laid in natural cement, cement 1, sand 3	150	40
	laid in lime and cement; cement 1, lime 1, sand 1	125	30
	laid in lime; lime 1, sand 4	100	15
Concrete,	Portland cement, cement 1, sand 2, stone 4	200	30
	Portland cement; cement 1, sand 2, stone 5	150	20
	natural cement; cement 1, sand 2, stone 4	125	16
	natural cement; cement 1, sand 2, stone 5	100	10

96. The values given in Tables I, II, and III are the average ultimate and breaking loads for the different materials. They are the results of tests made at different times on specimens prepared for the purpose. It will be noticed that the strength values of squared masonry are not given, and though conservative practice recommends that masonry of squared stone may be considered as having an ultimate strength equal to four-tenths of the strength of the stone, this is only an assumption that has not been substantiated by tests. The scarcity of reliable tests on masonry piers and walls is due to the fact that in order to obtain accurate results of the test, specimens must be of full-size dimensions and when thus built their strength is so great as to resist the ultimate power of the testing machine.

97. In using the values given in Tables I, II, and III, factors of safety of not less than 10 for compression and 15 and 20 for tension and transverse stress, respectively, should be employed. The usual practice in structural and architectural engineering is to use the allowable unit values for masonry and masonry materials given in Table IV. These values are considered good practice, and, in most materials, correspond with values recommended by the building laws of several cities.

SELECTION OF BUILDING STONES

98. In the building of important masonry structures, it is of primary importance that the stone employed shall be of sufficient strength and durability. Probably nothing in engineering construction is so neglected as the inspection of the building stone that is to be used.

Where it is necessary to employ great quantities of building stone in important situations, with reference to the stability of the structure, an inspection of the quarry from which the stone is to be obtained should be made. Besides, it should be the effort of the person who is to decide on the merits of the stone to inspect some building or structure that has been erected of the same material for a considerable length of time.

It is well, however, not to depend wholly on either inspection at the quarry or at the building but to subject the stone to laboratory tests, when it should be tested both chemically and physically as well as subjected to microscopic inspection.

99. The inspection at the quarry, when conservatively made, will frequently reveal the durability of the stone as well as its uniformity. Exposed quarry faces will sometimes show the weathering properties of the stone, besides its liability to disintegration through moisture and running water containing deleterious acids and alkalies. Such inspection will also determine whether there is sufficient stone of a uniform texture and color in sight to supply the amount of material required for the work. By quarry inspection likewise, the several grades of stone are known, and in first-class work it is imperative that the best grade or run of the quarry be insisted on. Frequently, the third-grade stone is employed in the structure, and on showing deterioration and poor weathering qualities causes otherwise excellent building stone, when of first-class cuttings, to be condemned.

100. By the inspection of stone that is in place in a building or structure for a considerable length of time, an excellent idea may be had of its durability as to structure, color, and weathering properties. If, after years of exposure in the severe atmosphere of an industrial city situated in a temperate zone, the building stone shows no disintegration or exfoliation and has retained its original luster and color, but for the soil of dust and smoke stains, due to its environments, it certainly can be considered of the best structural value for building purposes.

101. While the quarry and building inspections of stone are of the utmost practical importance, they should, wherever possible, be augmented by laboratory tests; in fact, these tests are absolutely necessary. When the stone to be used is from a new quarry, the characteristics of the product are little known. The laboratory tests usually consist of chemical analysis, microscopic examinations, and physical tests.

The *chemical analysis* determines both qualitatively and quantitatively the chemical constituents of the stone. Examined qualitatively, the mineral elements and chemical combinations comprising the stone, together with the impurities and original matter, are determined; while the quantitative analysis shows the proportion of the different elements and chemical combinations. When the chemical composition of a stone is in this way determined, conclusions can usually, though not always, be drawn as to the quality of durability and the weathering properties of the stone.

The *microscopic examination* of building stone is of even more importance and is less expensive to conduct than the chemical analysis, for by it is revealed the structure of the stone. By the microscope may be observed whether the stone is igneous, metamorphic, or unaltered sedimentary rock. It also shows the size and shape of the particles or crystals composing the stone, their relative closeness, and the character and compactness of the cementing material holding them together. Usually the mineral constituents of the stone may be determined likewise by microscopic examination and frequently their proportions may be estimated, together with the percentage of impurities contained in the stone. Likewise by the microscope may be detected any flaws in the structure, such as cracks, cavities, incipient fractures, and gas bubbles.

The *physical tests* of stone are of great practical value in determining its strength and durability. When the strength values of a building stone are not known, it is frequently necessary to make tests in order to determine the crushing and transverse unit stress of the material, for though the unit crushing strength of building stone is usually greatly in excess of that required, the transverse strength is frequently barely sufficient to sustain the load when a fair factor of safety is allowed. An estimation of the durability of a building stone may be deduced from data collected from physical tests made to ascertain the specific gravity, porosity, and weight of the stone and from such tests as will demonstrate the effects of extreme heat and cold and the actions

of carbonic and sulphurous acids. In making these last tests, the object is to impose on the stone, as nearly as possible, conditions that in a few hours' time will approach the effect produced by climatic changes and vitiated atmosphere through a lapse of years.

MORTARS

CHARACTERISTICS OF MATERIALS

102. Mortars for structural purposes are composed of lime or cement and sand mixed to the proper consistency with water. When lime and sand are used, the mixture is known as *lime mortar;* when the mixture is of cement and sand, it is designated as *cement mortar.* Frequently in building work, lime mortar is mixed with cement mortar, when the term *lime-and-cement mortar* is used. Small percentages of other materials, such as salt, sugar, brick, and volcanic dust, are sometimes mixed with mortar in order to accomplish specific purposes, such as the prevention of freezing, hastening of setting, increasing of strength, and the creating of hydraulic properties, though with doubtful effectiveness. In mixing mortar, it is usual to designate the amount of such ingredients by a notation, such as 1 to 1, 1 to 2, or 1 to 3, which signifies that the mortar is composed of one part of cement or lime to one, two, or three parts of sand, respectively. The first number of the notation always indicates the amount of cement, which for convenience is taken at unity. In proportioning the mixture, the parts of the ingredients are generally measured by volume and just sufficient water supplied to work the mortar to the proper consistency.

103. Sand.—This material is an important part of all mortar mixtures, for in lime mortar it prevents excessive shrinkage and adds crushing resistance and tenacity, while in cement mortar it is principally used for economic reasons and to provide additional resistance to crushing, though it decreases tenacity of cement.

It is important that sand for mortars shall be clean and sharp. *Clean sand* when rubbed between the fingers will not soil them, but sand containing impurities will leave an earthy stain. When clean sand is dampened and squeezed in the palm of the hand, on the release of the pressure, it will fall apart, while if it contains soil, clay, or earthy substance, the particles will be held together by being cemented by the impurities. Sand for structural purposes should be *sharp;* that is, the grains should be cubical and angular rather than globular or rounded. The sharpness is best tested by rubbing or rolling the sand between the hands and if there is a distinct grating sound, it indicates that the sand grains are angular. Sand that is sharp and angular lies closer together and offers more surface of adhesion for the cement composing the matrix of the mortar.

Much importance has unwarrantedly been placed on the size of the grains of sand for structural purposes. Very fine sand and sand so coarse as to contain grains larger than the joints of masonry should not be used. A sand containing a mixed size of grain is the most economical and the best for structural purposes. When the usual specification is complied with, namely, that the sand shall be sifted through a screen having a mesh $\frac{1}{8}$ inch square, grains of not more than $\frac{1}{16}$ inch on a side will be passed, and sand of such size gives excellent results.

The usual sources from which building sand is obtained are the seashore, the river bank, and sand deposits or pockets. The first, which is known as *sea sand*, is usually objected to on account of the salt that it contains and should not be used where the appearance of the work is a factor unless it is well washed. The second, which is termed *river sand*, is generally composed of rounded particles rather than angular and may be either clean or dirty. The third, or *pit sand*, is usually sharp, though it is likely to contain clayey and earthy substances, which destroy its cleanliness.

104. Lime.—By calcining or burning calcium carbonate or limestone at a high temperature, **quicklime** or

commercial lime is produced. In the process of burning, the water that the stone contains is driven off, together with carbonic-acid gas or carbon dioxide. Lime has a great affinity for water and should be used soon after being made, as it is likely to become *air-slaked*, when it is unfit for use in making lime mortar. When lime is brought in contact with water, it takes it up rapidly, swelling and generating considerable heat, the lime falling apart and producing a fine white powder; this process is known as *slaking*. When more water is added to this powder, a *lime paste* is formed, which by the addition of sand produces lime mortar. The best limes slake vigorously and completely, the *lime flour*, or precipitate, being fine and free from refractory lumps and cores. The presence of such unslaked fragments indicates that the limestone was not pure or that the process of calcination was imperfect.

In slaking lime, a volume of water equal to two or three times the volume of the lime is used. The entire amount of water necessary for slaking should be used at one time, as it is deleterious to turn in cold water after the lime has commenced to slake.

105. Cement.—Commercial cements used for making mortar for structural purposes may be classified as *Portland* and *natural-rock cements*. The difference in their manufacture consists more essentially in the method of procuring the raw material rather than in the process of preparation. The name **Portland cement** is given to any cement that is manufactured from the several necessary materials carefully gathered and proportioned, while the term **natural,** or **natural-rock, cement** is given to those cements that are manufactured direct from a natural rock containing the necessary ingredients.

Portland cement is obtained by mixing lime or marl with clay in the proper proportion. The mixture is made into balls, or nodules, which are calcined in a furnace and ground to a fine powder. The natural-rock cements are manufactured from a natural rock that contains lime, alumina, and silica.

Rocks containing the proper materials for making these cements are of a mixed limestone and clayey nature. These rocks are calcined at a high temperature approaching vitrification and are broken, pulverized, and finally ground to form the commercial cement. Both Portland and natural-rock cements are *hydraulic*, that is, will set under water, and are consequently invariably used for work in damp situations and below the water-line.

When cement is brought in contact with water, it hardens rapidly even when air is not present. This hardening or *setting*, as it is called, is caused by a chemical reaction that takes place and forms with the water of crystallization double silicates of lime and alumina. Some cements set more rapidly than others and cements that take an initial set in from 5 to 30 minutes, or even 1 hour after mixing, are known as *quick-setting cements*, while those that require from several hours to a day or longer, in order for the initial set to take place, are known as *slow-setting cements*.

Usually natural-rock cements set in less time than the Portland cements; besides they are lighter in weight and do not obtain the same degree of strength in as short a time after mixing. A barrel of natural-rock cement weighs about 300 pounds, while Portland cements are usually one-third heavier.

Portland cement is considered, in many respects, superior to natural-rock cements for structural purposes, yet in this country great quantities of natural cement have been used for the most important engineering work and have been found, after years of service, to have been exceedingly durable, attaining great strength.

The natural-rock cements are found and manufactured in New York, Pennsylvania, Maryland, Virginia, and Kentucky, as well as in several of the western states. The principal brands are many. Those having an extended reputation are known as Rosendale, Louisville, Cumberland, James River, and Round Top. The Portland cements that are used in this country are both domestic and imported. Some of the principal domestic Portland cements are known as the

Vulcanite and Alpha, manufactured in New Jersey; the Dexter, Lehigh, Atlas, and Whitehall, manufactured in Pennsylvania. The imported cements come from Germany, France, and England. Lagerdorfer German cement is used to some extent, it being a strong, finely ground, and uniform material. Vicat cement, which is imported from France, is a high-priced cement and owes its use in this country to the fact that it is not as likely to stain fine ashlar work as some of the domestic cements. English Portland cement is also used to some extent, though it is doubtful whether it possesses any qualifications of excellency over the best American Portland cements. The distinction that the New York building laws make with regard to brands of cements sold under the general nomenclature of Portland and natural is interesting, though it does not determine actually whether the cement is Portland or natural.

The requirements of the building laws are as follows:

"Cement classified as Portland cement shall be considered to mean such cement as will, when tested neat, after 1 day set in air, be capable of sustaining, without rupture, a tensile stress of 120 pounds per square inch, and after 1 day in air and 6 days in water be capable of sustaining, without rupture, a tensile stress of at least 300 pounds per square inch.

"Cements other than Portland cement shall be considered to mean such cement that when tested neat, after 1 day set in air, be capable of sustaining, without rupture, a tensile stress of at least 120 pounds per square inch."

The building laws continue to state that the test shall be made under the supervision of the Commissioner of Buildings, and that those tests shall be made at such a time as he shall deem advisable. They also require that records of the tests shall be kept for public information.

106. Lime Mortar.—Common mortar of quicklime and sand, though formerly used for all classes of work, is now supplanted by cement mortar for the better class of structural work. In mixing **lime mortar**, the lime is

thoroughly slaked to a powder and wetted to a paste when the sand is mixed and thoroughly incorporated with the paste by working and kneading. While lime on exposure to the atmosphere will become air-slaked and unfit for use, the slaked lime, when mixed with sand, is not so impaired and it is usually considered beneficial to mix up large batches of the mortar some time before using it, when it only requires tempering with water and some working to be ready for immediate use.

Sand is mixed with lime to form lime mortar in the proportions of one part of lime to from three to six parts of sand. With rich limes, that is, those that slake freely and completely, the proportion of three parts of sand to one of lime is not sufficient, and four parts of sand makes a better mixture. The proportion of six parts of sand to one of lime is never used except in the poorest work, and in fact the building ordinances of the several cities require that mortar shall be made of one part of lime and not more than four parts of sand, the lime to be in all cases thoroughly burnt, of good quality, and properly slaked before it is mixed with the sand.

Lime mortar sets by the drying out of the mortar and by a chemical action produced through the absorption of carbon dioxide, or carbonic-acid gas, from the air. By taking up this gas, the slaked lime is again converted to calcium carbonate, which surrounds each particle of sand, thus converting the mortar into artificial stone. Since the absorption of carbon dioxide is necessary, in order that lime mortar may harden, this mortar will not set in water and should not be used in foundation work below the water-line or in extremely damp situations. Lime mortar used in the interior of very thick walls has been known to remain soft for years; consequently, it cannot be used in concrete.

Though lime mortar will not harden in wet situations, the process of crystallization caused by the absorption of carbon dioxide in the air is facilitated by keeping the work damp for a short time after laying. For this reason, brickwork should not be laid in very hot or dry weather unless the bricks are thoroughly soaked in water.

When lime mortar does not retain sufficient moisture to properly promote the chemical action of hardening, it forms a chalky mass that is easily pulverized between the fingers, showing that it has lost most of its properties of adhesion and cohesion.

107. Cement Mortar.—Mortar composed of either Portland or natural-rock cement is being used extensively for all structural purposes, owing to the reduction in the price of these cements, and their excellent quality.

In this mortar, only cement and sand are used with just sufficient water to mix it to a mealy consistency; an excess of water is detrimental to its qualities of strength. The usual proportions of sand and cement, when natural-rock cements are used, are one part of cement and one or two parts of sand, the usual practice being to use the latter proportion. In mixing cement mortar with Portland cement, it is customary to mix the cement and sand in the proportion of 1 to 3, for this mixture is considered equal in strength and durability to a mixture of one part of natural rock cement and two parts of sand.

The New York building laws specify that cement mortar shall be made of cement and sand in the proportion of one part of cement and three parts of sand and shall be used immediately after being mixed. The cement and sand are to be measured and thoroughly mixed before adding water. They also stipulate that the cements must be finely ground and free from lumps. The usual practice in making cement mortar is to mix the sand and cement together dry, measuring the parts by volume, adding the water as required in order to reduce the mixture to work consistently, care being exercised in supplying the water that the amount added is not excessive.

Owing to the fact that cement mortar sets rapidly, it should never be mixed, except immediately before being used; and after a batch of cement mortar has once taken its initial set no attempt should be made to retemper it, but the whole mass should be discarded. The natural-rock cement

mortars set more quickly than Portland cement mortars, but with care can be conveniently used in any class of work. Both natural-rock and Portland cement mortars have hydraulic properties and set rapidly and completely under water. This property of hydraulicity is not destroyed by the addition of lime in the proportion of not more than one part of lime to two of cement. Because of the hydraulic properties of cement mortar and on account of the great strength that it attains when it sets in water, this mortar should always be used for masonry in damp situations or for foundations that are likely to be immersed in water. The usual cement mortars are durable even when submerged in sea-water.

One barrel of cement to two barrels of sand will make about 8 cubic feet of cement mortar and will lay 1 cubic yard of brickwork or rubble, while one barrel of cement and three barrels of sand will make about 12 cubic feet of mortar, which will lay 1½ cubic yards of ordinary masonry, or provide sufficient matrix for 1½ cubic yards of concrete. The quantity of water that is required in mixing cement mortar varies considerably with the temperature and with the dryness of the materials. Ordinarily, one-third of a barrel of water mixed with one barrel of cement, will make two-thirds of a barrel of stiff paste.

In making cement mortar, sometimes the cement is mixed to a paste with the water and the sand added, but the usual practice is to mix the cement and sand dry and to introduce sufficient water in order to work it to a mealy and pasty condition. In the manufacture of all cement mortar, the sand and cement must be thoroughly mixed, as otherwise the substance will not have a uniform texture and its mass will incorporate spots of dry cement, and frequently there will be pockets of sand without the matrix of cement.

There is much difference of opinion as to the advisability of using cement mortar in freezing weather. Undoubtedly, the best practice is to suspend all masonry operations when the thermometer is below freezing, but when necessary, however, cement mortar may be used in temperatures several degrees below freezing. It is well, however, in laying work

with cement mortar in freezing weather to heat the sand, brick, and stone, if possible, and to use hot water in mixing. With ordinary precautions and by covering the work when operations are suspended at night, cement mortar may be used in low temperatures with safety. The frost may have some action on the exterior edges of the joints, but the action will not penetrate to the interior of the wall.

Some authorities recommend the introduction of salt in cement mortar when it is used in freezing weather. This practice is not to be recommended, however, for the salt retards the setting and is liable to stain the masonry or brickwork.

108. Cement-and-Lime Mortar.—Frequently mortar for building work is composed of a mixture of both cement and lime with sand. The lime introduced with cement in mortar makes the material more tractable by increasing the plasticity of the mortar, and thus facilitates the work of laying the masonry or brickwork. It also retards the setting, which is frequently requisite, and lowers the freezing point of the mortar. Besides, it does not shrink, like lime mortar, in setting and is therefore desirable in laying the backing of ashlar walls. Its use also promotes economy, for lime is much less costly than cement.

The proportion of the ingredients used in mixing this mortar varies considerably with the class of work and the judgment of the designer. For brick backing, piers, and walls, one part of Portland cement, two parts of good, fresh, wood-burned lime, and three parts of clean, sharp sand will give satisfaction. When this mixture is to be used for ashlar facing one part of Vicat Portland cement, two parts of good, fresh, wood-burned lime, and three parts of clean, sharp sand, mixed together as soon as prepared, have been found to give excellent results. Frequently only a small percentage of lime is added to the cement mortar to give it plasticity, and in this instance the specifications usually require that the mortar shall be composed of one part of Portland cement, three parts of clean, sharp sand, and just sufficient lime to make the mortar pasty. Lime-and-cement

mortar is admirably adapted to the construction of brick arches used in fireproof-floor construction.

The New York building laws stipulate that cement-and-lime mortar mixed shall be made of one part of lime, one part of cement, and not more than three parts of sand to each.

In mixing lime-and-cement mortar, the slaked lime is mixed in bulk with the sand and the cement added, after tempering it with water, immediately before being required. The lime mortar and the cement mortar made in this way must be thoroughly incorporated by working, though this must be done rapidly before the cement has had time to set.

109. Comparison of Lime and Cement Mortars.—It is interesting to compare lime and cement mortars so as to see wherein one is more advantageous than the other. The points in favor of lime are: its cheapness; the large quantity of mortar that can be made from a given quantity of lime; the fact that if, before use, it is kept damp, it remains good indefinitely; and the ease and simplicity of mixing. Against its use may be mentioned these points: in damp situations, it will not harden; in the interior of walls of any considerable thickness, it hardens (if at all) but very slowly; and it is much weaker than cement.

Among the advantages of cement mortar are: its hydraulic quality, in virtue of which it hardens even quicker in water than in air; its greater adhesive power; its rapidity of setting; and the great ultimate strength that it attains. The points against it are: the extra care required in mixing and handling; and its greater cost, which is, of course, the controlling factor. Summing up, it is evident that cement is greatly preferable to lime, and the quality of masonry laid with it is so much superior as to more than counterbalance the difference in cost over lime mortar.

Cement mortar possesses an advantage over lime mortar, in that in setting it does not shrink. Because of this property, and because of the fact that cement mortar sets rapidly, walls can be laid up in cement mortar much more rapidly than they can in lime mortar and with greater safety.

A mixture of cement and lime mortars is frequently used, and is a considerable improvement on ordinary lime mortar, in that, if the quality of lime is not greater than one-fourth or one-fifth that of the cement, the strength will be practically that of cement mortar, and the mixture will set quickly in damp places, while the cost will be materially less than that of cement mortar.

It may be interesting to compare the relative cost of masonry laid in lime and in cement mortar. For this purpose the following figures are given, which, being obtained from a very reliable source, will undoubtedly be found quite accurate. To work from a common basis, the estimates are made per 1,000 brick, and per perch of rubble masonry, all laid in 1-to-3 mortar. The quantities given, of course, remain constant, while the prices vary according to local rates. These figures represent actual cost.

BRICKWORK IN LIME MORTAR

1,000 brick	$6.00
3 bushels lump lime, at $.25	.75
½ load (½ cubic yard) sand, at $1.50	.75
Bricklayer, 7 hours, at $.35	2.45
Helper, 7 hours, at $.15	1.05
Cost, per 1,000 laid	$11.00

BRICKWORK IN CEMENT MORTAR

1,000 brick	$6.00
1½ barrels Rosendale cement, at $1 50	2.25
½ load sand, at $1.50	.75
Labor, same as before	3.50
Cost	$12.50

NOTE —If Portland cement is used, the cost will be about $13 63 per thousand brick.

RUBBLE MASONRY IN LIME MORTAR

1 perch stone	$1.25
1 bushel lime	.25
⅙ load sand, at $1.50	.25
Mason, ⅓ day, at $3.00	1.00
Helper, ¼ day, at $1.50	.38
Cost, per perch	$3.13

RUBBLE MASONRY IN CEMENT MORTAR

1 perch stone	$1.25
½ barrel Rosendale cement, at $1.50	.75
⅙ load sand, at $1.50	.25
Labor, same as before	1.38
Cost	$3.63

NOTE.—Using Portland cement, at $2 25 per barrel, the cost per perch will be $4.01.

From these figures, it is evident that a considerable difference exists between the cost of lime and cement mortars, in favor of the former. A large part of this difference may be overcome by replacing some of the cement by lime, which will cheapen the mortar, and at the same time give it the valuable qualities possessed by both materials.

MATERIALS OF STRUCTURAL ENGINEERING

(PART 3)

TIMBER AND METALS

CHARACTERISTICS OF TIMBER

1. **Wood,** as a building material, is divided into three general groups; namely, the *evergreen*, the *tropical*, and the *hardwood*. In the first of these are classed pine, spruce, hemlock, cedar, cypress, etc.; in the second, palm, rattan, bamboo, etc.; in the third, oak, chestnut, walnut, locust, maple, hickory, ash, boxwood, whitewood, and a number of others. Each of these woods has peculiarities and characteristics which render it fit and useful for some building purposes, and utterly unfit and useless for others.

2. **White pine,** commonly known as *pine*, or sometimes referred to as *northern pine*, to distinguish it from the species described below, is a tree common in the northern part of the United States and in Canada. It furnishes a light, soft, and straight-grained wood of a yellowish color, but is not so strong as other woods of the same class, and in building is used principally as a finishing material, where a good, durable, but inexpensive job is required. As a material for patternmaking it has no equal, and its power of holding glue renders it invaluable to the cabinetmaker and joiner.

3. **Georgia pine,** also known as *hard pine*, *pitch pine*, and occasionally as *long-leafed pine*, which is really the

best name for it, is a large forest tree growing along the southern coast of the United States, from Virginia to Texas, and extending only about 150 miles inland. Its annual rings are smaller than those of the white pine, and have a dense, dark-colored, resinous summer growth, which gives the wood a well-marked grain.

The wood is heavy, hard, strong, and, under proper conditions, very durable. For heavy framing timbers and floors it is most desirable, and on account of its grain is sometimes used for the trim of unimportant rooms. It rapidly decays in a damp location, and therefore cannot be used for house sills, or as sleepers or posts that are in contact with the ground, but if situated in a dry, well-ventilated place, it will remain practically unchanged for over a century.

Great care should be exercised in obtaining Georgia pine, as in many localities this wood is confused with another material variously known as *Carolina pine* and *Northern yellow pine*, which is greatly inferior to it in every respect.

The *Carolina pine* is not a long-leafed pine, and is neither so strong nor so durable as the Georgia or Southern pine. In appearance it is somewhat lighter than the long-leafed pine, and the fiber is softer and contains less resin than the regular hard, or pitch, pine.

4. **Spruce** is a name given to all the wood furnished by the various species of the spruce fir tree. There are four varieties of the wood, known as *black spruce*, *white spruce*, *Norway spruce*, and *single spruce*.

Black spruce grows in the northern half of the United States and throughout British America. Its wood is light in weight, reddish in color, and, though easy to work, is very tough in fiber and highly desirable for joists, studs, and general framing timber. It is also greatly used for piles and submerged cribs and cofferdams, as it not only preserves well under water but also resists the destructive action of parasitic crustacea, such as barnacles and mussels, longer than any other similar wood.

White spruce is not so common as the black variety, though, when sawed into lumber, it can scarcely be distinguished from it. Its growth is confined to the extreme northern part of the United States and to British America. Another variety of white spruce is a large-sized tree growing in the central and southern parts of the Rocky Mountains, from Mexico to Montana.

Norway spruce is a variety growing in Central and Northern Europe and in Northern Asia, and its tough, straight grain makes it an excellent material for ships, masts, spars, etc., as well as the more ordinary purposes of house building. Under the name of *white deal*, it fills the same place in the European woodworking shops as white pine does in America.

Single spruce grows in the central and the western part of the United States. It is lighter in color, but otherwise its properties are similar to the black and the white spruce.

5. Hemlock is similar to spruce in appearance, though much inferior as a building material. The wood is very brittle, splits easily, and is liable to be *shaky*. Its grain is coarse and uneven, and though it holds nails much more firmly than pine, the wood is generally soft and not durable.

Some varieties of it are better than others, but in commerce they are so mixed that it is difficult to obtain a large quantity of even quality. Hemlock is used almost exclusively as a cheap, rough framing timber.

6. White cedar is a soft, light, fine-grained, and very durable wood, but lacks both strength and toughness. Its durability makes it a desirable material for shingles, and also for tanks in which water is stored; these are about the only purposes for which it is used in building construction, though it is used largely in boat building, cigar-box manufacture, and cooperage.

7. Red cedar is a smaller tree than white cedar, and of much slower growth. The wood is very similar in texture to white cedar, but even more compact and durable. It is of a reddish-brown color, and possesses a strong, pungent

odor, which repels insects. Its extreme durability makes it valuable for posts, sills, sleepers, etc. in contact with the ground, and its strong odor renders it extremely serviceable as shelving for closets and linings for chests and trunks, where the exclusion of moths and other insects is desired.

8. Cypress is a wood very similar to cedar, growing in Southern Europe and in the southern and western portions of the United States. It is one of the most durable woods, and is well adapted for outside use.

In the northern part of the United States its use is confined almost exclusively to shingles, but in the South it is used as extensively as pine is in the North.

9. Redwood is the name given to one of the species of giant trees of California, and is the most valuable timber grown in that state. It grows to a height of from 200 to 300 feet, and its trunk is bare and branchless for one-third of its height. The color is a dull red, and while the wood resembles pine and is used generally in the West for the same purposes as pine is in the East, it is inferior to pine on account of its peculiarity of *shrinking lengthwise* as well as crosswise. It is used largely for railroad ties, fence posts, telegraph poles, and other purposes where durability under exposure is required. As an interior finishing material it is highly prized, as it takes a high polish, and its color improves with age.

10. The **hardwood** group is headed by the **oak** as typical of its class, nearly all others being compared with it in regard to hardness, durability, and strength.

White oak is the hardest of the several American species of the oak tree, and it grows in abundance throughout the eastern half of the United States. It furnishes a wood that is heavy, hard, cross-grained, strong, and of a light yellowish-brown color. It is used where great strength and durability are required, as in framed structures, ship building, cooperage, and carriage making.

Red oak is similar in nearly every respect to white oak, except in its grain and color, the grain usually being

coarser and the color darker and redder. It is also about 12 per cent. softer.

English oak is similar to the American oaks in color, texture, and appearance, but is superior to them for such structural purposes as ship building and house framing.

The structure of the fiber, and the large, thick, and numerous medullary rays, make oak especially prized as a material for cabinetwork and furniture when the log is quarter-sawed. The silver grain and the high and durable polish that the wood is capable of receiving, make it one of the most beautiful used in joinery and cabinetwork.

11. Ash, the wood of a large tree growing in the colder portions of the United States, is heavy, hard, and very elastic. Its grain is coarse, and its color is very similar to that of red oak, which it also resembles in strength and hardness.

Ash is sometimes used for furniture and cabinetwork, making a good imitation of oak, but it is never so strongly marked in the silver grain as oak, and its tendency, after a few years, to become decayed and brittle renders it unfit for structural work.

12. Hickory is the heaviest, hardest, toughest, and strongest of all the American woods. The medullary rays are very numerous and distinct, and produce a fine effect in the quarter-sawed plank. The flexibility of the wood, together with its toughness and strength, render it valuable in the manufacture of carriages, sleighs, and implements requiring bent-wood details.

As a building material, it is unfit for use: first, on account of its extreme hardness and difficulty of working; and second, on account of its liability to the attacks of boring insects, even after the fibers have been filled and varnished.

13. Locust is one of the largest forest trees in the United States, and furnishes a wood that is as hard as white oak. It is composed of very wide annual layers, in which the vessels are few, but very large, and are arranged in rows, giving the wood a peculiar striped grain. Its principal use is in exposed places where great durability is required, while

for posts for buildings and fences in damp locations it has no superior. Its hardness increases with age, and on this account it is used for turned ornaments and occasionally in cabinetwork.

14. **Maple** is a large-sized timber tree that furnishes a light-colored, fine-grained, hard, strong, and heavy wood. The annual growth is narrow and close, but on careful examination small vessels may be seen scattered through it. The medullary rays are small and distinct, giving to the quarter-cut lumber a clearly defined silver grain. Two other characteristics of the grain are observed, especially in old trees, and are known as *curly maple* and *bird's-eye maple*. The former is a waviness of the grain similar to the burl obtained from the root timber of the walnut tree, while the bird's-eye is an effect produced in old trees by the circular inflexion of the fibers. The plank appears to be covered with numerous small spots, similar to minute knots, and strongly resembling birds' eyes, whence it derives its name. Though the appearance of both the curly maple and the bird's-eye maple is practically due to distorted fibers, which materially reduce the strength of the wood, they are highly prized in the cabinetmaker's art, as they lend to the polished surface a variegation and impart a beauty equaled by few other materials.

15. **Chestnut,** a large forest tree common to the eastern part of the United States, produces a comparatively soft, coarse-grained wood that, though very brittle, is exceedingly durable when exposed to the weather. It will not stand variations of slowly evaporating moisture as well as locust, and is therefore not so well suited for fence posts and sills laid in contact with the earth; but for exposed structures and sleepers laid in concrete or sandy soil, it affords a material much more easily worked than locust and nearly as durable as cedar.

At the age of 50 years the tree is in fine condition for cutting, previous to which the wood is likely to be composed of large cells filled with moisture, that do not dry out without

impairing the quality of the timber. On the other hand, if the tree is not cut at 50 years, it is almost sure to become decayed in the heart wood, and thereby rendered unfit for use.

16. Beech is the wood of a large forest tree growing in the eastern part of the United States, and in Europe. It is used but slightly in building, owing to its tendency to rot in damp situations, but it is often used, especially in European countries, for piles, in places where it will be constantly submerged. It is very hard and tough, and of a close, uniform texture, which renders it a desirable material for tool handles and plane stocks, a use to which it is often put. It is occasionally used for furniture on account of its susceptibility to a high polish, but is too brittle for very fine work requiring strength.

17. Whitewood, so called from the purity of its color, is the lumber of the tulip tree, a large, straight forest tree abundant in the United States. It is light, soft, very brittle, and shrinks excessively in drying. When thoroughly dry it will not split with the grain, and in even slight atmospheric changes will warp and twist exceedingly. Its cheapness, ease of working, and the large size of its boards cause it to be used in carpentry and joinery, in many places where it is utterly unsuited.

18. Buttonwood, also called **sycamore,** is the name given in the United States to the wood of a species of tree generally known as **plane tree.** The wood is heavy and hard, of a light brown color, and very brittle. Its grain is fine and close, but, though susceptible to a high polish, it is not much used in general carpentry or joinery, as it is very hard to work and has a strong tendency to warp and twist under variations of temperature. In damp places it will soon show signs of decay and is therefore unfitted for any but the most protected positions.

19. Lignum vitæ is an exceedingly heavy, hard, and dark-colored wood, with an almost solid annual growth. It

is very resinous, difficult to split, and has a soapy feeling when handled. Its color is dark brown, with lighter brown markings, and it is used mostly for small turned articles, tool handles, and the sheaves of block pulleys.

STRENGTH VALUES

20. The **ultimate strength** of any material is that unit stress that is just sufficient to break it.

The **ultimate elongation** is the total elongation produced in a unit of length of the material having a unit of area, by a stress equal to the ultimate strength of the material.

21. Modulus of Rupture.—When a simple beam breaks, the fibers at the top are in *compression*, and those at the bottom in *tension*, as shown in Fig. 1. By actual tests, it has been found that though some of the different fibers of materials under transverse stresses are in compression and some in tension, the ultimate resistance of the material does not agree with the ultimate resistance of the fibers to either tension or compression. Though many attempts have been made to account for it, this fact remains; hence, it becomes necessary to obtain some constant, or value, more closely agreeing with the strength of materials under transverse stress. It is usual, therefore, where the cross-section of the beam is uniform, to obtain, by actual tests, the constant or value for each material. This value is called the **modulus of rupture** and is generally expressed in pounds per square inch.

FIG. 1

22. Table I gives values of the strength of timber when subjected to different stresses. The column in the table, headed Compression, With Grain, will be found useful in computing the strength of columns. The values in the column headed Compression, Across Grain, are used in cases similar

TABLE I

AVERAGE ULTIMATE BREAKING UNIT STRESSES, IN POUNDS, PER SQUARE INCH

Kind of Timber	Tension		Compression			Transverse		Shearing	
	With Grain	Across Grain	With Grain: End Bearing	With Grain: Columns Under 15 Diams	Across Grain	Modulus of Rupture	Modulus of Elasticity	With Grain	Across Grain
White oak	10,000	2,000	7,000	4,500	2,000	6,000	1,100,000	800	4 000
White pine	7,000	500	5,500	3,500	800	4,000	1,000,000	400	2,000
Southern long-leaf or Georgia yellow pine	12,000	600	8,000	5,000	1,400	7,000	1,700,000	600	5,000
Douglas, Oregon, and yellow fir	12,000		8,000	6,000	1,200	6,500	1,400,000	600	
Washington fir or pine (red fir)	10,000					5,000			
Northern or short-leaf yellow pine	9,000	500	6,000	4,000	1,000	6,000	1,200,000	400	4,000
Red pine	9,000	500	6,000	4,000	800	5,000	1,200,000		
Norway pine	8,000		6,000	4,000	800	4,000	1,200,000		
Canadian (Ottawa) white pine	10,000			5,000				350	
Canadian (Ontario) red pine	10,000			5,000		5,000	1,400,000	400	
Spruce and eastern fir	8,000	500	6,000	4,000	700	4,000	1,200,000	400	3,000
Hemlock	6,000			4,000	600	3,500	900,000	350	2,500
Cypress	6,000		6,000	4,000	700	5,000	900,000		
Cedar	8,000		6,000	4,000	700	5,000	700,000		1,500
Chestnut	9,000			5,000	900	5,000	1,000,000	600	1,500
California redwood	7,000			4,000	800	4,500	700,000	400	
California spruce				4,000		5,000	1,200,000		
Factor of safety	10	10	5	5	4	6	2	4	4

to Fig. 2, and are such as will not produce an indenture of more than $\frac{1}{100}$ inch in the surface of the timber, a value well within the safe limit. The values under the heading, Shearing, With Grain, are used in computing the strength of the end of the tie-beam at the heel of the main rafter in a

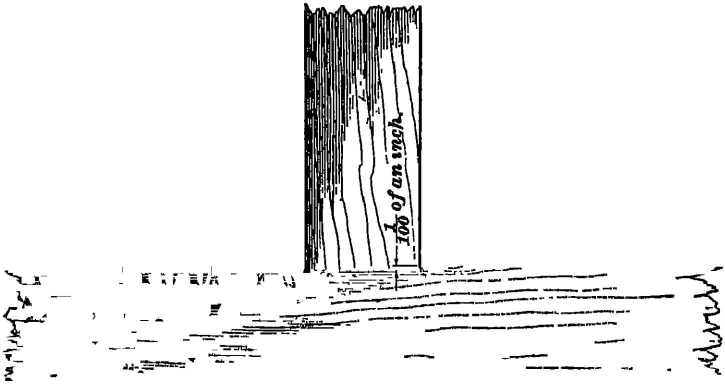


FIG. 2

roof truss, as shown in Fig. 3. The tendency is to shear off the piece *h* parallel to the grain along the line *a b*. The figures in the column headed Transverse, Modulus of Rupture, are the constants, or values, used when computing the strength of beams.

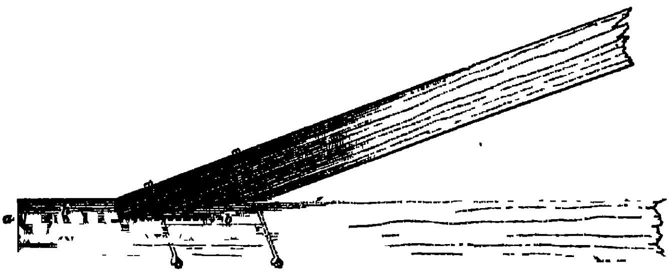


FIG. 3

The values in Table I were recommended by the committee on Strength of Bridge and Trestle Timbers of the Association of Railway Superintendents of Bridges and Buildings in its fifth annual convention, which was held in New Orleans, October, 1895.

CHARACTERISTICS OF IRON AND STEEL

23. **Iron** is very widely distributed in nature and its compounds are abundant. Probably no portion of the earth's crust is free from it, yet it occurs native only in very small quantities, and the iron thus found is probably of meteoric origin and is always alloyed to some extent with other metals, as nickel, cobalt, copper, etc. Its strong affinity for non-metals explains its infrequent occurrence in the native condition; and the dissimilarity between the metal and its ores may explain why it was among the later useful metals to be discovered, if, as is generally believed, such is the case. It may be mentioned, however, that some writers think that iron was known and used at a much earlier period in the world's history than is generally believed, but that its tendency to corrode has destroyed all traces of its use in ancient times, while instruments of brass and bronze remain.

Chemically pure iron is valuable only for experimental purposes and as a curiosity, as it has no use in the arts except, perhaps, in medicine. It may be obtained on a small scale in several ways, among which may be mentioned the reduction of pure ferric oxide by heating it in a current of hydrogen, and by the electrical decomposition of a solution of pure ferrous sulphate or chloride. While pure iron is devoid of value, when it contains small quantities of other elements it is the most useful and widely used of all metals; in fact, it is almost impossible to overestimate its importance in the arts.

24. The manufacture of iron from its ores depends on chemical principles with which we are familiar. As iron does not occur native, it is necessary to reduce its compounds, and this is done in such a manner that the resulting metal will contain the elements necessary to give it the properties that have made it so valuable. The method almost universally employed is to charge in the ore, together with the fuel—which at present is nearly always

either coke, coal, or charcoal—at the top of a tall furnace; as the ore always contains extraneous matter, a flux is added in the proper amount to form a fusible slag with these impurities. Hot air blown into the furnace near the bottom, on coming in contact with the highly heated fuel in excess, forms carbon monoxide, which passes up through the descending charge of ore, fuel, and flux. At the temperature of the furnace, both the carbon of the fuel and the carbon monoxide thus formed act as reducing agents on the ore, removing the oxygen and leaving metallic iron, which, at the intense heat near the bottom of the furnace, melts and drops to the bottom, taking up some carbon from the fuel, and silicon, sulphur, phosphorus, and manganese from the ore, fuel, and flux. At the same time, the silica, alumina, lime, and magnesia of the ore, fuel, and flux unite, forming a fluid slag that, being lighter than iron, floats on the molten metal in the bottom. The iron and slag thus formed are drawn out at proper intervals through openings provided for them in the bottom of the furnace.

When the ascending gas reaches the top of the furnace it contains considerable carbon monoxide, which is very combustible. It passes through an opening near the top of the furnace and is led through the "down-comer" to a feedpipe Part of it is then used to heat the stoves that are employed to heat the blast of air blown in near the bottom of the furnace by means of blowing engines. The part of the gas not used in the stoves is burned under the boilers that produce steam to run the blowing engines.

25. Definition of Steel.—While at first thought it seems to be a simple matter to define *steel* properly, the more familiar one becomes with the subject, the more perplexing is it to write a concise definition that will apply to the wide range of steels produced, or even to the greater part of them. Before the introduction of the modern methods of manufacture, the distinction between steel and wrought iron was sharp and well marked, and steel could be defined as any alloy of iron with carbon that would take

a temper on quenching. Wrought iron does not sensibly harden on sudden cooling in water from a red heat. Modern methods of manufacture, however, have produced a metal that largely partakes of the nature of wrought iron, though it is made by the same processes that give a metal that hardens on quenching. For this reason such a classification as the above would now throw out the greater amount, or at least a very large tonnage, of the material classed and accepted by the metallurgical and commercial world as steel. The Bessemer converter and open-hearth furnace early showed an adaptability to produce a soft metal having great strength, elasticity, and ductility, capable of displacing wrought iron, and, for most purposes, far superior to it. It is impossible, therefore, to offer a thoroughly comprehensive definition of steel that is not easily assailable and whose inapplicability is shown from some standpoint.

Steel may be defined as a metal produced by the complete fusion of materials in a bath, the necessary properties being given, after conversion, by additions of carbon or carbon alloys. **Wrought iron** may be defined as a metal produced by the partial fusion, or bringing to a pasty condition, of materials on a hearth.

Blister, or *cementation*, *steel*, which is made by soaking bars of iron, at or above a red heat, in charcoal or carbon, would seem to be a notable exception; but as this is mainly an intermediate product for remelting in crucibles, and as its production is of little importance, it will be disregarded in this treatment of the subject.

The question of the proper classification of steels is one to which much attention has been given in the past, an international committee at one time having been selected from the metallurgical and technical societies of the principal steel-producing countries to adopt a universal classification. While much good came of their work, and strenuous efforts were made to adopt their classification, it was never generally used metallurgically nor commercially.

Many theories have been advanced as to what steel is. One that is held by many practical metallurgists is that the

ideal steel is an alloy of pure iron and carbon only, all other elements being regarded as impurities. From this point of view, all grades of steel can be produced by simply varying the amount of carbon; but as impurities are necessarily present, all steel contains varying, and usually very small, amounts of sulphur, phosphorus, silicon, metallic oxides, and gases, which require other additions for their neutralization or elimination. Again, special alloys are required for giving steels characteristic qualities for particular purposes; such are the nickel, tungsten, chrome, manganese, and molybdenum steel.

26. Processes of Manufacture.—There are only three processes for the manufacture of steel: The *crucible*, the oldest of present methods; the *Bessemer;* and the *open-hearth.* The last two were developed almost simultaneously. The **Bessemer** was first perfected, and for the first 35 years, or up to about 1890, led the open-hearth, both as to tonnage produced and in the perfection of methods and appliances—both metallurgical and mechanical. While the Bessemer process still produces the greater tonnage, this is the only direction in which it can claim superiority over the open-hearth. In the order of their metallurgical and commercial importance today the processes rank: first, the open-hearth; second, the Bessemer; and third, the crucible.

While the **crucible process** is of the least consequence, it holds the most distinctive field metallurgically, and one from which the others seem unlikely to crowd it out. Given the same composition, it is well established that crucible steel is superior to either of the others, but owing to the much higher cost of production, its use is now restricted mainly to the making of high-grade tools, certain mining drills, parts of intricate machines, and, in general, where the first cost of the steel can be ignored.

The **open-hearth process** can claim as its own a larger field than the Bessemer. Open-hearth steel is now used for the better grades of plate steel, forgings, car axles, and structural steel. The basic open-hearth process is used

where an extra-soft, pure steel is required, as in plates, sheets, rods, wires, etc. Bessemer steel is used for rails, nails, tin plate, light axles, in fact, for those articles where cheapness is desired. It is, however, being rapidly replaced by steel produced by the basic open-hearth process. The basic process, by cheaper production than was possible in the acid open-hearth, makes this a formidable rival of the Bessemer and seems practically sure to largely supplant it in the next few years. Owing to lower cost of production, the Bessemer process held undisputed sway for years in all lines using a large tonnage of steel. The open-hearth gradually demonstrated its superior fitness for special lines. While both the crucible and open-hearth processes have distinctive fields, held from the cheaper metal by the superior quality of their product, the Bessemer has no field the open-hearth cannot fill, and only by lower cost does it still produce the greater tonnage. Practically all rails are as yet made of Bessemer metal; also, most of the "billets" and "slabs" for merchant bar, tin plate, sheets, nails, light axles, and some ship and tank plates, etc.

27. Table II gives the average ultimate strengths of the various metals employed in building construction.

EXAMPLE 1.—What pull will be required to break a 2-inch diameter rod of wrought iron?

SOLUTION.—The area of the rod is equal to the area of a 2-in. circle, which is $2^2 \times .7854 = 3.14$ sq. in.; the ultimate tensile, or breaking, strength of wrought iron, according to Table II, is 48,000 lb. per sq. in. Therefore, the ultimate strength of the rod in question is $3.14 \times 48{,}000 = 150{,}720$ lb. Ans.

EXAMPLE 2.—What length of wrought-iron bar, if hung by one end, will break of its own weight, assuming the weight of 1 cubic inch of wrought iron to be .277 pound.

SOLUTION.—Assume any size of bar; say, $1\frac{1}{8}$ in. in diameter. The area of this bar is .99 sq. in., which may, for convenience, be called 1 sq. in. Now, as there is just 1 cu. in. in each lineal inch in the rod, a length of 1 ft. weighs $.277 \times 12 = 3.32$ lb The tensile strength of wrought iron being 48,000 lb per sq in., and 1 ft. of its length weighing 3.32 lb., the length of rod required is $\frac{48{,}000}{3.32} = 14{,}458$ ft. Ans.

TABLE II

AVERAGE ULTIMATE STRENGTHS OF MATERIALS IN POUNDS PER SQUARE INCH

	Compression	Tension	Elastic Limit	Shearing	Modulus of Rupture	Modulus of Elasticity
Brass, Bronze, and Copper						
Brass, cast	(30,000)	24,000	6,000	36,000	20,000	9,000,000
Brass wire, annealed		50,000				
Brass wire, unannealed		80,000	16,000			14,000,000
Bronze, aluminum	120,000	75,000				
Bronze, gun metal	(20,000)	32,000	10,000		53,000	10,000,000
Bronze, manganese	120,000	60,000	30,000			
Bronze, phosphor		50,000	24,000			14,000,000
Bronze, Tobin		66,000	40,000			4,500,000
Copper, bolts	30,000	30,000				
Copper, cast	(40,000)	24,000	6,000	30,000	22,000	10,000,000
Copper wire, annealed		36,000				15,000,000
Copper wire, unannealed		60,000	10,000			18,000,000
Cast and Wrought Iron						
Iron, cast	80,000	15,000	6,000	18,000	30,000	12,000,000
Iron wire, annealed		60,000				15,000,000
Iron wire, unannealed		80,000	27,000			25,000,000
Iron, wrought, shapes	46,000	48,000	26,000	40,000	44,000	27,000,000
Iron, wrought, rerolled bars	48,000	50,000	27,000	40,000	48,000	26,000,000
Cast and Structural Steel						
Steel, castings	70,000	70,000	40,000	60,000	70,000	30,000,000
Steel, structural, soft	56,000	56,000	30,000	48,000	54,000	29,000,000
Steel, structural, medium	64,000	64,000	33,000	50,000	60,000	29,000,000
Steel wire, annealed		80,000	40,000			29,000,000
Steel wire, unannealed		120,000	60,000			30,000,000
Steel wire, crucible		180,000	80,000			30,000,000
Steel wire, for suspension bridges		200,000	90,000			30,000,000
Steel wire, special tempered		300,000				

NOTE —Compression values enclosed in parentheses indicate loads producing 10 per cent. reduction in original lengths.

FACTOR OF SAFETY

28. The **factor of safety,** or, as it is sometimes called, the **safety factor,** is the ratio of the breaking strength of the structure to the load that, under usual conditions, it is called on to carry. Suppose that the load required to break, dismember, or crush a structure is 5,000 pounds, and that the load it is called on to carry is 1,000 pounds, then the factor of safety may be obtained by dividing 5,000 pounds by 1,000 pounds, or $\frac{5000}{1000} = 5$, the factor of safety in this structure.

The factor of safety depends on the conditions, circumstances, or materials used; in other words, it is the *factor of ignorance.* When a piece of steel, wood, or cast iron is used in a building, the engineer does not know the exact strength of that particular piece of steel, wood, or cast iron. From his own experience, and that of others, he knows the approximate tensile strength of structural steel to be 60,000 pounds per square inch, and that it varies more or less from this value. In regard to timber, the uncertainty is much greater, because of knots, shakes, and interior rot, not always evident on the surface. Cast iron is even more unreliable, on account of almost indeterminable blowholes, flaws, and imperfections in the castings.

29. Deterioration.—Another factor to be considered is **deterioration** in the material, due to various causes. In metals there is corrosion on account of moisture and gases in the atmosphere, especially noticeable in the steel trusses over railroad sheds, where the sulphur fumes from the stacks of the locomotives unite with the moisture in the air, forming free sulphuric acid, which attacks the steel vigorously, and it demands constant painting to prevent its entire destruction. Wood is subject to decay from either dry or wet rot, caused by local conditions; it may, like iron and steel, be subjected to *fatigue*, produced by constant stress due to the load it is required to sustain. Cast iron does not deteriorate to any great extent, its corrosion not being as

rapid, possibly, as that of steel or wrought iron. However, there are internal strains produced in cast iron by the irregular cooling of the metal in the mold. Under the slightest blow, castings will sometimes, owing to these internal stresses, snap and break in a number of places.

These reasons are, in truth, sufficiently cogent to require the factor of safety now adopted in all engineering work. Table III gives the factor of safety commonly employed by conservative constructors for various materials used in building work.

TABLE III

FACTOR OF SAFETY FOR DIFFERENT MATERIALS USED IN CONSTRUCTION

Materials	Factor of Safety
Structural steel and wrought iron	3 to 4
Wood	4 to 5
Cast iron	6 to 10
Stone	10 at least

In this table, the factor of safety generally used for structural steel is 3 to 4, which simply means that the steel structure should not break until it bears a load three or four times greater than it is designed to carry.

EXAMPLE.—If the breaking strength of a cast-iron column is 200,000 pounds, what safe load will the column sustain if a factor of safety of 6 is used?

SOLUTION.— $200{,}000 \div 6 = 33{,}333$ lb. Ans.

EXAMPLES FOR PRACTICE

1. Provided a factor of safety of 3 is adopted, what will be the safe working strength of a 2-inch diameter tension rod of medium structural steel? Ans. 67,020 lb.

2 The bottom of the notch in a spruce timber 10 inches wide and 12 inches deep, forming the tie-member in a roof truss, is 18 inches from the end What resistance will the end of the tie offer to the thrust of the rafter? Ans 72,00C

3. A short block of Northern yellow pine, 10 inches by 10 inches in section, standing on end, supports 50,000 pounds. What is its factor of safety? Ans. 12

BEAMS AND GIRDERS

(PART 1)

ELEMENTS OF BEAMS

DEFINITIONS

1. Any bar resting on supports and liable to be subjected to transverse stresses is called a **beam.**

A large beam that carries smaller, or secondary beams, is a **girder.**

A beam resting on two supports very near its end is a **simple beam.**

A beam resting on one support at its middle, or having one end fixed (as in a wall) and the other end free, is a **cantilever.**

A beam that has both ends firmly secured is a **fixed beam.**

A beam that rests on more than two supports is a **continuous beam.**

The distance between the supports of a simple beam is its **span.**

MOMENTS OF FORCES

2. In order to calculate the stresses produced in beams under different conditions, and the forces at the points of support, it is necessary to understand the theory of the *moments of forces.*

In Fig. 1, W is a weight that acts downwards with a force of 10,000 pounds. If some fixed point a, not in the line along which the weight W acts, is connected with the line of

action of W by a rigid arm, so that W pulls on one end of this arm, while the other end is firmly held at a, experience teaches that the pull of W tends to turn, or rotate, the arm around the point a. The tendency of a force to produce rotation around a given point is called the **moment of the force** with respect to that point.

The point a that is taken as the center around which there is a tendency to rotate, is called the **center of moments.**

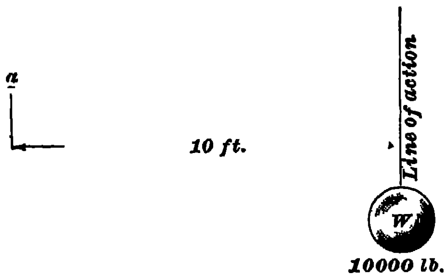


FIG. 1

The perpendicular distance from the center of moments to the line along which the force acts, is the **lever arm** of the force, also called the **leverage** of the force.

3. The measure of the moment of a force, that is, of the tendency of the force to produce rotation around a given center, is the product of the magnitude of the force multiplied

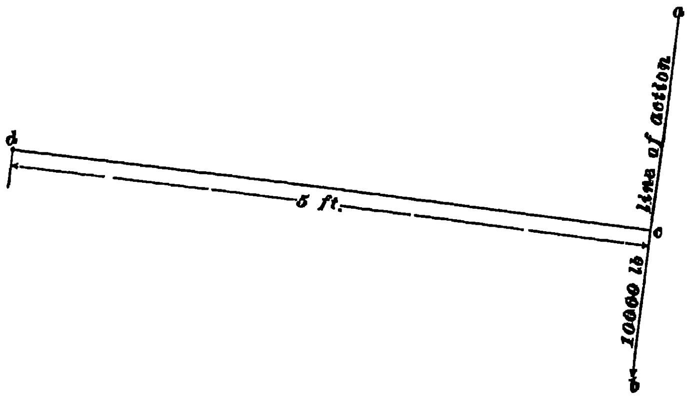


FIG. 2

by the length of its lever arm. In Fig. 1, for example, the magnitude of the force is equal to the weight of 10,000 pounds, and the length of the lever arm is 10 feet; therefore, the moment of the force W, with respect to the center a, is

10,000 × 10 = 100,000, which, since the factors by which it is produced represent feet and pounds, is called **foot-pounds.** Thus, the moment of a force of 10,000 pounds, whose lever arm is 10 feet long, is 10,000 × 10 = 100,000 foot-pounds.

EXAMPLE.—What is the moment of the force of 10,000 pounds whose line of action is *a b*, Fig. 2, the center of moments being at *d*?

SOLUTION.—The perpendicular distance *c d* from the line of action of the force to the center of moments being 5 ft., and the magnitude of the force 10,000 lb., the moment is 10,000 × 5 = 50,000 ft.-lb. Ans.

4. In Fig. 3, the line of action of the force of 10,000 pounds passes directly through the point *d*; consequently, the

FIG. 3

perpendicular distance from the line of action to the point *d* is *zero* and there is no tendency to rotate around that point; therefore, there is no motion.

5. The moment of a force may be expressed in *inch-pounds*, *foot-pounds*, or *foot-tons*, depending on the unit of measurement used to designate the magnitude of the force and the length of its lever arm. For instance, if the magnitude of a force is measured in pounds, and the lever arm through which it acts, in inches, the moment will be in *inch-pounds;* if a force of 10 tons acts through a lever arm of 20 feet, the moment of the force is 10 × 20 = 200 *foot-tons.*

EXAMPLE—What is the moment, in inch-pounds, of a force of 8,000 pounds, if the length of the lever arm is 13 feet?

SOLUTION—Since the moment is to be in inch-pounds, the length of the lever arm must be in inches. 13 ft. = 13 × 12 = 156 in., and the moment is 8,000 × 156 = 1,248,000 in.-lb. Ans.

6. Equilibrium of Moments.—When a body is at rest, the forces that act on it must balance one another; the forces are then said to be in **equilibrium.** That there may be perfect balance among the forces, it is necessary that there be not only no unbalanced force tending to move the body along some given line, but that there be, also, no

unbalanced moment, the effect of which would turn the body about some point.

Fig. 4 shows a beam or lever, resting on the support *c*, on the right-hand end of which a force *b* of 5 pounds acts downwards tending to turn it around the point of support *c* in the direction traveled by the hands of a clock, that is, to produce **right-hand** rotation. The measure of this tendency is $5 \times 10 = 50$ foot-pounds. Another force *a* acts downwards on the left-hand end of the lever, tending to produce **left-hand** rotation, or to turn the lever in the direction opposite to that traveled by the hands of a clock. Since the force *a* is 10 pounds, and it acts with a lever arm of 5 feet, its moment is $10 \times 5 = 50$ foot-pounds, the same as the moment of the force *b*. There are thus two equal moments, one tending to turn the lever to the right and the other to the left;

FIG. 4

as a result, the effect of one is neutralized by the effect of the other, and the second condition of equilibrium is fulfilled; that is, there is *equilibrium of moments.*

7. Positive and Negative Moments.—It is customary to distinguish between the directions in which there is a tendency to produce rotation by the use of the signs + and −. Thus, if a force tends to produce right-hand rotation, its moment may be called **positive** and be given the plus sign, while a force that tends to produce rotation in the opposite direction is called **negative**, and its moment is given the minus sign. That there may be equilibrium of moments, the above considerations show that the difference between the sum of the positive moments and the sum of the negative moments must be zero; this difference is called the **algebraic sum** of the moments. The following principle is then evolved: *In order that there may be equilibrium, the algebraic sum of the moments of all the forces acting on a body must be zero*

8. Resultant and Reactionary Moments.—In Fig. 5 is shown a lever composed of two arms at right angles to each other and free to turn about the center *c*. A force *a*, whose moment is $10 \times 5 = 50$ foot-pounds, acts on the horizontal arm in such a manner that it tends to produce left-hand rotation. Another force *b*, whose moment with respect to the center *c* is $12 \times 3 = 36$ foot-pounds, tends to produce right-hand rotation. These two forces, therefore, will not produce equilibrium, but their combined efforts will be equal to the algebraic sum of the moments. This sum is the **resultant moment** of the two forces *a* and *b*. Thus, $-50 + 36 = -14$ foot-pounds, which is the resultant moment of the forces considered, because it produces the same effect as the two moments combined.

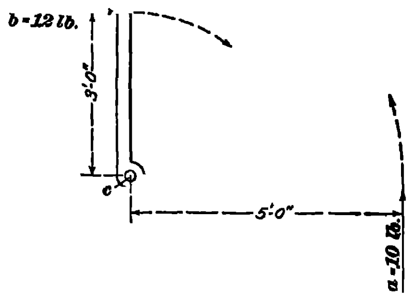


FIG. 5

In order to secure equilibrium, there must be another force acting in such a manner as to overcome the algebraic sum of these two moments, and consequently its moment must be equal and opposite in effect to the resultant moment. The name given to the moment of this force is **reactionary moment.**

If the length of the lever arm of the force that acts to produce the reactionary moment is known, the magnitude of the force may be readily found. Thus, in the present case, the resultant moment is -14 foot-pounds; let it be required to find the force to produce equilibrium, when acting with a lever arm 7 feet long. Since the moment is the product of the force multiplied by its lever arm, it follows that the required force may be found by dividing the given moment by the length of the lever arm; consequently, the required force is $14 \div 7 = 2$ pounds.

If, instead of the two forces just considered, we have a body acted on by a number of forces whose moments about

a given center are known, the reactionary moment of these forces, that is, the moment of the force required to produce equilibrium, is the algebraic sum of the moments of the given forces with the sign changed; and, further, if the length of the lever arm of the reactionary moment is known, the magnitude of the required force can be found by dividing the moment by the length of the lever arm.

9. The above principles may be expressed as follows:

Rule.—*To find the force required to produce equilibrium of moments, when the moments of any number of given forces and the lever arm of the required force are given, divide the algebraic sum of the given moments by the length of the given lever arm. If the algebraic sum is positive, the required force must tend to produce left-hand rotation; if negative, the force must tend to produce right-hand rotation.*

EXAMPLE.—In Fig. 6, a system of forces, shown by the arrows, acts in various directions and at various distances from the center O. The force F' is 25 pounds and its lever arm Op' is 8 feet, F'' is 16 pounds with a lever arm Op'' of 12 feet, F''' is 40 pounds with a lever arm Op''' of 6 feet, and the force F^{v} is 100 pounds, acting directly through the center O. If the distance Op^{iv} is 12 feet, what must be the magnitude of the force F^{iv} in order to produce equilibrium of moments?

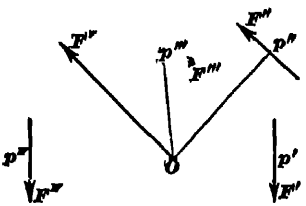


FIG. 6

SOLUTION.—As shown by the arrows, the forces tending to produce right-hand rotation are F' and F''', and their moments, called positive, are, respectively, $25 \times 8 = +200$ ft.-lb., and $40 \times 6 = +240$ ft.-lb. The lever arm of the force F^{v} is zero; consequently, it has no moment with respect to the center O. The force F'' tends to produce left-hand rotation and its moment is $16 \times 12 = -192$ ft.-lb. The algebraic sum of the moments of the given forces is $+200 + 240 - 192 = +248$ ft.-lb.; therefore, according to the rule, the force F^{iv} must be $248 \div 12 = 20\frac{2}{3}$ lb., which, since the algebraic sum of the given moments is positive, must tend to produce left-hand rotation, as shown by the arrow. Ans.

10. The principles involved in the theory of moments are among the most simple in mechanics, and, at the same time, of the greatest practical importance in the solution of

problems relating to the strength of beams, girders, and trusses.

EXAMPLE.—In Fig. 7, the lower tie-member in the roof truss has been raised to secure a vaulted ceiling effect in the upper story of the building that the truss covers. The weight transmitted through this member to the pier wall is 30,000 pounds; there is, consequently, an equal upward force due to the reaction of the wall. This force of 30,000 pounds tends to break the truss by producing rotation about the point b. What is its moment around the point b?

SOLUTION.—Since the perpendicular distance from the line of action of the force is 3 ft., the moment of the force a around the point b is $30,000 \times 3 = 90,000$ ft.-lb. Ans.

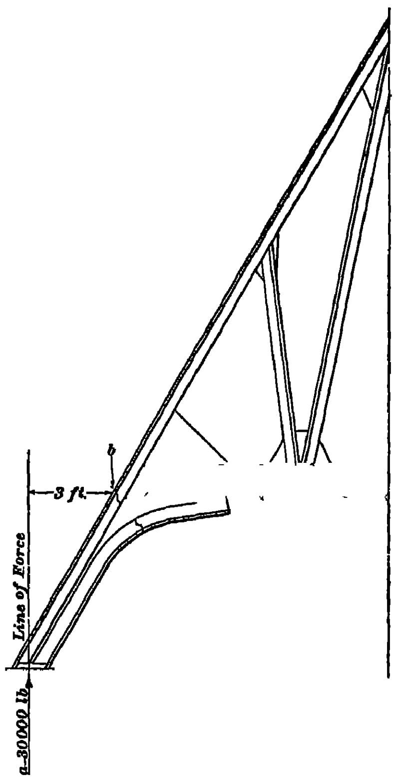


FIG. 7

THE LEVER

11. A **lever** is a bar capable of being turned about a pin, pivot, or point, as in Figs. 8, 9, and 10.

The object W to be lifted is called the **weight**; the force P used is called the **power**; and the point or pivot F is called the **fulcrum**.

That part of the lever between the weight and the fulcrum, or Fb, is called the **weight arm**, and the part between the power and the fulcrum, or Fc, is called the **power arm**.

Take the fulcrum, or point F, as the center of moments; then, in order that the lever shall be in equilibrium, the moment of P about F, or $P \times Fc$, must equal the moment of W about F, or $W \times Fb$. That is, $P \times Fc = W \times Fb$, or, in

other words, the power multiplied by the power arm equals the weight multiplied by the weight arm.

If F be taken as the center of a circle, and arcs be described through b and c, it will be seen that, if the weight arm is moved through a certain angle, the power arm will

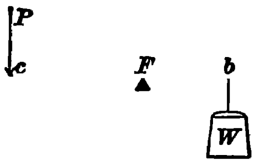


FIG. 8

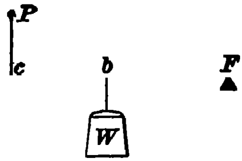


FIG. 9

move through the same angle; also, that the vertical distance that W moves will be proportional to the vertical distance that P moves. From this it is seen that the power arm is proportional to the distance through which the power moves, and the weight arm is proportional to the distance through which the weight moves.

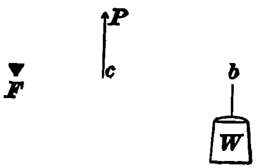


FIG. 10

Hence, instead of writing $P \times Fc = W \times Fb$, it might have been written $P \times$ distance through which P moves $= W \times$ distance through which W moves. This is the general law of all machines, and can be applied to any mechanism, from the simple lever to the most complicated arrangement. Stated in the form of a rule, it is as follows:

Rule.—*The power multiplied by the distance through which it moves equals the weight multiplied by the distance through which it moves.*

EXAMPLE 1.—If the weight arm of a lever is 6 inches long and the power arm is 4 feet long, how great a weight can be raised by a force of 20 pounds at the end of the power arm?

SOLUTION.— 4 ft. = 48 in. Hence, $20 \times 48 = W \times 6$, or $W = 160$ lb. Ans.

EXAMPLE 2.—(*a*) What is the ratio between the power and the weight in the last example? (*b*) In the last example, if P moves 24 inches, how far does W move? (*c*) What is the ratio between the two distances?

SOLUTION.—(*a*) 20 : 160 = 1 : 8; that is, the weight moved is eight times the power.

(*b*) $20 \times 24 = 160 \times x$. $x = \frac{480}{160} = 3$ in., the distance that W moves. Ans.

(*c*) 3 : 24 = 1 : 8, or the ratio is 1 : 8. Ans.

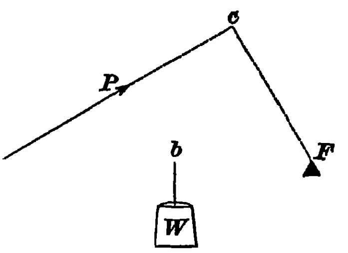


FIG. 11

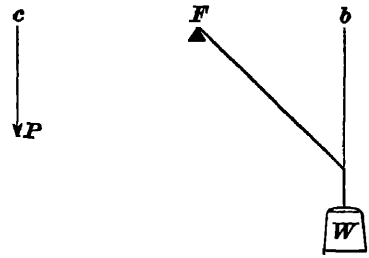


FIG. 12

The law that governs the straight lever also governs the bent lever; but care must be taken to determine the true length of the lever arm, which is in every case the

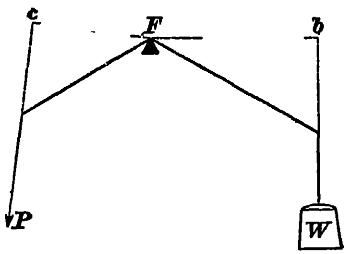


FIG 13

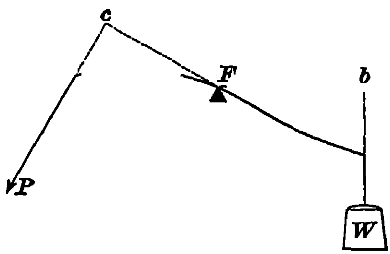


FIG. 14

perpendicular distance from the fulcrum to the line of direction of the weight or power.

Thus, in Figs. 11, 12, 13, and 14, Fc in each case represents the power arm, and Fb the weight arm.

EXAMPLES FOR PRACTICE

1. A lever arm has a length of 10 feet; the load acting on the end of the lever is 6,000 pounds. What is the moment of this load, in inch-pounds? Ans 720,000

2. A piece of timber 20 feet long is balanced at a point 8 feet from one end, the load at this end being 9,000 pounds. What is the load at the other end? Ans 6,000 lb.

3. The one support of a beam 20 feet long is 8 feet from the left-hand end; at this end is a load of 25 pounds; at the right of the support, 3 feet distant, is a load of 5 pounds; and at 7 feet to the right of the support is a load of 10 pounds. What load is required, and at which end should it be placed, to produce balance, or equilibrium, in the beam? Ans. 9.58 lb. at right-hand end

4. A steel I beam that extends 6 feet outside of the center of a building wall, and 3 feet inside, is required to support a load on the outside end of 4,000 pounds. What load on the inner end will keep the beam from tilting? Ans. 8,000 lb.

REACTIONS

12. Since one condition of equilibrium requires that the sum of all the forces acting on a body in one direction must be balanced by an equal set of forces acting in the opposite direction, it follows that, in order that any body may be kept from falling, there must be an upward pressure, or thrust, against it, just equal to the downward pressure due to its weight; this upward thrust is called a **reaction.**

In accordance with this principle, it is evident that the simple beam shown in Fig. 15 is supported by the sum

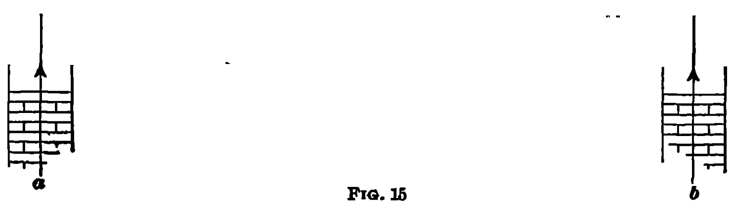


FIG. 15

of the upward pressures exerted on it by the two brick piers on which it rests; also, that this sum is equal to the weight of the beam plus the weight of any load it may carry. This is expressed by the statement: The sum of the reactions at the supports of any beam is equal to the sum of the loads.

13. Relation Between the Reactions.—If the load on a simple beam is either uniformly distributed over the entire length of the beam, applied at the center of the span, or

symmetrically placed on each side of the center of the span, the reaction at each support is equal to one-half of the total load. When, however, the loads are not symmetrically

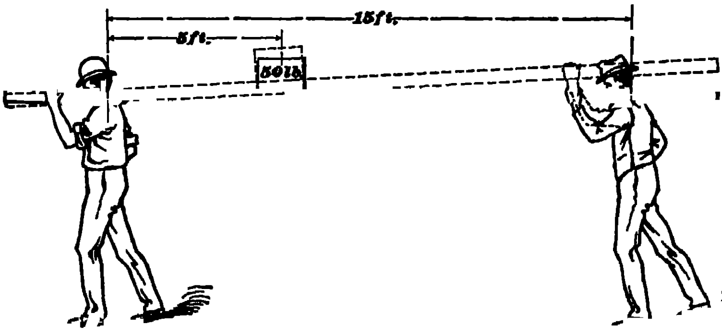


FIG. 16

placed, the reactions are unequal and must be determined before the first step toward obtaining the strength of the beam can be taken. The reactions at the points of support

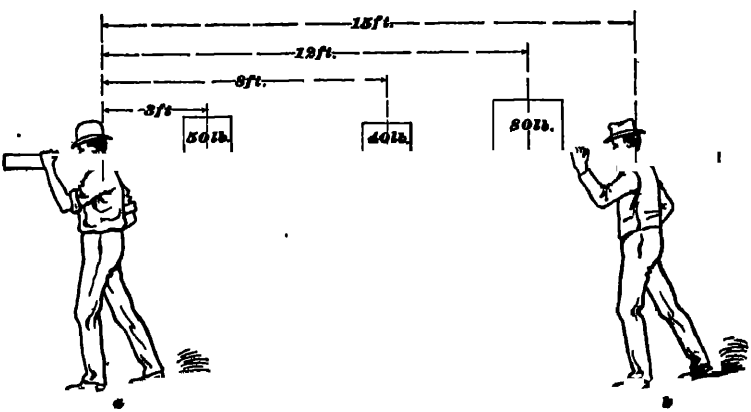


FIG. 17

of a beam carrying a number of loads irregularly placed, are determined by applying the principle of moments, as shown in the following illustrative examples:

14. Two men, *a* and *b*, 15 feet apart, carry a 50-pound weight between them on a plank, as shown in Fig. 16. What part of the load does each man carry?

If the load had been placed midway between them, it is quite evident that each man would have half the weight of the plank and load to support. But, since the load is moved until within 5 feet of *a*, he must support a greater proportion of the load than *b*. If *b* raises his end of the plank, as shown in dotted lines, it is evident that *a* simply acts as a hinge while *b* raises the weight with a lever arm 15 feet long. The weight of 50 pounds acts down with a leverage of 5 feet; its moment about *a* as a center is, therefore, $50 \times 5 = 250$

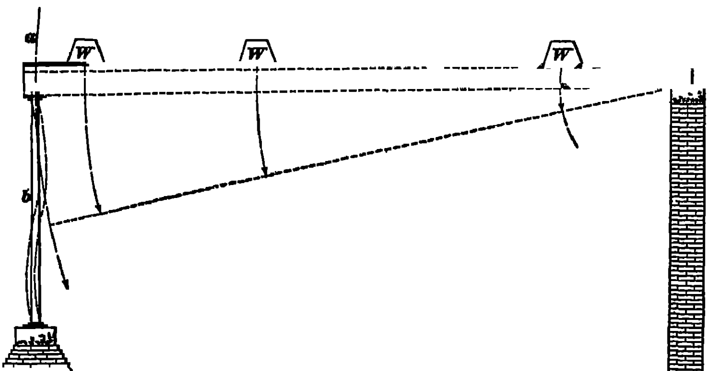


FIG. 18

foot-pounds. That there may be equilibrium of moments, it is evident that the man *b* must exert an upward force whose moment with a lever arm of 15 feet equals 250 foot-pounds; that is, he must exert a force of $250 \div 15 = 16\frac{2}{3}$ pounds to support his share of the weight. Since the sum of the reactions must equal the sum of the loads, it follows that if *b* supports $16\frac{2}{3}$ pounds, *a* must support the difference between the load of 50 pounds and $16\frac{2}{3}$ pounds, or $33\frac{1}{3}$ pounds.

15. Fig. 17 shows the men *a* and *b* supporting three loads of 50, 40, and 80 pounds, respectively. It is desired to estimate the force that each must exert to sustain the weights, leaving the weight of the plank out of the question. Assuming the

center of moments to be at a, find the resultant moment of all the weights about this point, as follows:

$$
\begin{array}{rl}
50 \times 3 = & 150 \text{ foot-pounds} \\
40 \times 8 = & 320 \text{ foot-pounds} \\
80 \times 12 = & 960 \text{ foot-pounds} \\
\text{Total,} & 1430 \text{ foot-pounds}
\end{array}
$$

This is the moment of all the loads on the beam about the point a as a center. Hence, the force that b must exert, in order to produce equilibrium, is $1,430 \div 15 = 95\frac{1}{3}$ pounds. The part of the load that a supports is the difference between the total load, $50 + 40 + 80 = 170$ pounds, and the part of the load supported by b; that is, $170 - 95\frac{1}{3} = 74\frac{2}{3}$ pounds.

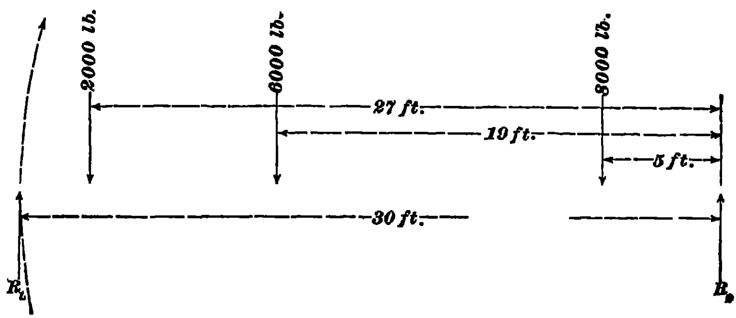


FIG. 19

16. Take a more practical example. In Fig. 18, let it be required to find the reactions R_1 and R_2. (In all the subjoined problems, R_1 and R_2 represent the reactions.) The center of moments may be taken at either R_1 or R_2. Taking R_2 as the center in this case, construct a diagram as in Fig. 19. The three loads are forces acting in a downward direction; the sum of their moments, with respect to the assumed center, may be computed as follows:

$$
\begin{array}{rl}
8,000 \times 5 = & 40000 \text{ foot-pounds} \\
6,000 \times 19 = & 114000 \text{ foot-pounds} \\
2,000 \times 27 = & 54000 \text{ foot-pounds} \\
\text{Total,} & 208000 \text{ foot-pounds}
\end{array}
$$

The magnitude of the reaction R_1 acting in an upward direction with a lever arm of 30 feet is therefore 208,000 ÷ 30 = 6,933⅓ pounds. The sum of all the loads is 2,000 + 6,000 + 8,000 = 16,000 pounds. Then, 16,000 − 6,933⅓ = 9,066⅔ pounds, the reaction at R_2.

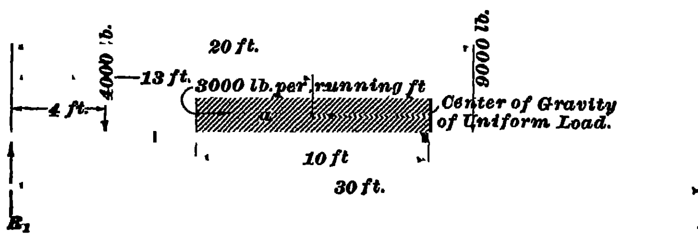


FIG. 20

EXAMPLE 1.—What is the reaction at R_2 in Fig. 20?

SOLUTION.—In computing the moment due to a uniform or evenly distributed load, as at a, the lever arm is always considered as the distance from the center of moments to the center of gravity of the load. The amount of the uniform load a is 3,000 × 10 = 30,000 lb , and the

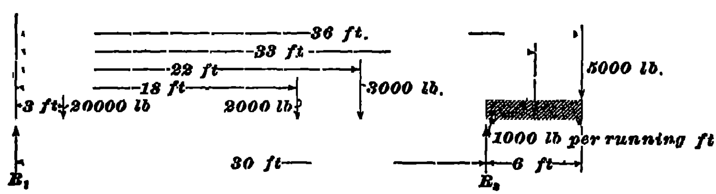


FIG. 21

distance of its center of gravity from R_1 is 13 ft. The moments of the loads on this beam may then be seen from the following:

$$
\begin{aligned}
30{,}000 \times 13 &= 390000 \text{ ft.-lb.}\\
4{,}000 \times 4 &= 16000 \text{ ft.-lb.}\\
9{,}000 \times 20 &= 180000 \text{ ft.-lb.}\\
\text{Total,} &\quad 586000 \text{ ft.-lb.}
\end{aligned}
$$

This is the sum of the moments of all the loads about R_1 as a center The leverage of the reaction R_2 is 30 ft Hence, 586,000 ÷ 30 = 19,533⅓ lb , the reaction at R_2. Ans.

EXAMPLE 2.—A beam is loaded as shown in Fig. 21. What is the amount of each of the reactions R_1 and R_2?

SOLUTION.—Considering R_1 as the center of moments, the moments of the loads about it are:

$$
\begin{aligned}
20{,}000 \times 3 &= 60000 \text{ ft.-lb.}\\
2{,}000 \times 18 &= 36000 \text{ ft.-lb.}\\
3{,}000 \times 22 &= 66000 \text{ ft.-lb}\\
5{,}000 \times 36 &= 180000 \text{ ft.-lb.}\\
1{,}000 \times 6 \times 33 &= 198000 \text{ ft.-lb.}\\
\text{Total,}\quad & \ 540000 \text{ ft.-lb.}
\end{aligned}
$$

This, divided by 30, the length of the lever arm of the reaction R_2, equals 18,000 lb., the reaction at R_2. The sum of the loads is 20,000 + 2,000 + 3,000 + 5,000 + 6,000 = 36,000 lb.; and 36,000 − 18,000 = 18,000 lb., the amount of the other reaction, or R_1. Ans.

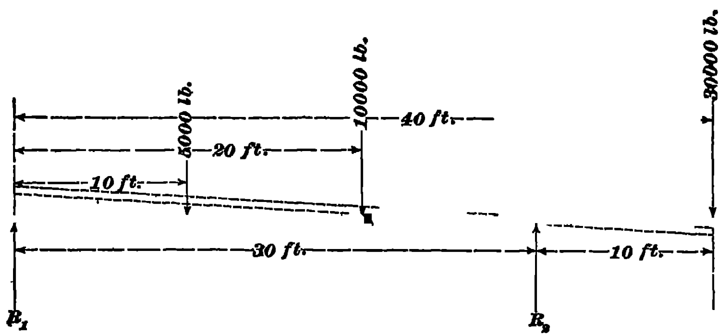


FIG. 22

EXAMPLE 3.—Compute the reactions at the supports R_1 and R_2 in a beam loaded as shown in Fig. 22.

SOLUTION.—Letting R_1 represent the center of moments, the moments of the loads are:

$$
\begin{aligned}
5{,}000 \times 10 &= 50000 \text{ ft.-lb.}\\
10{,}000 \times 20 &= 200000 \text{ ft.-lb.}\\
30{,}000 \times 40 &= 1200000 \text{ ft.-lb}\\
\text{Total,}\quad & \ 1450000 \text{ ft.-lb.}
\end{aligned}
$$

Now, 1,450,000 ÷ 30, the distance between the supports, equals 48,333⅓ lb., the required reaction at R_2. The sum of the loads is 5,000 + 10,000 + 30,000 = 45,000 lb.; therefore, the reaction R_2 is greater than the sum of the loads. This shows that the force at R_1 must act in a downward direction in order that the sum of the downward forces may equal the upward force at R_2. Since this is opposite to the usual direction, the reaction at R_1 is called negative or minus. In other words, instead of an upward reaction at R_1, there must be a downward force at this point, or the beam will, as shown by the dotted lines, rotate

around the support R_2. The magnitude of this downward force is the difference between the upward reaction at R_2 and the sum of the downward pressures due to the loads; that is, $48{,}333\frac{1}{3} - 45{,}000 = 3{,}333\frac{1}{3}$ lb Compute the reaction at R_1 by taking the center of moments at R_2 and applying the rule in Art **9** to find the magnitude and direction of action of the force at R_2 whose moment is the resultant of the moments of the loads on the beam. The load of 30,000 lb. tends to produce right-hand rotation around the center R_2, hence, its moment, $30{,}000 \times 10 = 300{,}000$ ft.-lb., is positive The 10,000-lb load is 10 ft to the left of R_2 and its tendency is to produce left-hand rotation about R_2; consequently, its moment is negative and equal to $10{,}000 \times 10 = 100{,}000$ ft -lb. In a similar manner, the moment of the 5,000-lb. load is found to be negative and equal to $5{,}000 \times 20 = 100{,}000$ ft -lb. These results may be collected thus:

Positive moment:
$30{,}000 \times 10 =$ $+300000$ ft.-lb.
Negative moments:
$10{,}000 \times 10 = -100000$ ft.-lb.
$5{,}000 \times 20 = -100000$ ft.-lb.
-200000 ft.-lb.
Difference, $+100000$ ft.-lb.

the resultant of the moments of the three loads. Since the positive moment is greater than the sum of the negative moments, the force at R_1 must tend to produce left-hand rotation; that is, it must act downwards; its lever arm being 30 ft. long, its magnitude must be $100{,}000 \div 30 = 3{,}333\frac{1}{3}$ lb , the same result as was obtained before.

EXAMPLES FOR PRACTICE

1. The span of a simple beam is 25 feet; at distances of 9 feet, 16 feet, and 18 feet from the left-hand end are placed concentrated loads of 8,000, 4,000, and 16,000 pounds, respectively What is the amount of the left reaction? Ans. 11,040 lb.

2. The two reactions supporting a beam are 2,500 and 3,000 pounds; what is the amount of a single concentrated load necessary to produce these reactions? Ans 5,500 lb.

3. A 30-foot beam overhangs the right-hand support 6 feet, on this end is a weight of 6,000 pounds; 10 feet, 12 feet, and 18 feet from the left-hand support are loads of 8,000, 6,200, and 7,800 pounds, respectively. What is the amount of the right-hand reaction? Ans. $19{,}783\frac{1}{3}$ lb.

4. If for a distance of 10 feet from the left-hand end of a beam there is distributed a load of 1,000 pounds per running foot, and at the center of the beam is located the concentrated load of 16,500 pounds, what is the amount of the left-hand reaction, provided that the beam is supported at both ends and is 30 feet long? Ans. $16{,}583\frac{1}{3}$ lb.

STRESSES IN BEAMS

17. It has been seen that a beam is a body acted on by various external forces so related as to be in a condition of equilibrium; so far, however, the effect of these forces on the beam itself has not been considered.

In a body subjected to a direct pull or thrust, as a rope or a column, the external forces are directly opposed to each other, and the resultant stresses in all sections are of the same kind, tension, or compression. In a beam, however, the external forces, while they generally act in parallel lines, are not directly opposed to each other, and it is the function of the beam to transfer these forces from one line of action to another. Take, for example, the case of a weight suspended from a pin driven in a wall, as shown in Fig. 23. The downward force of 20 pounds due to the action of the weight is balanced by the upward pressure, or reaction, of the pin on the rope; the rope is thus subjected to the action of two directly opposing forces, the result being a tensile stress that is the same for each section of the rope between the weight and the pin. The pin acts as a cantilever beam that transfers the downward pressure, due to the pull of the rope, horizontally to the wall, where it is balanced by an equal upward pressure, or reaction. The pin is thus subjected to two opposing forces that, however, act in different lines; these forces produce a set of opposing forces, or stresses, in the pin itself, which are different in kind for different parts of the pin, and vary in magnitude for each section between the rope and the wall.

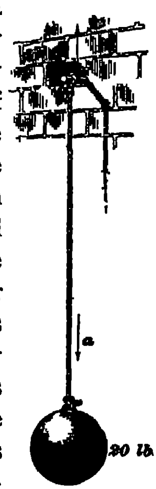


FIG. 23

SHEAR

18. An inspection of Fig. 23 shows that if a vertical plane is passed through any point between the rope and wall, the part of the pin between this plane and the rope will be

acted on by a downward force, due to the pull of the rope, while the other part is subjected to an equal upward force due to the reaction of the wall; the action of these two forces tends to slide the two parts of the pin past each other, along the section formed by the cutting plane. The pin is thus subjected to a stress which, from its similarity to a shearing action, is called **shear.**

19. Shear in a Simple Beam.—By observation of Fig. 24 (*a*), which shows a simple beam having a concentrated load at the center, the only apparent stress in the beam is that of bending. However, if the fibers of the beam were not continuous, and instead of being one piece it were composed of short blocks, the tendency would be for the blocks to slip past each other, as shown at (*b*). This tendency would exist in all parts of the beam, producing vertical shear, which must be considered in calculating its strength. If the beam were laminated in horizontal layers, the effect would be as shown at (*c*); that is, one layer would slide on the next one as the beam deflects. In a solid beam there is the same tendency to shear the beam longitudinally.

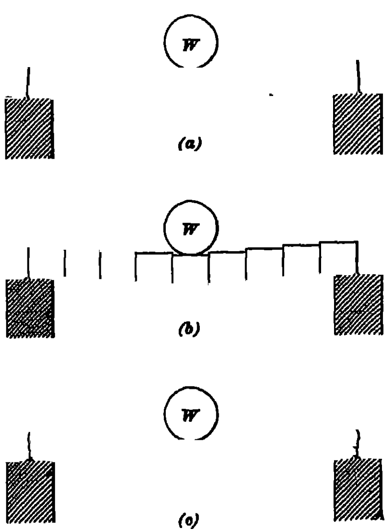


FIG. 24

Consider now the simple beam shown in Fig. 25. Since the loads are symmetrically applied, each reaction is equal to 40 pounds, one-half of the total load on the beam. Beginning at the left reaction R_1, there is an upward force of 40 pounds acting on the beam; since the forces are in equilibrium, this upward force is balanced by an equal downward

force, which is the vertical resultant of the loads and the reaction R_2. Considering, therefore, any section of the beam between R_1 and the point of application of the load n, it is seen that the part of the beam at the left of this section is subjected to an upward thrust of 40 pounds, while the part at the right is subjected to an equal downward thrust; the result is a shearing stress on this section, whose magnitude is equal to the reaction R_1.

When the point of application of n is reached, the effect of the upward force R_1 is partly balanced by the downward force of 10 pounds due to the load n; considering, therefore, any section of the beam $a\ b$ between the points of application of the loads n and m, it is seen that the part of the beam at the left is acted on by the vertical resultant of the reaction R_1 and

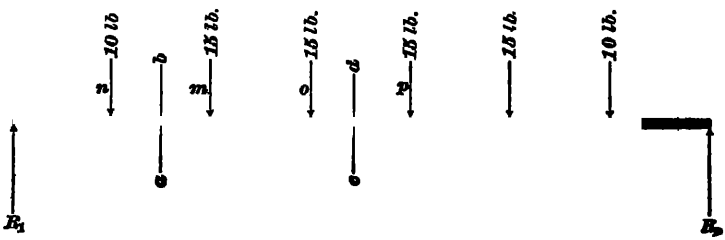


FIG. 25

the load n; that is, by an upward force of $40 - 10 = 30$ pounds, while the part at the right is acted on by an equal downward force, which is the vertical resultant of the remaining loads and the reaction R_2. Any section between the points of application of n and m is therefore subject to a shearing stress equal to the difference between the reaction R_1 and the load n; that is, to $40 - 10 = 30$ pounds. In the same way, it follows that the shearing stress for any section between m and o is $40 - (10 + 15) = 15$ pounds. For any section $c\ d$ between the points of application of o and p, the shearing stress is $40 - (10 + 15 + 15) = 0$; in other words, on each side of this section the downward forces and the reactions are equal, and their resultant is zero; it is, therefore, a section in which there is no shear.

For convenience, it is customary to call the reactions, or forces, acting in an upward direction, positive, and the loads, or downward forces, negative; since the difference between the sums of the positive and negative numbers representing a given set of values is called their algebraic sum, it follows that *the shear for any section of a beam is equal to the algebraic sum of either reaction and the loads between this reaction and the given section.*

In nearly all cases the external forces—loads and reactions—act on a beam along vertical lines; the shearing stress just considered is called the vertical shear, because it is the resultant of these forces along a section formed by an imaginary vertical cutting plane.

20. Maximum Shear.—From what has been said, it is evident that the shear in any simple beam is always greatest between the reactions and the nearest loads, and that in any case the **maximum shear** is equal to the greater reaction.

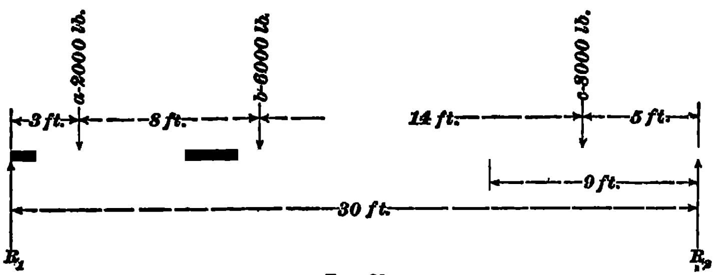


FIG. 26

21. Positive and Negative Shear.—If a section of the beam near the left reaction is taken and the forces acting on the part of the beam at the left are considered, it is seen that their resultant is positive; the shear at this section is therefore called **positive shear**. If, however, a section near the right reaction is taken, the resultant of the forces at the left is found to be negative, and in consequence the shear is called **negative**. It is also evident that there is a section between the two, where the resultant of the forces changes from positive to negative; at such a section the shear is said to **change sign**.

EXAMPLE 1.—(*a*) What is the maximum shear on the beam shown in Fig. 26? (*b*) What is the shear at a point 9 feet from the right support? (*c*) What is the shear at a point 18 feet from the right support?

SOLUTION.—(*a*) First estimate the reactions as follows: Taking the center of moments at the left support, the moments of the loads are:

$$\begin{aligned} 2{,}000 \times 3 &= 6000 \text{ ft.-lb.} \\ 6{,}000 \times 11 &= 66000 \text{ ft.-lb.} \\ 8{,}000 \times 25 &= 200000 \text{ ft.-lb.} \\ \text{Total,} & \quad 272000 \text{ ft.-lb.} \end{aligned}$$

272,000 ÷ 30 = 9,066$\frac{2}{3}$ lb., the reaction at R_2. The sum of the loads equals 2,000 + 6,000 + 8,000 = 16,000 lb.; 16,000 − 9,066$\frac{2}{3}$ = 6,933$\frac{1}{3}$ lb., the reaction at R_1. The maximum shear is therefore 9,066$\frac{2}{3}$ lb. Ans.

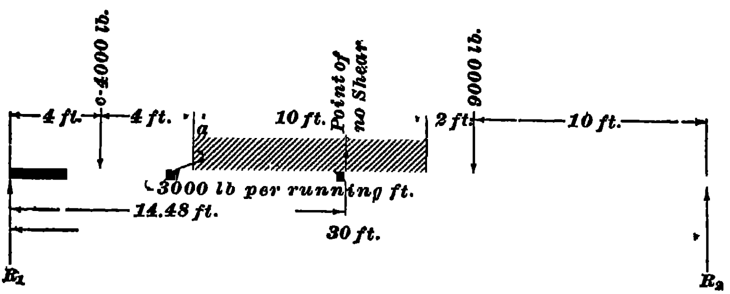


FIG. 27

(*b*) As the reaction R_2 at the right support is equal to 9,066$\frac{2}{3}$ lb., and as there is only the one load *c* of 8,000 lb., between R_2 and a point 9 ft. away, the shear at this point must equal 9,066$\frac{2}{3}$ − 8,000 = 1,066$\frac{2}{3}$ lb. Ans.

(*c*) The shear at 18 ft. from the reaction R_2 is also 1,066$\frac{2}{3}$ lb., because there is no other weight occurring between this point and R_2.

EXAMPLE 2.—At what point in the beam loaded as shown in Fig. 27 does the shear change sign?

SOLUTION.—Compute the reaction R_1 as follows: With the center of moments at R_2 the moments of the loads are:

$$\begin{aligned} 9{,}000 \times 10 &= 90000 \text{ ft.-lb.} \\ 4{,}000 \times 26 &= 104000 \text{ ft.-lb.} \\ 3{,}000 \times 10 \times 17 &= 510000 \text{ ft.-lb.} \\ \text{Total,} & \quad 704000 \text{ ft.-lb.} \end{aligned}$$

704,000 ÷ 30 = 23,466$\frac{2}{3}$ lb., the reaction at R_1. The first load that occurs, working out on the beam from R_1, is *c* of 4,000 lb. Then, 23,466$\frac{2}{3}$ − 4,000 = 19,466$\frac{2}{3}$ lb. The next load that occurs on the beam is the uniform load of 3,000 lb , per running ft. There being altogether

30,000 lb. in this load, it is evident that it will more than absorb the remaining amount of the reaction R_1; the point where the change of sign occurs must consequently be somewhere in that part of the beam covered by the uniform load. The load being 3,000 lb. per running ft., if the remaining part of the reaction, 19,466$\frac{2}{3}$ lb., be divided by the 3,000 lb., the result will be the number of feet of the uniform load required to absorb the remaining part of the reaction, and this will give the distance of the section, beyond which the resultant of the forces at the left becomes negative, from the edge of the uniform load at a; thus, 19,466$\frac{2}{3}$ $\div$ 3,000 = 6.48 ft. The distance from R_1 to the edge of the uniform load is 8 ft. The entire distance to the section of change of sign of the shear is therefore 8 + 6.48 = 14.48 ft. from R_1. Ans.

EXAMPLES FOR PRACTICE

1. The uniformly distributed load on a beam supported at both ends is 40,000 pounds; what is the maximum shear on the beam?

Ans. 20,000 lb.

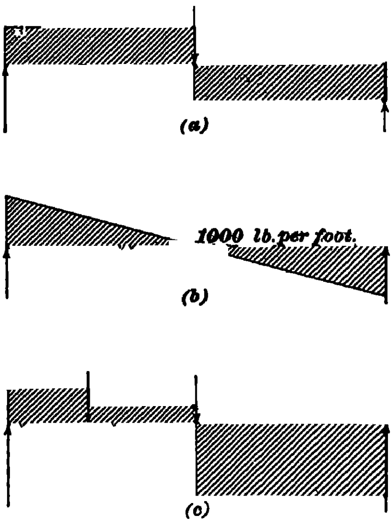


FIG. 28

2. A beam supported at both ends has three concentrated loads: A of 2,000 pounds, B of 6,000 pounds, and C of 8,000 pounds, located 10 feet, 12 feet, and 18 feet, respectively, from the left-hand end of the beam, the span of which is 40 feet; what is the shear between the loads C and B?

Ans. 2,100 lb.

3. The span of a beam is 20 feet, and there is a uniformly distributed load on three-quarters of this distance from the left-hand support, of 9,000 pounds; at distances of 8 feet and 12 feet from the right-hand support are located concentrated loads of 5,000 pounds and 6,000 pounds, respectively. At what distance from the left-hand end of the beam does the shear change sign?

Ans. 8 ft. 8$\frac{1}{2}$ in.

22. The shear on a beam may be represented graphically as shown in Fig. 28, in which the shaded portion designates the amount of the shear along the beam. The portion above

the horizontal line represents the positive shear, while the negative shear is indicated by the portion below the line; (*a*) shows the shear on a simple beam loaded at the center; the shear is uniform at all points except directly under the load, where it is zero. The shear on a beam uniformly loaded is illustrated at (*b*); in this case the shear is maximum at the supports and decreases gradually until it becomes zero at the center. The shear on a beam having concentrated loads may be represented as shown at (*c*). These diagrams are simply representations of the results obtained by analysis, but a method of obtaining the amount of shear graphically will be given later.

BENDING STRESSES

23. Bending Moment.—If, in a cantilever loaded as in Fig. 29, any point x on the center line $a\,b$ is taken as a center of moments, and a section made by a vertical plane $c\,d$ through this center is considered, it is evident that the moment of the force due to the downward thrust of the load tends to turn the end of the beam to the right of $c\,d$, around the center x; the measure of this tendency is the product of the weight W and its distance from $c\,d$; and, since it is the moment of a force that tends to bend the beam, it is called the **bending moment.**

24. Resisting Moment.—An inspection of Fig. 29 shows that if the end of the beam turns around the center x until it takes the position shown by the dotted lines, the parts of the two surfaces formed by the cutting plane $c\,d$ that are above the center x must be pulled from each other, while those below are pushed closer together. Thus, it is seen that if a vertical section is considered through any point on the center line $a\,b$ between the load and the point of support, the tendency of the load is to separate the particles in this section above the center line, and to push those below the center line closer together; in other words, through the bending action of the load, the upper part of the beam is subjected to a tensile stress, while the lower part is subjected to a compressive stress.

Fig. 29 also shows that the greater the distance of the particles in the assumed section above or below the center x, the greater will be their displacement; since the stress in a loaded body is directly proportional to the strain, or relative displacement, of the particles, it follows that the stress in a particle of any section is proportional to its distance from the center line, and that the greatest stress is in the particles composing the upper and lower surfaces of the beam.

In accordance with the conditions of equilibrium, the algebraic sum of the moments of all the forces tending to produce rotation around a given center must be zero; it is evident that the weight of the load is a force that tends to produce right-hand rotation around the center x; therefore, if the beam does not break under the action of the load, there must be forces acting whose moments, with respect to the center x, balance the moment of the load. These forces are the resistances with which the particles of the beam oppose any effort to change their relative positions. The tensile stresses in the particles above the center x and the compressive stresses in those below it, are a set of forces that resist the tendency of the load to turn the end of the beam, and, when the effect of the load is just balanced by the effect of these forces, it is evident that the sum of the moments of these resisting stresses is equal to the moment of the load. The sum of the moments of the stresses of all the particles composing any section of a beam is called the **resisting moment,** or **moment of resistance,** of that section.

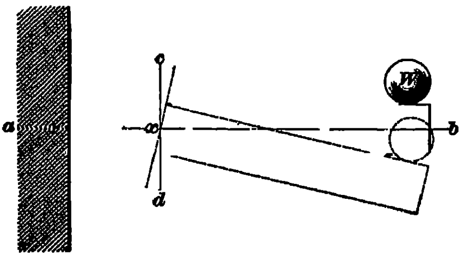


FIG. 29

25. The relations between the effect of a load and the resulting stresses in a beam have been thoroughly proved, both by mathematical investigations and numerous

experiments. The results of these experiments on beams may be briefly expressed by the following:

Experimental Law.—*When a beam is bent, the horizontal elongation or compression of any fiber is directly proportional to its distance from the neutral surface, and, since the strains are directly proportional to the horizontal stresses in each fiber, they are also directly proportional to their distances from the neutral surface, provided the elastic limit is not exceeded.*

26. Bending Moments in Simple Beams.—Referring to the simple beam shown in Fig. 26, take the center of moments on the neutral axis directly under the load a, and consider the effect produced on a vertical section of the beam through this center, by the reaction R_1. It was shown in Art. **21**, that the reaction R_1 is an upward force of $6{,}933\frac{1}{3}$ pounds; it therefore has a tendency to turn the end of the beam upwards around the assumed center with a moment of $6{,}933\frac{1}{3} \times 3 = 20{,}800$ foot-pounds. It is evident that, to prevent this turning from actually taking place, the positive moment of the reaction must be balanced by a negative moment that can be produced only by a set of internal stresses. The condition that the moment of the stresses must be negative makes it plain that the upper fibers must be in compression and the lower in tension, a result exactly opposite to the effect produced by the bending moment on the fibers in the cantilever.

27. Effect of the Moments Due to Loads.—The only force acting on the beam at the left of the section considered in the last article was the reaction R_1. The load a acted downwards directly through this section, but its lever arm, and consequently its moment, with respect to the assumed center, was zero. Take now a point on the center line of the beam directly under the positive load b. The reaction has a moment, with respect to this center, of $6{,}933\frac{1}{3} \times 11 = 76{,}266\frac{2}{3}$ foot-pounds, while the load a, which acts downwards with a lever arm of 8 feet, has a negative moment of $2{,}000 \times 8 = 16{,}000$ foot-pounds. The bending moment at the assumed section is the algebraic sum of these moments, that is, $76{,}266\frac{2}{3} - 16{,}000 = 60{,}266\frac{2}{3}$ foot-pounds. Again, taking the

center of moments on a section 9 feet from the right reaction R_2, the moments are as follows:

Positive moment:		
Reaction R_1, $6{,}933\frac{1}{3} \times 21 =$		145600 ft.-lb.
Negative moments:		
Load a, $2{,}000 \times 18 =$	36,000 ft.-lb.	
Load b, $6{,}000 \times 10 =$	60,000 ft.-lb.	96000 ft.-lb.
	Difference,	49600 ft.-lb.

This resultant moment is the bending moment at the given section.

28. The illustrations show that the bending moment varies from point to point in a beam, and depends on the length of the beam, and on the size as well as position of the loads. Since the stresses in the beam, and consequently its ability to carry its loads, depend directly on the bending moment, it follows that it is important to find not only the bending moment for any assumed section but also the section where the bending moment is greatest. It is, in this connection, useful to note the relation between the bending moment and the shear.

29. The shear in a simple beam is always greatest at the greater reaction, being equal to that reaction. In passing along the beam from either reaction, there is no change in the shear until a load is reached; at each point where a load is added, the shear is diminished by an amount equal to the load. At the point where the sum of the added loads equals or exceeds the reaction, the shear is said to change sign. The section where the change in sign in the shear takes place, depends on the method of loading. With a uniformly distributed load, the shear diminishes uniformly from each reaction, and the section where the sign changes is the section of the beam midway between the supports. With a single concentrated load, the shear is equal to each reaction at all sections between that reaction and the point where the load is applied, and the section where the shear changes sign is directly under the load. With any system of loading, the

section where the shear changes sign can be found by adding the successive loads from either reaction toward the center of the beam, until a sum is obtained that equals or exceeds the reaction; the section where the shear changes sign is under the point of application of the last load added.

30. The bending moment in a simple beam increases as the shear decreases; it is zero at either reaction and increases toward the center, becoming greatest at the section where the shear changes sign. With a uniformly distributed load, the greatest bending moment is at the section of the beam midway between the supports; with a single concentrated

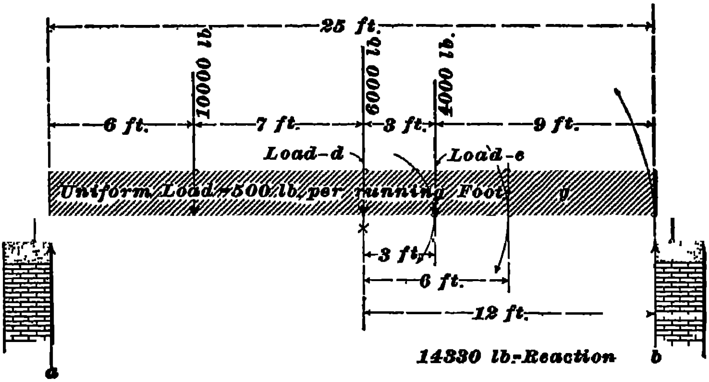


FIG. 30

load, the greatest bending moment is directly under the load; and with any system of loading, the greatest bending moment occurs at the section where the shear changes sign. Having located the section of greatest bending moment, the magnitude of this moment can be readily computed, by taking the center of moments, on the section of greatest bending moment and computing the resultant moment of either reaction and all the loads between it and the center in question.

EXAMPLE.—A wooden beam, supported on two brick piers, is loaded as shown in Fig 30: (*a*) What is the greatest shear? (*b*) Where does the shear change sign? (*c*) What is the greatest bending moment, in inch-pounds?

SOLUTION.—(*a*) Since the greatest shear is equal to the greater reaction, the reactions will first be computed. Take the center of moments at the edge of pier *a*, and as the moment of the uniform load is the same as the moment of an equal concentrated load acting at the center of gravity of the uniform load, the moments of the loads are:

10,000-lb. load	$10{,}000 \times 6$	=	60000 ft.-lb.
Load *d*	$6{,}000 \times 13$	=	78000 ft.-lb.
Load *e*	$4{,}000 \times 16$	=	64000 ft.-lb.
Uniformly distributed load . .	$12{,}500 \times 12\frac{1}{2}$	=	156250 ft.-lb.
	Total,		358250 ft.-lb.

This also equals the moment of the reaction of the pier *b*. The reaction at *b* is therefore $358{,}250 \div 25 = 14{,}330$ lb The total load is $10{,}000 + 6{,}000 + 4{,}000 + 12{,}500 = 32{,}500$ lb; the reaction at *a* is therefore $32{,}500 - 14{,}330 = 18{,}170$ lb. This, being the greater reaction, is the greatest shear. Ans.

(*b*) Beginning at the left reaction and adding the loads in succession toward the right, the load of 10,000 lb., plus the uniformly distributed load between the reaction and the point of application of the load *d*, is $10{,}000 + 500 \times 13 = 16{,}500$ lb. This is less than the left reaction, but, when the load *d* is added, the sum of the loads is greater than the reaction; consequently, the shear changes sign under the load *d*. Ans.

(*c*) Taking the center of moments under the load *d*, and considering the forces at the left, the moments are:

Positive moment:

$18{,}170 \times 13 =$ 236210 ft.-lb.

Negative moments:

$6{,}500 \times 6\frac{1}{2} =$ 42250 ft.-lb.
$10{,}000 \times 7 =$ 70000 ft.-lb.

112250 ft. lb.

Difference (bending moment in ft.-lb.), 123960 ft.-lb.,

The bending moment, in inch-pounds, is therefore $123{,}960 \times 12 = 1{,}487{,}520$ in.-lb. Ans.

31. Formulas for Maximum Bending Moments and Safe Loads.—The following table gives formulas for obtaining the maximum safe loads that may be supported by beams under different conditions and the maximum bending moments produced by these loads. In these formulas, W is the weight, in pounds; S, the section modulus; s_a, the safe unit fiber stress; and l, the span in inches.

In Case XVI, the greatest load can be supported and consequently the least bending moment occurs when the distance x is .207 l, and the formulas given apply only to this condition. The bending moment for any point is found by the formula $M = \frac{Wc}{2}\left(\frac{c}{l} - 1 + \frac{x}{c}\right)$, in which the value c is the distance of the point at which the amount of the bending moment desired is located from the left-hand end of the beam.

When the end of a beam extends into the wall it is considered as being fixed, if the masonry is built around it tightly; while if the end rests on a wall or other support it is considered as being simply supported. When the load is represented by cross-sectioning, it denotes that it is a distributed load; the round weight represents a concentrated load. A load such as shown in Cases III and X is sometimes encountered, for instance, in a case where a beam is required to support the side walls or curbing of a stairway. A triangular load, as shown in Case IX, is considered when the beam is to support a solid brick wall.

32. The rule given in Case VII is that most used, as it applies to a beam uniformly loaded, such as floor joists, girders, and, in some cases, the rafters of a roof. The rule in Case IX is convenient in calculating the bending moment on lintels supporting brickwork or masonry over openings. It will be observed that if the beam supporting a concentrated load at the center is firmly fixed or fastened at both ends, as in Case XIII, instead of being simply supported as designated in Case IV, the bending moment under the same load will be only half as much. Also, in Case XIV, where the ends of the beam are firmly fixed, the uniformly distributed load that may be supported is one and one-half times as great as where the ends merely rest on supports, as in Case VII. It is seldom advisable, in ordinary building practice, to consider the ends of a beam fixed, it being good practice to assume the ends of the beam as simply bearing on the wall, using the rules and formulas in Table I. However, it should be understood that all of these rules and formulas apply to static loads. The

TABLE I

FORMULAS FOR MAXIMUM SAFE LOADS AND MAXIMUM BENDING MOMENTS ON BEAMS

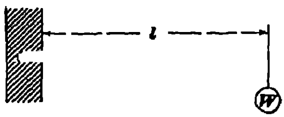
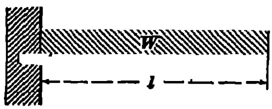
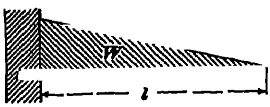
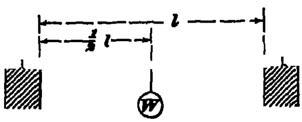
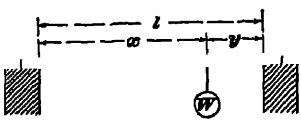
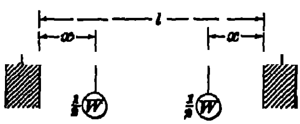
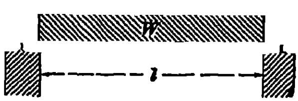
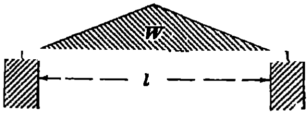
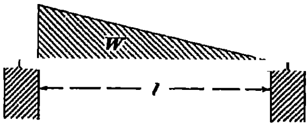
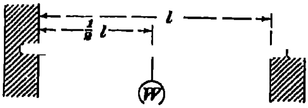
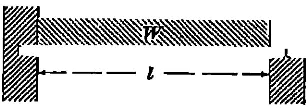
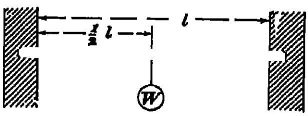
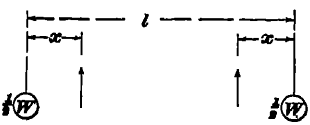
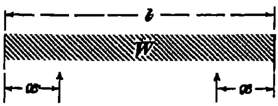
Case	Method of Loading	Maximum Bending Moment		Maximum Safe Load	
I		Wl	(1)	$\frac{S s_a}{l}$	(17)
II		$\frac{Wl}{2}$	(2)	$\frac{2 S s_a}{l}$	(18)
III		$\frac{Wl}{3}$	(3)	$\frac{3 S s_a}{l}$	(19)
IV		$\frac{Wl}{4}$	(4)	$\frac{4 S s_a}{l}$	(20)
V		$\frac{Wxy}{l}$	(5)	$\frac{l S s_a}{xy}$	(21)
VI		$\frac{Wx}{2}$	(6)	$\frac{2 S s_a}{x}$	(22)
VII		$\frac{Wl}{8}$	(7)	$\frac{8 S s_a}{l}$	(23)
VIII		$\frac{Wl}{12}$	(8)	$\frac{12 S s_a}{l}$	(24)

TABLE I—(Continued)

Case	Method of Loading	Maximum Bending Moment		Maximum Safe Load	
IX		$\frac{Wl}{6}$	(9)	$\frac{6\,S\,s_a}{l}$	(25)
X		$\frac{104\,Wl}{810}$	(10)	$\frac{810\,S\,s_a}{104\,l}$	(26)
XI		$\frac{3\,Wl}{16}$	(11)	$\frac{16\,S\,s_a}{3\,l}$	(27)
XII		$\frac{Wl}{8}$	(12)	$\frac{8\,S\,s_a}{l}$	(28)
XIII		$\frac{Wl}{8}$	(13)	$\frac{8\,S\,s_a}{l}$	(29)
XIV		$\frac{Wl}{12}$	(14)	$\frac{12\,S\,s_a}{l}$	(30)
XV		$\frac{Wx}{2}$	(15)	$\frac{2\,S\,s_a}{x}$	(31)
XVI		$\left\{\begin{matrix}\frac{Wx^2}{2\,l}\\ \text{or}\\ \frac{W}{2}\left(x-\frac{l}{4}\right)\end{matrix}\right\}$	(16)	$\left\{\begin{matrix}\frac{2\,l\,S\,s_a}{x^2}\\ \text{or}\\ \frac{2\,S\,s_a}{x-\frac{l}{4}}\end{matrix}\right\}$	(32)

same load suddenly applied produces a stress in the beam twice as great as that of a static load. The safe suddenly applied load is therefore only half as much.

A graphical method for determining the bending moment on a beam is explained in the following article:

EXAMPLE.—What will be the bending moment, in inch-pounds, on a wooden girder supporting a floor area of 150 square feet, the dead and live load being 100 pounds per square foot, and the span of the girder 20 feet?

SOLUTION.—The total uniformly distributed load is $150 \times 100 = 15{,}000$ lb.; therefore, by applying the formula in Case VII, Table I, the bending moment

$$M = \frac{Wl}{8} = \frac{15{,}000 \times 20 \times 12}{8} = 450{,}000 \text{ in.-lb.} \quad \text{Ans.}$$

EXAMPLES FOR PRACTICE

1. A beam has a span of 20 feet, and is loaded with a uniformly distributed load of 2,500 pounds per lineal foot; what is the greatest bending moment, in inch-pounds, on the beam? Ans. 1,500,000 in.-lb.

2. What is the bending moment, in inch-pounds, on a cantilever beam, securely fastened into a wall, extending from the point of support 10 feet and loaded with a uniformly distributed load of 1,000 pounds per lineal foot? Ans 600,000 in.-lb.

3. What is the bending moment, in inch-pounds, on a girder having a span of 30 feet, if there is a uniformly distributed load of 1,500 pounds per lineal foot and a load of 20,000 pounds concentrated at the center? Ans. 3,825,000 in.-lb.

4. A plate girder in a building is required to support a uniformly distributed load of 2,000 pounds per lineal foot, extending 20 feet each side of the center of the girder; in addition, it is required to support a load of 30,600 pounds, concentrated 10 feet from one end of the girder, and another load of 43,000 pounds, located 22 feet from the same end. What will be the greatest bending moment on the girder, in foot-pounds, if the span is 60 feet? Ans. 1,528,933 ft.-lb.

GRAPHICAL METHOD OF OBTAINING BENDING MOMENT AND SHEAR

33. The graphical method of obtaining bending moment and shear on a beam can best be explained by assuming an example and solving it.

Fig. 31 (a) shows a simple beam having three concen trated loads, W_1, W_2, and W_3. First lay out, as in (b), the *force polygon*, or *stress diagram*, as it is sometimes called; the

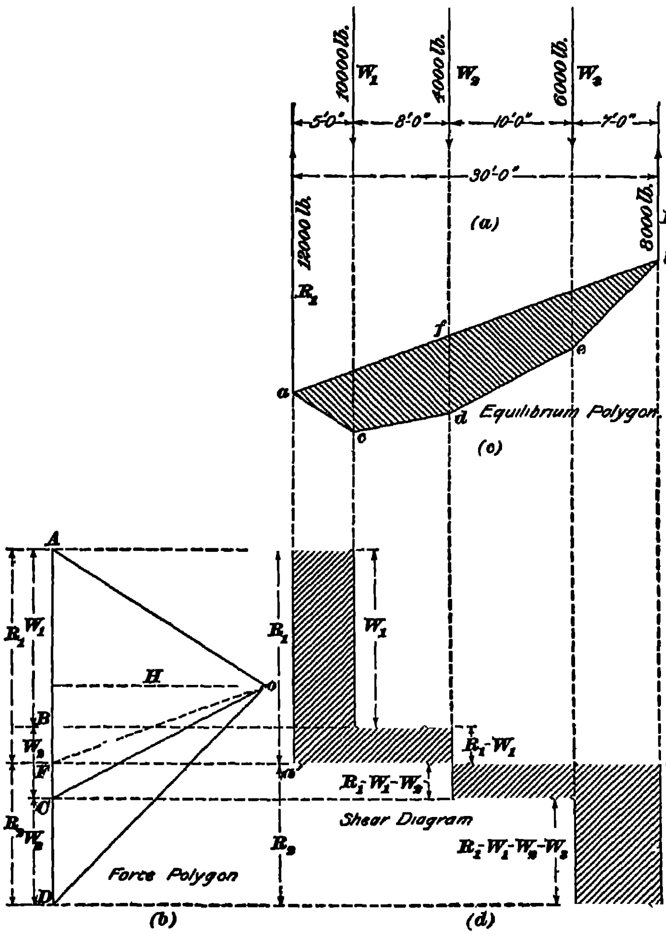


FIG 31

loads are laid off to scale at A B, B C, and C D. From any point o, outside of the load line, draw lines to A, B, C, and D. If a cord is supported at two points, a and b, in Fig. 31 (c), and the loads W_1, W_2, and W_3 are hung at the points c, d, and e,

respectively, the cord will be in equilibrium, provided that the portions *a c*, *c d*, *d e*, and *e b* are parallel to *A o*, *B o*, *C o*, and *D o*, respectively.

The figure in (*c*) is called the *equilibrium*, or *funicular*, *polygon*, and is a representation of the variation of the bending moment along the beam. Therefore, the bending moment at any point is proportional to the ordinate in the funicular polygon below that point, or the vertical line intercepted between the sides of the polygon. For instance, the bending moment under the load W_2 is proportional to the ordinate *f d*. The reactions R_1 and R_2 may be obtained from the force polygon by drawing the line *o F* parallel to *a b* in the equilibrium polygon; *A F* and *F D* represent the reactions R_1 and R_2, respectively.

In the force polygon, a horizontal line drawn from the point *o* to the load line represents the horizontal component of the tensions *a c*, *c d*, *d e*, and *e b* in the funicular polygon, and is designated as *H*. The bending moment at any point is equal to the product of the vertical ordinate in the funicular polygon at that point and the force *H*. The ordinate should be measured by the scale to which the beam was drawn and the line *H* by the scale to which the load line was laid off. The explanation of this is given in Art. **35**.

34. To construct the shear diagram shown in (*d*), project downwards the points at which the loads and reactions occur. The point *F* in the force polygon is projected horizontally, thus giving *a′ b′*, the base line of the polygon. Project the point *A* until it intersects the line drawn from the load W_1 in (*a*), and the point *B* until it intersects the line drawn from the load W_2. The shear at this point changes sign and consequently the remainder of the diagram will be below the base line *a′ b′*. Project the points *C* and *D* until they intersect the lines projected from the load W_3 and the reaction R_2, respectively. This completes the shear diagram, as shown by the shaded section in (*d*). The shear at any point is found by measuring the ordinate at that point by the scale used in laying off the load line *A D*.

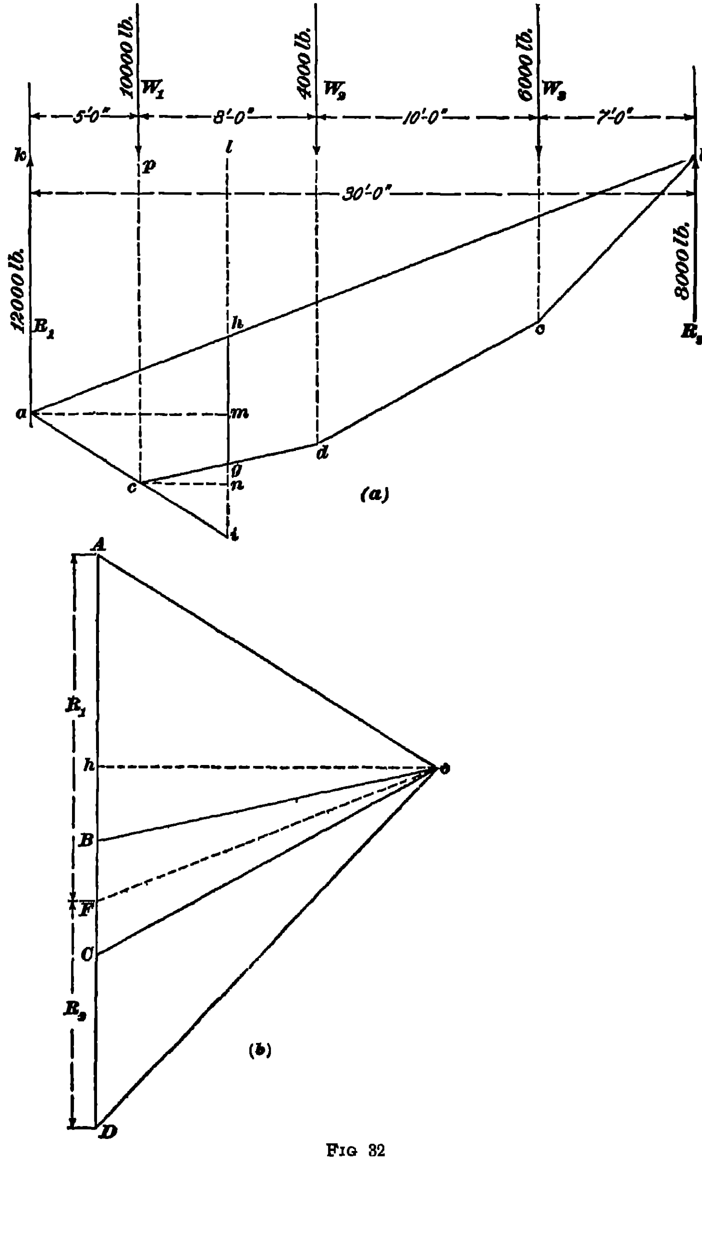

10000 lb.
W_1
4000 lb.
W_2
6000 lb.
W_3
5'-0"
8'-0"
10'-0"
7'-0"
30'-0"
12000 lb.
R_1
8000 lb.
R_2
k
p
l
b
h
e
a
m
d
g
c
n
i
(a)
A
R_1
h
o
B
F
C
R_2
D
(b)

FIG 32

35. The graphical determination of the bending moment may be explained by means of Fig. 32, in which the beam shown in Fig. 31 is considered.

On the beam kb it is desired to find the bending moment at the point l, 9 feet from k. Let the bending moment be considered first with reference to the reaction R_1 alone, omitting the effect of the load W_1. Construct the triangle ahi by producing the lines gh and ac until they intersect at i. As the triangle ahi in (a) is similar to AoF in (b), it follows that $ah : hi = Fo : AF$, and as the altitudes of the two triangles are proportional to their bases, the following proportion may be written: $am : hi = ho : AF$; therefore, $AF \times am = ho \times hi$. But $am = kl$ and $AF = R_1$, hence, $R_1 \times kl = ho \times hi$.

As the load W_1 acts in opposition to the reaction R_1, its moment must be subtracted from that of R_1. In the similar triangles cgi and AoB, $cg : gi = oB : AB$ and consequently $cn : gi = oh : AB$, but $AB = W_1$ and $cn =$ its moment pl; therefore, $W_1 \times pl = gi \times oh$. The resulting moment is then found by subtracting that of W_1 from R_1, as follows: $hi \times ho - gi \times ho$, or $ho(hi - gi) = ho \times hg$. Thus, it is proved that the bending moment at any point on a beam is equal to the product of the line representing the horizontal force oh and a vertical line intercepted between the sides of the funicular polygon and passing through the point considered.

In Fig. 32 (a) the line gh measures 6.14 feet and in (b) the line ho measures 11,000 pounds, according to the scales to which the two figures have been drawn; the product $6.14 \times 11{,}000 = 68{,}154$ foot-pounds is the bending moment at l. By calculation, the bending moment is found to be $9 \times 12{,}000 - 4 \times 10{,}000 = 68{,}000$ foot-pounds. The difference is caused by inaccuracies in the drawing by reason of the small scale to which it is drawn.

CONTINUOUS BEAMS

36. When beams or girders extend over three or more supports in one piece, that is, are not jointed over the supports, they are said to be *continuous*, and the strains produced are very different from those in ordinary beams.

Fig. 33 shows the action of a beam fixed at each end and having one intermediate support. The points *b*, *c*, *e*, and *f*, where the direction of the curve changes and, consequently, where the stress in the upper and lower portions of the beam changes, are called the points of **contraflexure.** The portions *a b*, *c d*, *d e*, and *f g* act as cantilevers and the upper edges of the beam are in tension while the lower are in compression.

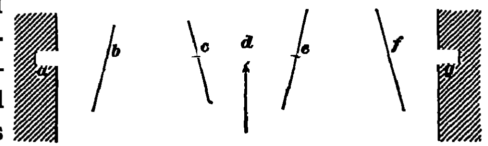


FIG 33

The portions *b c* and *e f* resemble simple beams and their upper edges are in compression while the lower are in tension. If the ends of the girders are not fixed, but are simply supported, the curves will be as shown in Fig. 34. The curves of each span produced by a uniform load resemble those of a beam fixed at one end and supported at the other. The location of the points of contraflexure and the value of the bending moments are affected by the distribution of the load and the section of the girder.

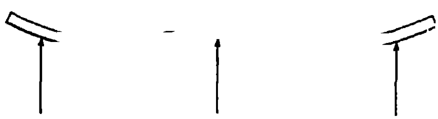


FIG 34

The calculations in regard to continuous girders are very complicated and consequently will not be discussed here, but the following formulas will give all the information required in proportioning such beams to the load they are required to carry.

Tables II and III give the reactions and bending moments at the supports for continuous beams uniformly loaded and extending over a number of equal spans. In these tables, l equals the length of each span and w the weight per unit of

TABLE II

REACTIONS FOR CONTINUOUS BEAMS OVER EQUAL SPANS

Coefficients of w l

Number of Span	Number of Each Support									
	1st	2d	3d	4th	5th	6th	7th	8th	9th	10th
1	1/2	1/2								
2	3/8	10/8	3/8							
3	4/10	11/10	11/10	4/10						
4	11/28	32/28	26/28	32/28	11/28					
5	15/38	43/38	37/38	37/38	43/38	15/38				
6	41/104	118/104	100/104	106/104	100/104	118/104	41/104			
7	56/142	161/142	137/142	143/142	143/142	137/142	161/142	56/142		
8	153/388	440/388	374/388	392/388	386/388	392/388	374/388	440/388	153/388	
9	209/530	601/530	511/530	535/530	529/530	529/530	535/530	511/530	601/530	209/530

TABLE III

BENDING MOMENTS FOR CONTINUOUS BEAMS OVER EQUAL SPANS

Coefficients of $w l^2$

Number of Span	Number of Each Support									
	1st	2d	3d	4th	5th	6th	7th	8th	9th	10th
1	0	0								
2	0	1/8	0							
3	0	1/10	1/10	0						
4	0	3/28	2/28	3/28	0					
5	0	4/38	3/38	3/38	4/38	0				
6	0	11/104	8/104	9/104	8/104	11/104	0			
7	0	15/142	11/142	12/142	12/142	11/142	15/142	0		
8	0	41/388	30/388	33/388	32/388	33/388	30/388	41/388	0	
9	0	56/530	41/530	45/530	44/530	44/530	45/530	41/530	56/530	0

length; hence, the load on each span is equal to $w\,l$. The reactions are expressed in terms of $w\,l$ and the bending moments in terms of $w\,l^2$. Only the fractional coefficients are given.

To illustrate the method of using the tables, the following example will be assumed:

EXAMPLE.—The lower tie-member of an A-shaped truss, *a a* Fig. 35, supported by the four reactions R_1, R_2, R_3, and R_4, sustains a uniformly distributed load of 2,000 pounds per lineal foot. (*a*) What will be the theoretical amount of the reactions? (*b*) What is the amount of the greatest bending moment from this uniformly distributed load?

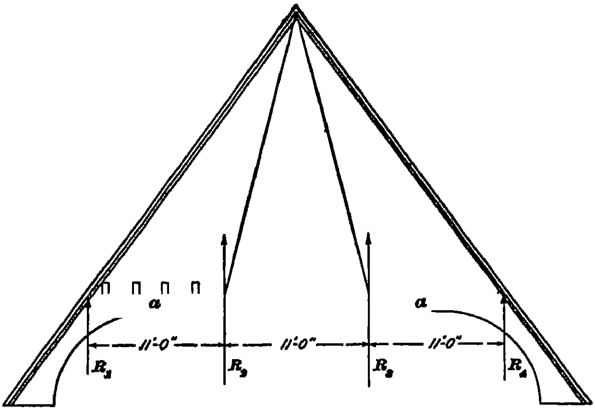


FIG. 35

SOLUTION.—(*a*) From Table II, the reactions at the ends are equal to $\frac{4}{10}$ of $w\,l$, and for the intermediate supports, $\frac{11}{10}$ of $w\,l$. Substituting these values, the reactions at the ends are equal to $\frac{4}{10} \times 2{,}000 \times 11 = 8{,}800$ lb. Ans.

The reactions at the intermediate supports are equal to $\frac{11}{10} \times 2{,}000 \times 11 = 24{,}200$ lb. Ans.

(*b*) The greatest bending moment occurs at the intermediate supports; from Table III, this moment is equal to $\frac{1}{10}\,w\,l^2$, or $\frac{1}{10} \times 2{,}000 \times 11 \times 11 = 24{,}200$ ft.-lb. Ans.

EXAMPLES FOR PRACTICE

1. The first floor of a building used as a store was supported on 12-inch I beams having a clear span of 30 feet. A subsequent tenant desired to use the first floor of the building as a storage warehouse and it was decided to place a row of piers down the center of the

basement to further support these girders By what percentage is the carrying capacity of the floor increased by the introduction of the central pier? Ans. 400%

2. What will be the load from the girder on the central pier mentioned in the above problem, if the load is 2,000 pounds per lineal foot? Ans. 37,500 lb.

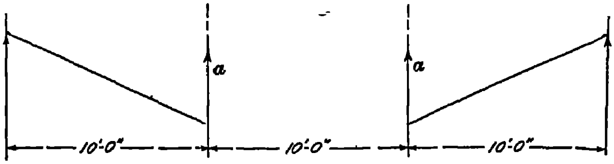


FIG 36

3. A trussed girder, Fig. 36, is loaded with a uniformly distributed load of 1,500 pounds per foot; what will be the amount of the compressive stress in the straining posts *a*, *a*. Ans. 16,500 lb.

DEFLECTION OF BEAMS

37. Elasticity is that property which a body possesses of returning to its original form, after being strained or distorted by the application of a stress. This property is possessed by all bodies in a greater or less degree. If, after being distorted, a body does not perfectly resume its original form, it is said to have a **permanent set.** It is believed that the elasticity of all solids is more or less imperfect, and that the slightest strain produces a corresponding permanent set. It is customary, however, to consider the elasticity of all building materials as practically perfect within certain limits. Under this assumption, stresses, up to a certain limit, may be applied and removed, and the resulting strain or alteration of form will be only temporary, with no appreciable permanent set. However, stresses above this limit will cause permanent sets.

38. The *elastic limit* of any material is the maximum unit stress that may be applied to it without causing any apparent permanent set; or, it is that point at which the strain ceases to be proportional to the stress.

To illustrate: Consider a piece of steel wire supported at one end and loaded by a weight suspended from the other.

The wire is found to stretch under the action of the load, and by varying the weight or stress, the strain in each case is found to vary in the same proportion, so long as the weight is not greater than one-fourth of the breaking strength of the wire; within this limit it is found, on removing the weight, that the wire resumes its original length.

If the load is made considerably greater than one-fourth of the breaking strength of the wire, it is found that when the load is removed the wire has taken a permanent set; in other words, it will not return to its original length. If the wire remains permanently longer than it was before the load was applied, it has been strained beyond the limit of elasticity.

Suppose that a weight of 2,000 pounds is hung from the end of a wrought-iron rod having a sectional area of 1 square inch, and that the rod stretches about $\frac{1}{13000}$ of its original length. When the weight is removed, the bar resumes its original length, as far as can be measured by ordinary instruments. Now, instead of 2,000 pounds, attach a weight of 24,000 pounds to the rod and it stretches about $\frac{1}{1000}$ of its length; when this weight is removed, we find that the bar does not return to its original length, but that it is slightly longer than it was before; that is, the bar has a permanent set.

The unit stress where the weight on the rod is just sufficient to produce the least permanent set is called the **elastic limit.**

39. The **modulus of elasticity** is the ratio of the unit stress to the unit strain for loads within the elastic limit.

For example, if the weight of 2,000 pounds on an iron bar, whose section is 1 square inch, produces an elongation of $\frac{1}{13000}$ of the original length of the bar, the unit stress is 2,000 pounds per square inch; the unit strain is $\frac{1}{13000}$; and the modulus of elasticity is $2{,}000 \div \frac{1}{13000} = 26{,}000{,}000$ pounds per square inch.

In most building materials, the modulus of elasticity for tension and the modulus for compression may be considered as practically equal. The moduli of elasticity of some of the principal building materials are: yellow pine, 1,200,000 pounds

basement to further support these girders. By what percentage is the carrying capacity of the floor increased by the introduction of the central pier? Ans. 400%

2. What will be the load from the girder on the central pier mentioned in the above problem, if the load is 2,000 pounds per lineal foot? Ans. 37,500 lb.

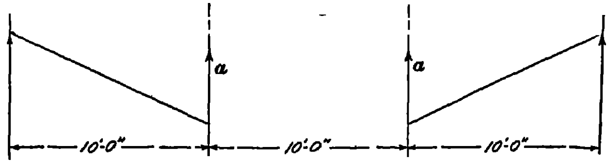


FIG 36

3. A trussed girder, Fig. 36, is loaded with a uniformly distributed load of 1,500 pounds per foot; what will be the amount of the compressive stress in the straining posts *a*, *a*. Ans. 16,500 lb.

DEFLECTION OF BEAMS

37. Elasticity is that property which a body possesses of returning to its original form, after being strained or distorted by the application of a stress. This property is possessed by all bodies in a greater or less degree. If, after being distorted, a body does not perfectly resume its original form, it is said to have a **permanent set.** It is believed that the elasticity of all solids is more or less imperfect, and that the slightest strain produces a corresponding permanent set. It is customary, however, to consider the elasticity of all building materials as practically perfect within certain limits. Under this assumption, stresses, up to a certain limit, may be applied and removed, and the resulting strain or alteration of form will be only temporary, with no appreciable permanent set. However, stresses above this limit will cause permanent sets.

38. The *elastic limit* of any material is the maximum unit stress that may be applied to it without causing any apparent permanent set; or, it is that point at which the strain ceases to be proportional to the stress.

To illustrate: Consider a piece of steel wire supported at one end and loaded by a weight suspended from the other.

The wire is found to stretch under the action of the load, and by varying the weight or stress, the strain in each case is found to vary in the same proportion, so long as the weight is not greater than one-fourth of the breaking strength of the wire; within this limit it is found, on removing the weight, that the wire resumes its original length.

If the load is made considerably greater than one-fourth of the breaking strength of the wire, it is found that when the load is removed the wire has taken a permanent set; in other words, it will not return to its original length. If the wire remains permanently longer than it was before the load was applied, it has been strained beyond the limit of elasticity.

Suppose that a weight of 2,000 pounds is hung from the end of a wrought-iron rod having a sectional area of 1 square inch, and that the rod stretches about $\frac{1}{13000}$ of its original length. When the weight is removed, the bar resumes its original length, as far as can be measured by ordinary instruments. Now, instead of 2,000 pounds, attach a weight of 24,000 pounds to the rod and it stretches about $\frac{1}{1000}$ of its length; when this weight is removed, we find that the bar does not return to its original length, but that it is slightly longer than it was before; that is, the bar has a permanent set.

The unit stress where the weight on the rod is just sufficient to produce the least permanent set is called the **elastic limit.**

39. The **modulus of elasticity** is the ratio of the unit stress to the unit strain for loads within the elastic limit.

For example, if the weight of 2,000 pounds on an iron bar, whose section is 1 square inch, produces an elongation of $\frac{1}{13000}$ of the original length of the bar, the unit stress is 2,000 pounds per square inch; the unit strain is $\frac{1}{13000}$; and the modulus of elasticity is $2{,}000 \div \frac{1}{13000} = 26{,}000{,}000$ pounds per square inch.

In most building materials, the modulus of elasticity for tension and the modulus for compression may be considered as practically equal. The moduli of elasticity of some of the principal building materials are: yellow pine, 1,200,000 pounds

per square inch; cast iron, 17,000,000 to 20,000,000; steel, 28,000,000 to 30,000,000; neat cement, 3,000,000; concrete, 700,000.

40. The stresses of tension and compression created in a loaded beam cause elongation and shortening of the longitudinal elements above and below the neutral plane, the result of which is a curvature of the beam. The amount of this curvature depends on the amount and distribution of the load, the material of which the beam is composed, its span and manner of support, and on the dimensions and form of the cross-section.

Deflection is the name applied to the distortion or bending produced in a beam when subjected to transverse stresses. The measure of the deflection at any point on a beam is the perpendicular displacement of the point from its original position. If, on the removal of the transverse stresses or loads on the beam, it returns to the straight or original form, the material in the beam has not been strained beyond the elastic limit. On the other hand, if the internal stresses exceed the elastic limit of the material, a permanent set will be given the beam.

41. **Stiffness** is a measure of the ability of a body to resist bending; this property is very different from the strength of the material or its power to resist rupture.

The stiffness of a structure does not depend so much on the elasticity of the material of which it is composed as on its arrangement and form; for example, a floor may be built of shallow and wide joists that will be sufficiently strong to carry a given load, but it will not be nearly so stiff as a floor of equal strength built of narrow and deep ones. This property of stiffness is as important in building construction as mere strength, and the two should be considered together; thus, the floor joists of a building may be strong enough to resist breaking, but so shallow as to lack stiffness, in which case the floor will be springy and vibrate from people walking on it. If there is a plastered ceiling on the under side of the joists of such a floor, the deflection of the joists

may cause the plaster to crack and fall into the room below. Where stiffness is lacking in the rafters of a roof, they will be liable to sag, thereby causing unsightly hollows in the surface of the roof, in which moisture and snow may lodge, which would be very detrimental to the roof covering.

42. From the foregoing, it is evident that not only must the strength of the beams composing a structure be calculated to withstand rupture, but the beams must be stiff or rigid enough to resist bending. It is, therefore, important to be able to calculate the deflection of any beam under its load, and if found excessive, the size of the beam may be increased and the deflection reduced to working limits.

The amount of deflection that exists in beams loaded and supported in different ways may be calculated by the formulas given in Table IV. In using these formulas, all the loads should be expressed in pounds and the lengths in inches. The modulus of elasticity is denoted by E, and the moment of inertia of the section by I.

EXAMPLE 1.—A 10-inch steel I beam, supported at the ends, must sustain a uniformly distributed load of 10,000 pounds. The span of the beam is 20 feet, and its moment of inertia is 146.4; there is to be a plastered ceiling on its under side, the allowable deflection of which is $\frac{1}{30}$ inch for each foot of span. Will the deflection of the beam be excessive?

SOLUTION.—The formula for the deflection of a beam of this character, from the table, is $\frac{5\,W l^3}{384\,E\,I}$. From Art **39**, the modulus of elasticity of structural steel is 28,000,000 to 30,000,000; taking the average value, and substituting the other values of the example in the formula, the deflection equals

$$\frac{5 \times 10{,}000 \times 240^3}{384 \times 29{,}000{,}000 \times 146.4} = .42, \text{ or about } \tfrac{7}{16} \text{ in.}$$

Since the allowable deflection for each foot of span is $\frac{1}{30}$ in., the total allowable deflection is $\frac{1}{30}$ of $20 = \frac{2}{3}$ in. This is greater than the calculated deflection, and the beam therefore satisfies the required conditions. Ans.

EXAMPLE 2.—A $12'' \times 16''$ yellow-pine girder must support a symmetrically placed triangular piece of brickwork, which weighs about 12,000 pounds. What will be the deflection of the timber if the span is 20 feet?

TABLE IV

FORMULAS FOR DEFLECTION OF BEAMS

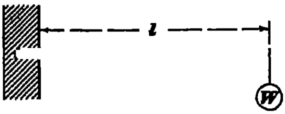
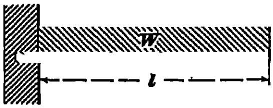
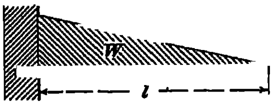
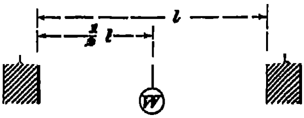
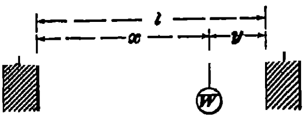
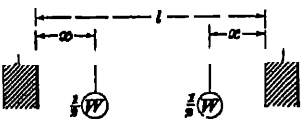
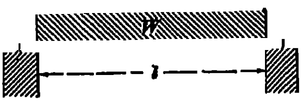
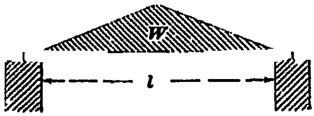
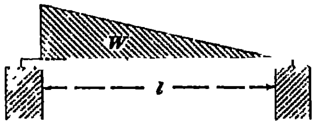
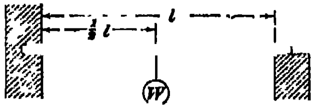
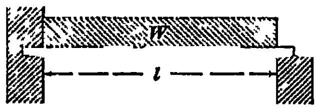
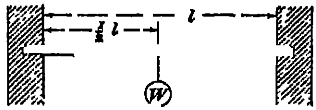
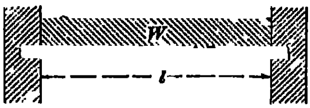
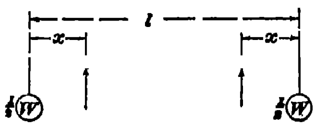
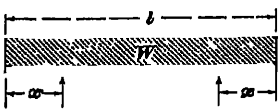
Case	Method of Loading	Deflection Inches	
I		$\frac{Wl^3}{3EI}$	(33)
II		$\frac{Wl^3}{8EI}$	(34)
III		$\frac{Wl^3}{15EI}$	(35)
IV		$\frac{Wl^3}{48EI}$	(36)
V		$\frac{Wxy(2l-x)\sqrt{3x(2l-x)}}{27\,l\,EI}$	(37)
VI		$\frac{Wx}{48EI}(3l^2-4x^2)$	(38)
VII		$\frac{5Wl^3}{384EI}$	(39)
VIII		$\frac{3Wl^3}{320EI}$	(40)

TABLE IV—(Continued)

Case	Method of Loading	Deflection Inches	
IX		$\frac{Wl^3}{60EI}$	(41)
X		$\frac{47\,Wl^3}{3{,}600EI}$	(42)
XI		$\frac{3\,Wl^3}{322\,EI}$	(43)
XII		$\frac{5\,Wl^3}{926\,EI}$	(44)
XIII		$\frac{Wl^3}{192\,EI}$	(45)
XIV		$\frac{Wl^3}{384\,EI}$	(46)
XV		For overhang: $\frac{Wx}{12\,EI}(3\,x\,l - 4\,x^2)$	(47)
		For part between supports: $\frac{Wx}{16EI}(l - 2\,x)^2$	(48)
XVI		Variable	

SOLUTION.—The formula for the deflection, in this case, from the table, is $\frac{W l^3}{60 E I}$. From Art. **39**, the value of the modulus of elasticity is found to be 1,200,000. The moment of inertia of the section, from the formula $I = \frac{b d^3}{12}$, is $I = \frac{12 \times 16^3}{12} = 4{,}096$. Then, by substituting the given values, the deflection is

$$\frac{12{,}000 \times 240^3}{60 \times 1{,}200{,}000 \times 4{,}096} = .56, \text{ about } \tfrac{9}{16} \text{ in. Ans.}$$

EXAMPLES FOR PRACTICE

1. The moment of inertia of a 12-inch steel I beam is 228 3, and its span is 25 feet. If the ends of the beam are simply supported, what will be its deflection under a concentrated load of 10,000 pounds suspended from its center? Ans. .85 in.

2. A cantilever beam of 12″×16″ yellow pine extends from a building wall 10 feet, and is loaded on the end with a concentrated load of 12,500 pounds; what will be the greatest deflection of the beam? Ans. 1.46 in.

3. The span of a 15-inch steel I beam is 30 feet, and the moment of inertia of its section is 455.8; the load on the beam is uniformly distributed and amounts to 3,000 pounds per lineal foot. If the ends of the beam are firmly fixed, what will be its deflection? Ans. .83 in

BEAMS AND GIRDERS

(PART 2)

WOODEN AND STEEL BEAMS

FORMULA OF GENERAL APPLICATION

1. In every section of a beam under stress, each fiber offers against rupture a resistance whose moment is equal to the resisting force of the fiber multiplied by its perpendicular distance from the neutral axis of the section. The sum of these fiber moments is called the **resisting moment** of the section, and is equal to the bending moment producing the stress. By higher mathematics, it is proved that the resisting moment is equal to the product of the greatest unit stress in any part of a section multiplied by a factor, called the **section modulus,** or the **resisting inches,** which depends on the shape of the section; the latter term is now rarely used.

If the greatest unit stress is assumed to be the **modulus of rupture** or ultimate fiber stress of the material composing the beam, the following rule may be stated:

Rule I.—*To find the ultimate resisting moment of a beam, multiply the section modulus by the modulus of rupture of the material of which the beam is composed.*

The moduli of rupture for the materials used in building construction may be obtained from the table in *Materials of Structural Engineering*, Part 3.

From rule I; the ultimate resisting moment, which is represented by M_1, is equal to the product of the section modulus and the modulus of rupture, or $S s$. If M represents the bending moment, the beam is loaded to its maximum capacity when M and M_1 are equal; that is, the load applied to produce this condition would be the breaking load. For a uniformly distributed load the formula for the bending moment on a beam is $M = \frac{W L}{8}$, in which W equals the total load in pounds, and L the span of the beam in feet.

The bending moment is expressed in foot-pounds while the resisting moment is in inch-pounds; therefore, in equating these values it is necessary to reduce both values to the same denomination, which is accomplished by dividing $S s$ by 12. This gives the equation

$$\frac{W L}{8} = \frac{S s}{12}, \text{ then, } W = \frac{8 \, S s}{12 \, L}, \text{ or}$$

$$W = \frac{2 \, S s}{3 \, L} \qquad (1)$$

in which S = section modulus;
s = modulus of rupture;
L = span of beam, in feet;
W = total load, in pounds.

The uniformly distributed load that will break a beam whose size is known is determined by formula **1**, which may be stated by the following rule:

Rule II.—*To determine the uniformly distributed load, in pounds, that will break a beam, multiply twice the section modulus by the modulus of rupture and divide this product by three times the span, in feet.*

EXAMPLE —Taking the modulus of rupture of hemlock at 3,500 pounds and using a factor of safety of 4, what uniformly distributed load can be safely carried by a hemlock beam 8 inches by 12 inches, the span being 25 feet?

SOLUTION —Section modulus $= \frac{b \, d^2}{6} = \frac{8 \times 12^2}{6} = 192$ The formula for the breaking load is $W = \frac{2 \, S s}{3 \, L}$; hence, using a factor of safety

of 4, the safe uniformly distributed load will be $\frac{2\,Ss}{4 \times 3\,L}$. Substituting the values in this formula, the safe load is

$$\frac{2 \times 192 \times 3{,}500}{4 \times 3 \times 25} = 4{,}480 \text{ lb. Ans.}$$

WOODEN BEAMS

2. In practice, it is often necessary to determine what size beam will be required to carry a given load; the formula to be used in such a case is obtained by transposing the values in formula **1**. Thus,

$$W = \frac{2\,Ss}{3\,L},\ 3\,WL = 2\,Ss,\ \frac{3\,WL}{2\,s} = S, \text{ or}$$

$$S = \frac{3\,WL}{2\,s} \qquad (2)$$

This formula will give the value of the section modulus that the beam must have in order to furnish the necessary resistance. Then, for a rectangular beam, the formula $S = \frac{b\,d^2}{6}$ is used, in which the value of S found from the previous formula is substituted and a width or depth is assumed. The other dimension may be readily determined. The following example will illustrate this method:

EXAMPLE.—What size yellow-pine beam is required to support a uniformly distributed load of 500 pounds per foot over a span of 10 feet, the modulus of rupture of yellow pine being 6,000, and a factor of safety of 4 being used?

SOLUTION.—The safe strength of the wood, using 4 as a factor of safety, is 6,000 ÷ 4 = 1,500 lb. per sq. in. Substituting the values of W, L, and s in the formula $S = \frac{3\,WL}{2\,s}$ gives

$$S = \frac{3 \times (500 \times 10) \times 10}{2 \times 1{,}500} = \frac{150{,}000}{3{,}000} = 50$$

the required section modulus. Assuming 6 in. as the width of the beam, the values of S and b may be substituted in the formula $S = \frac{b\,d^2}{6}$; thus, $50 = \frac{6\,d^2}{6}$; $d^2 = 50$; $d = \sqrt{50} = 7.07$ in. It will therefore be necessary to use a 6″ × 8″ beam. Ans.

3. When a beam is designed to resist the bending moment produced by the loads on it, it will usually be

strong enough to resist the vertical and horizontal shear, so that it is not necessary to take into account these stresses in proportioning the size of the beam. However, they must be considered to some extent in designing the beam at its bearing, and care should be taken that no material is cut away at the bottom. If the section is reduced at the bearing by cutting out the under side, the resistance to the horizontal shear at this point will be partially destroyed.

GENERAL NOTES RELATIVE TO WOODEN BEAMS

4. In the framing of floors it is frequently necessary to frame around chimney breasts, elevator shafts, and similar projections. This construction is generally accomplished by framing or hanging the joists from a beam or girder that extends in front and parallel to the face of the projection. This girder is known as a *header* and is, in timber framing, composed of two or more joists placed side by side and spiked together; it is supported at the ends by similar girders extending parallel with the joists, which are termed **trimmers**; the joists that are framed into the header are called **tail-beams.**

Wooden beams or any other timbers entering a party wall of a building constructed of stone, brick, or iron should be separated from the beam or timber entering in the opposite side of the wall by at least 4 inches of solid masonry.

A header or trimmer more than 4 feet long should be hung in stirrup irons of suitable thickness for the size of the timbers. Patent hangers are also frequently used. All beams, except headers and tail-beams, should have one end resting 4 inches in the wall, or on a girder.

The ends of all wooden floorbeams and roof beams resting on brick walls should be cut to a bevel sloping away from the vertical toward the top, this slope being not less than 3 inches for their depth. Except in framed buildings, a floorbeam or roof beam should not be supported on stud partitions. Also, all wooden beams should be bridged with cross-bridging placed not more than 8 feet apart.

Each tier of beams should be anchored to the side, front, rear, or party walls at intervals of not more than 6 feet, with good, strong, wrought-iron anchors not less than $1\frac{1}{2}$ inches by $\frac{3}{8}$ inch; these anchors should be fastened to the side of the beams by two or more wrought-iron nails at least $\frac{1}{4}$ inch in diameter. Where beams are supported by girders, the latter should be anchored to the walls and fastened to each other by iron straps.

Where the ends of wooden beams rest on girders, they should be butted together and strapped by wrought-iron straps, those beams being strapped that were anchored into the wall. The straps are secured to the beams in the same manner as the anchors. If the beams are not butted on the girder, they should lap each other at least 12 inches and should be well spiked or bolted together.

It is necessary to anchor front and rear walls and all piers to the beams of each story. The same size anchors as are required for the side walls should be used and should extend over four beams.

5. The safe uniformly distributed load that a floorbeam will support, as determined by the New York building laws, is found by multiplying the area, in square inches, by its depth, in inches, and dividing this product by the span of the beam, in feet. This result is then multiplied by the value given in Table I for the wood of which the beam is composed.

TABLE I

Wood	Factor of Strength
Hemlock	70
Spruce	90
Oak	120
White pine	90
Yellow pine	140

EXAMPLE.—What will be the safe uniformly distributed load, in pounds, for a yellow-pine beam 3 inches by 10 inches and having a span of 20 feet?

SOLUTION.—Substituting the values stated in the example and solving according to the method explained above gives

$$\frac{3 \times 10 \times 10}{20} \times 140 = 2{,}100 \text{ lb.}$$

the safe uniformly distributed load. Ans.

FLITCH-PLATE GIRDERS

6. A **flitch-plate girder** is composed of a steel plate having a wooden beam bolted on each side. In designing a girder of this kind, it is necessary to proportion it so that the different parts will deflect equally. The load distribution may be disregarded, as it is the same for the wood and steel; also, the span is the same for each portion of the beam, and hence this value and all constants may be eliminated when considering the tendency of the materials to deflect under the same conditions.

By referring to the deflection formula and disregarding the values just mentioned, it will be observed that in order to have equal deflection in the two materials, the ratio of the load to the product of the modulus of elasticity and the moment of inertia must be the same for each material. When the depth of the wooden beams and the plate are the same, the moment of inertia will vary with the breadth; the other values used in determining this property may be disregarded, as they are constant. Then the ratio will be the load divided by the product of the modulus of elasticity and the width of the section for each material.

As the bending moment M is equal to the section modulus multiplied by the modulus of rupture, the value of the latter may be found by dividing the bending moment by the section modulus. The cross-section of a flitch-plate beam is rectangular; therefore, the section modulus is obtained from the formula $\frac{b\,d^2}{6}$; as the width b is the only value that varies, the others may be disregarded, and it may be said that the modulus of rupture, or unit fiber stress, for each material varies as the ratio of the load to the width of the section for each. When s represents the unit fiber stress; W, the load; and b, the width of the section for one material; and s', W', and b' represent the same values for the other material, the relation stated above may be expressed by the proportion:

$$s : s' = \frac{W}{b} : \frac{W'}{b'}$$

It has been stated that the ratio of the load to the product of the modulus of elasticity and the width of the section for each material must be equal; or, when E and E' represent the moduli of elasticity for the materials, $\frac{W}{E\,b} = \frac{W'}{E'\,b'}$; but, as the unit fiber stress for each material varies in the ratio of the load to the width, the former may be substituted for this ratio in the proportion. Thus, $\frac{s}{E} = \frac{s'}{E'}$, or $s : s' = E : E'$; hence, the unit fiber stress varies as the modulus of elasticity for each material. This proportion must always be true in order to provide for a uniform deflection in the two materials.

EXAMPLE.—If a flitch-plate girder 12 inches in depth is required to resist a bending moment of 300,000 inch-pounds, what thickness of spruce planks will be required, the steel plate being $\frac{1}{2}$ inch thick? The modulus of elasticity for structural steel is 29,000,000 and for spruce 1,200,000, while the safe unit stress for steel may be taken at 15,000 pounds.

SOLUTION.—The ratio of the moduli of elasticity for the two materials is $\frac{1,200,000}{29,000,000} = \frac{12}{290}$. Then the safe unit stress that may be employed for spruce is $\frac{12}{290}$ of 15,000 = 620 lb. per sq. in. The bending moment is equal to the modulus of rupture multiplied by the section modulus, or $M = S\,s$. As in this instance there are two separate beams, there are also two section moduli, viz.: $\frac{b\,d^2}{6}$ and $\frac{b'\,d^2}{6}$, that have to be added together; hence, in this case, $M = \frac{b\,s\,d^2}{6} + \frac{b'\,s'\,d^2}{6}$. By transposing, it is found that $b = \frac{6\,M - b'\,s'\,d^2}{s\,d^2}$. Considering s and b as the values for wood, and substituting in the formula gives

$$b = \frac{6 \times 300,000 - .5 \times 15,000 \times 144}{620 \times 144} = 8+ \text{ in.}$$

Therefore, each plank should be $8 \div 2 = 4$ in. in width. Ans.

7. Flitch-plate girders should have two lines of bolts, as shown in Fig. 1, the distance between centers on each line being twice the depth of the girder, while at each end there should be two bolts in the same vertical line. When a flitch-plate girder is connected to a column and it is desired to have the column flanges concealed, the girder may be notched out to fit over the flange. The floor may then be laid up to

the column and the bolts in the girder arranged as shown in Fig. 2, which is a sectional plan and elevation of a column

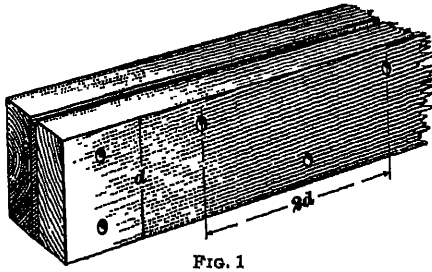


FIG. 1

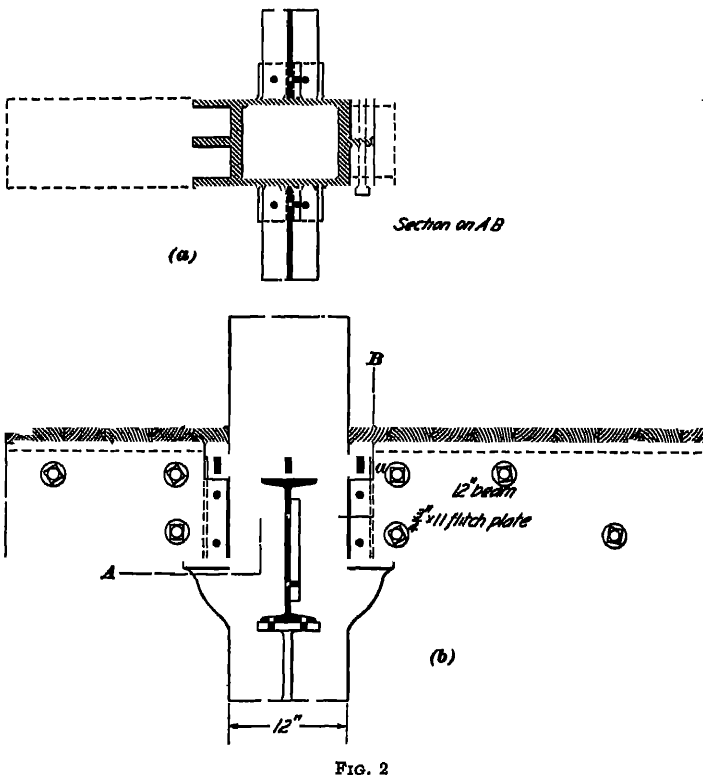


FIG. 2

with two steel and two flitch-plate girders. The flooring shown in the elevation is omitted in the plan.

If a lug projects from the column, the iron plate may be cut away sufficiently to permit the lug to slip between the wooden beams, which should be bolted to it. Where a flitch-plate beam is connected to an I beam or plate girder, an angle may be riveted to the web of the girder to support the flitch plate, and if the floor is to be raised above the top flange of the I beam, it will be necessary to cut the flitch plate in the same manner, as illustrated in Fig. 2 (*b*).

BUILT-UP WOODEN BEAMS

8. Difficulty is frequently experienced in obtaining timbers of the size required for very heavy work; therefore, the method of building up a beam of smaller pieces is sometimes resorted to. The most efficient built-up beam, and the form commonly used, is constructed by securing narrow pieces of material together so that the joints are vertical, as shown in Fig. 3. They may be fastened with through

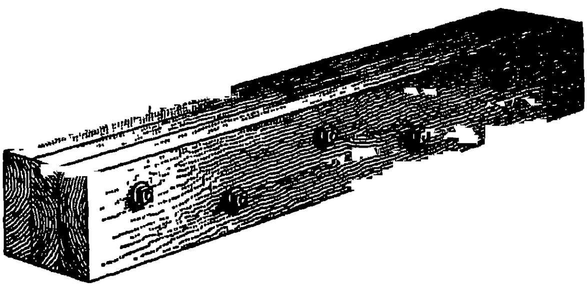


FIG. 3

bolts, which give the more efficient beam, or with lagscrews; in some cases they are simply spiked together. These built-up members are of advantage in trusses for church roofs, especially when it is desired to have the lower member of the truss curved. Fig. 4 shows a portion of a church roof truss, in which the rafter member, as well as the lower chord member, is built up of three pieces $1\frac{7}{8}$ inches thick, secured by lagscrews. Theoretically, a beam built up in this manner is as strong as a solid beam of

the same dimensions, and it is found in many cases to be even stronger. This is due to the fact that a solid beam of large dimensions is liable to have defects in the center of the timber or to be composed partly of the heart wood, which is not so strong as the outer layers. Better material

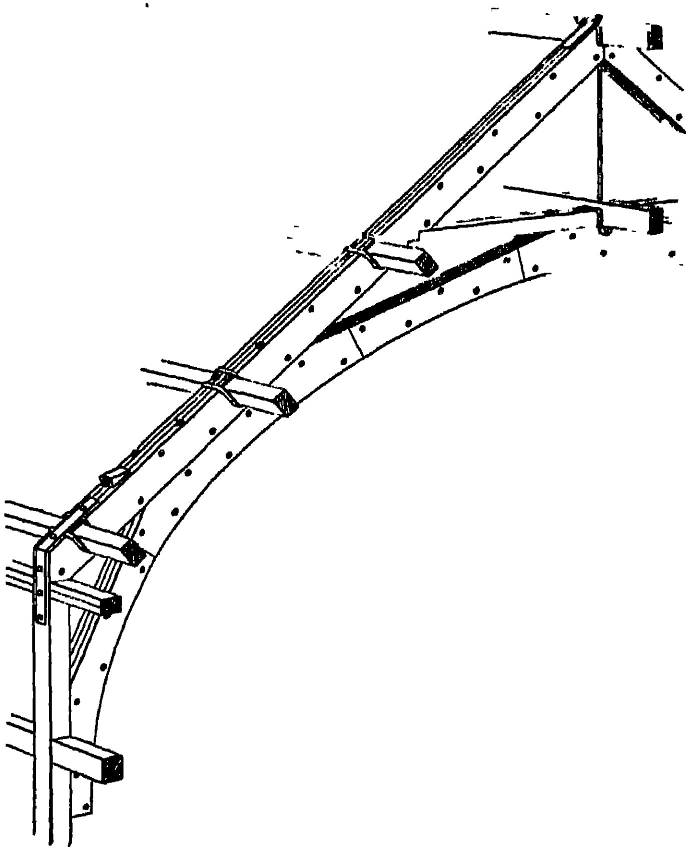


FIG. 4

can be obtained in small pieces and they are more likely to be well seasoned than the larger timbers. By having the lower chord member made up of sections it is also possible to cover up the tension bar entirely, which is sometimes desirable.

9. Another form of built-up beam is one in which the pieces are placed so that the joints are horizontal, as shown in Fig. 5 (*a*). The longitudinal shear produced in this beam

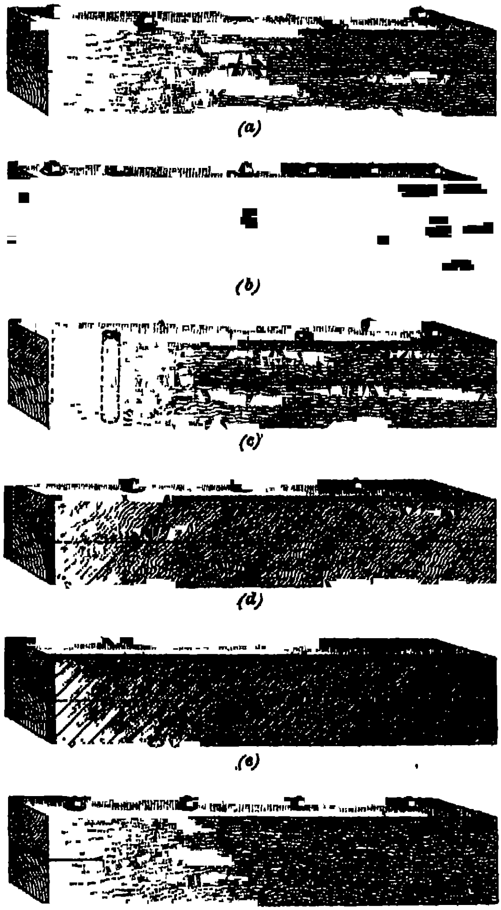


FIG. 5

when it is subjected to transverse stress is resisted only by the bolts. Probably a better construction is one in which hardwood or iron keys are used, in addition to the bolts. This is known as a *keyed beam*, and is shown in (*b*). Another

method of resisting the longitudinal shear is by inserting pieces of pipe in the beam, as shown in (*c*). This is a fairly cheap form, but is inferior to the keyed beam.

10. Clark's design for a built-up beam consists of two pieces, one above the other, fastened by narrow strips on each side, running diagonally, those on opposite sides of the beam running in opposite directions. Sometimes the diagonal strips are wide pieces and the heavy timbers are bolted together. These two forms of the Clark beam are shown in Fig. 5 (*d*) and (*e*), respectively. What is known as the *indented beam*, shown at (*f*), is sometimes used. The notches are intended to resist the horizontal shear, or the tendency of the two pieces to slide on each other, but when shrinkage occurs, the indented surfaces are not held tightly together and the efficiency of the beam is greatly diminished.

11. Efficiency of Built-Up Beams.—Table II gives the average efficiency of several kinds of built-up beams. These results were obtained from tests and the efficiency is the ratio of the strength of a built beam to a solid

TABLE II

EFFICIENCY OF BUILT-UP WOODEN BEAMS

Kind of Beam	Deflection	Efficiency Per Cent.
Indented	2.00	69.5
Clark's	2.00	76.0
Piped	1.70	84.6
Oak keys	1.25	90.6
Flat iron keys	1.50	78.6
Square iron keys	1.50	89.5

one of the same size and quality of material. The deflection is expressed in the same ratio, the deflection of a solid beam being the unit of comparison, so that from the table it may be observed that Clark's and the indented beams

have a deflection equal to twice that of a solid beam under the same condition of loading, while the deflection of the other beams tested and listed varies from $1\frac{1}{4}$ to $1\frac{3}{4}$ times the deflection of a solid beam.

DETAILS OF DESIGN

12. Beam Hangers.—Various devices for supporting the ends of beams where they are carried by headers and trimmers or where they run into the wall, have been manufactured; some of these give very good results, while others are not entirely satisfactory. When the **Goetz hanger,** shown in Fig. 6, is employed the beam is held in place by a nail driven up through the hole *a*; therefore, when the beams are placed opposite on each side of the girder, this arrangement ties the beams together. The pins, or lugs, that enter

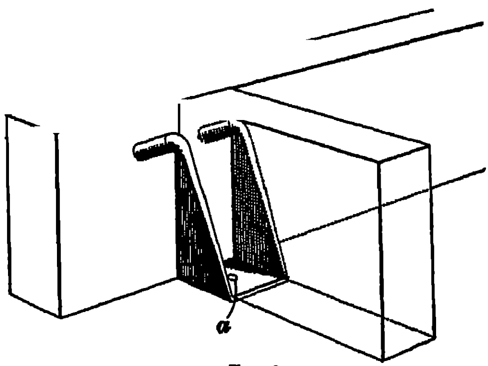
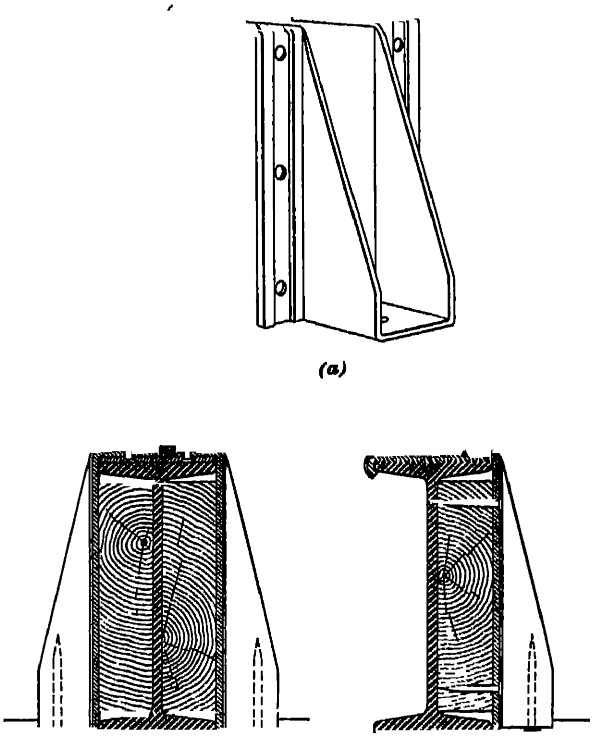


FIG. 6

the main beam are bent at an angle so that the hanger will lock against the beam when the weight is applied to the joist.

The **Van Dorn hanger** for wooden beams is shown in Fig. 7 (*a*), while the double and single hangers used on I beams are shown in (*b*) and (*c*), respectively. The double hanger is formed by bolting two hangers to an iron plate that rests on the top of the I beam; a block of wood is placed on each side of the web of the beam to keep the hanger in position. Fig. 8 shows a special Van Dorn hanger arranged to support joists on the opposite sides of a beam or girder.



(a)

(b) FIG. 7 (c)

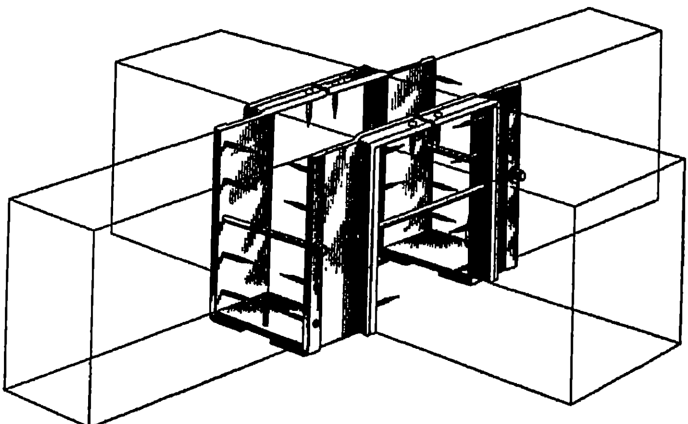


FIG. 8

A **stirrup iron hanger** is made by bending an iron strap to fit over the top of the beam and form a loop to hold the joist. Some special stirrup iron hangers are shown

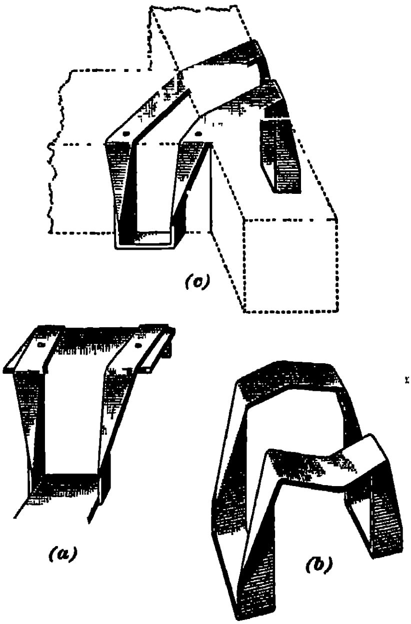


FIG. 9

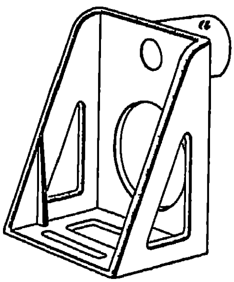


FIG 10

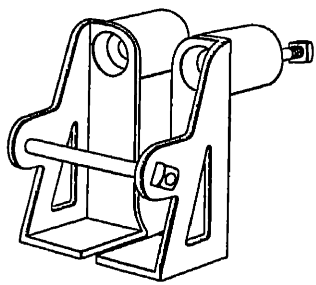
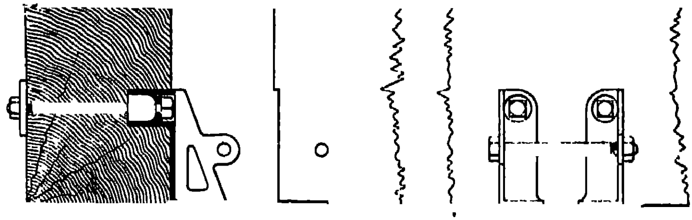
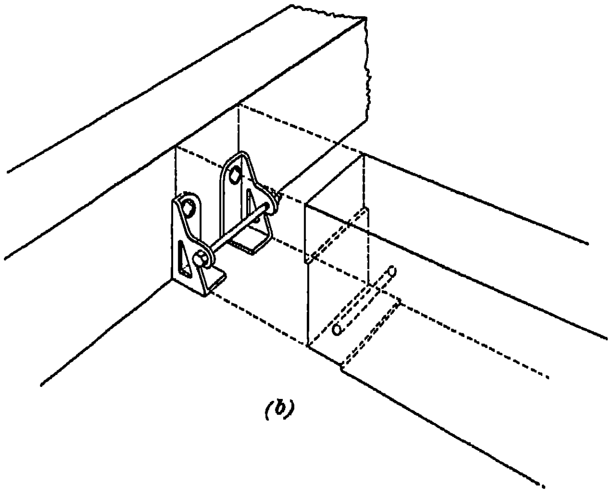


FIG 11

in Fig. 9, the one in (*a*) being a simple hanger with a plate riveted in the loop of the hanger to carry the joist and one on the top to rest on the beam or to be built into the wall.

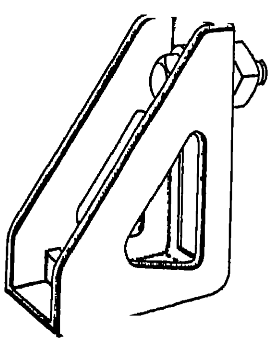


(a)

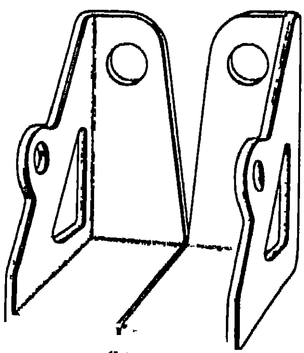


(b)

FIG. 12



(a)



(b)

FIG. 13

In (*b*) is shown a double hanger having one side shorter than the other, while (*c*) illustrates a double hanger that is used where the beams are not opposite each other and do not run in the same direction. These hangers are generally held in place by being nailed to the beams that support them.

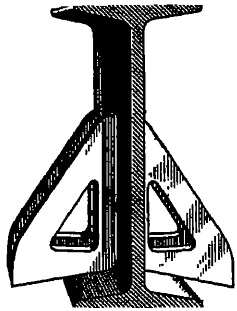
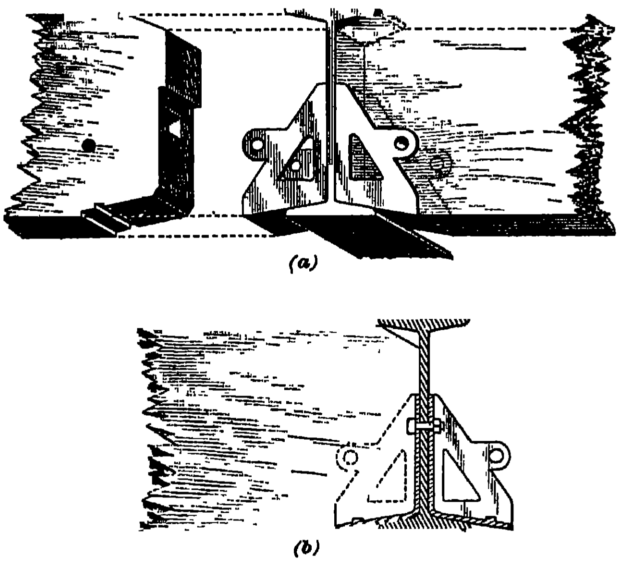


FIG. 14

The **Duplex hanger** for a small joist or beam is shown in Fig. 10; the girder on which it is hung has a hole drilled in it to receive the lug *a*. For wide beams, a double hanger, shown in Fig. 11, is used. The beam is bolted to the hanger, the latter being secured to the girder by bolts run through the lugs that enter the beam. Fig. 12 (*a*) shows a side and front elevation of a double hanger in position, while (*b*) shows a perspective of a



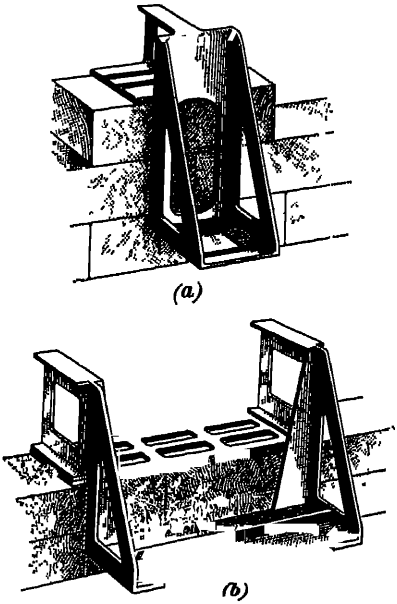
(*a*)

(*b*)

FIG. 15

connection made with such a hanger, the beam being moved out of position and shown in full lines, while the dotted lines

indicate its position when held by the hanger. In Fig. 13 (*a*) and (*b*) are shown the hangers used for I beams; these are made to fit exactly the flange of the I beam and are bolted through the web of the beam. Fig. 14 represents the single hanger, used for narrow beams, in position on the I beam. The double hanger used for heavier construction is shown in position in Fig. 15 (*a*) and (*b*), (*a*) showing the manner in which the end of the wooden beam should be cut to fit in the hanger, while (*b*) illustrates the method of bolting the



(*a*)

(*b*)

Fig 16

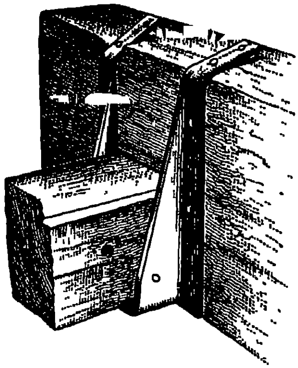


Fig 17

hangers to the I beams, and also shows the rib over which the joists must be cut and which serves as a tie when the latter are in place. The Duplex wall hangers, shown in Fig. 16 (*a*) and (*b*), are used extensively and give satisfactory results.

13. Tests Made on Beam Hangers.—The results of a test made on a Van Dorn hanger, a stirrup iron hanger, and a Duplex hanger are shown in Figs. 17, 18, and 19. In each case, one end of the I beam to which the load was applied rested on a pine block 7 inches thick, that was securely fastened in the hanger; the other end was supported on an iron bar. When 13,300 pounds had been applied to the beam

carried by the Van Dorn hanger, shown in Fig. 17, the hanger began to straighten out, and failed at a load of 18,750 pounds. The stirrup iron hanger failed at a load of 13,750 pounds, by pulling off from the header, as shown in Fig. 18; the crushed parts of the header are the points where the stirrup was hung for the test. The beam supported by the Duplex

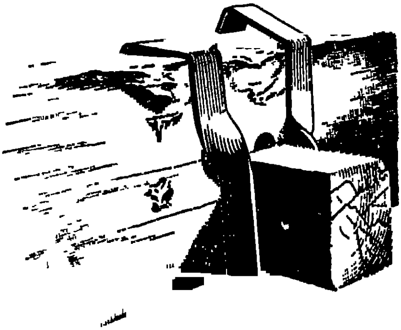


Fig. 18

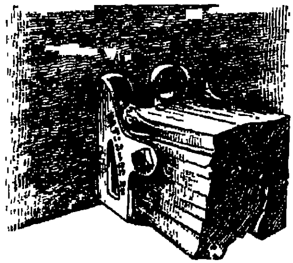


Fig. 19

hanger, Fig. 19, was loaded to 20,000 pounds, when the wood began to show signs of failing, but the hanger held until a load of 39,550 pounds was applied. At this point, one side of the hanger broke off short under the lug projecting into the header. The Duplex hanger was bolted to the header with two $\frac{3}{4}$-inch bolts through the lugs.

14. Heavy Wooden Construction.—In heavy wooden construction, the wooden beams are carried by iron girders and are sometimes supported on beam ledges or blocks bolted to the girder. Fig. 20 shows a 15-inch I beam that carries 8″ × 12″ wooden beams placed opposite each other. These beams rest on beam ledges *a*, of which only one is shown in the illustration. They extend the full length of the girder and are bolted to it with $\frac{3}{4}$-inch bolts. The ledges are notched out to receive the wooden beams and are also cut out to fit around the lower flange of the I beam, extending $\frac{3}{8}$ inch below it. The wooden beams, which are cut out to fit the upper flange of the girder, are notched at the top and a $1\frac{7}{8}$-inch anchor plate is set in the full width of the beams; this arrangement ties the beams together. For

additional stiffness, a $\frac{7}{8}$-inch cleat b is nailed on each side of the beam, extending from the beam ledge to within $\frac{3}{8}$ inch of the top of the wooden beam; the cleat is fastened to the anchor plate and the beam, thus forming a rigid connection. A $\frac{7}{8}$-inch casing is rabbeted into the beams and extends from one beam to the next, thus enclosing the girder and making a neat finish. The only part of the girder that is exposed is the lower flange, but even this is protected to some extent by the beam ledge. Fig. 21 shows an elevation of the girder with the finished casing at (a) and without it at (b).

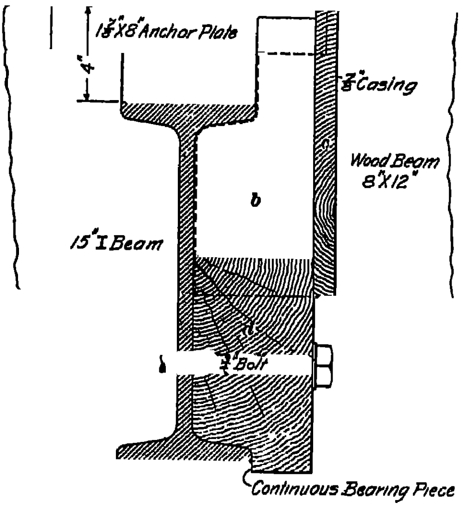


FIG. 20

The arrangement for the casing of a 20-inch girder carrying 8″ × 12″ wooden beams is shown in Fig. 22, in which a, a are the blocks that are placed under each beam and bolted through the web of the girder in two places, the blocks being counterbored to allow the bolt heads to sink below

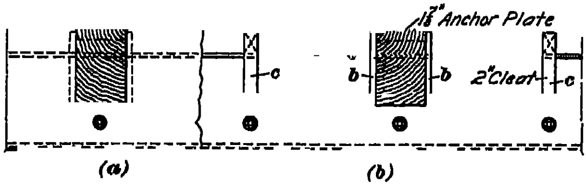


FIG. 21

the surface. The beams are notched and an anchor plate is set in as shown; a cleat, extending from the bottom flange of the I beam almost to the top of the wooden beam, is

spiked to each side, thus keeping the block, beam, and anchor plate in line and securing them rigidly together. The casings *b*, *c*, and *d* and the molding *e* are then put in place, the piece marked *d* being rabbeted into the beam. Though the arrangements shown in Figs. 20 and 22 can hardly be considered as strictly slow-burning construction, they are especially strong and rigid and may be employed with good results in buildings where it is necessary to provide against the effects of vibration.

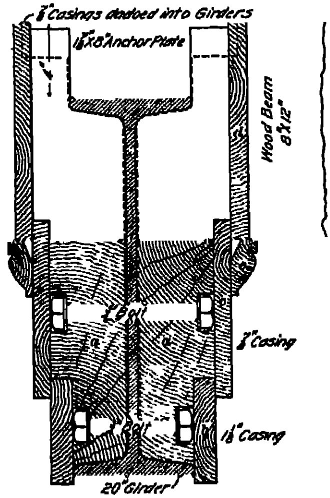


FIG. 22

STEEL BEAMS

15. The **steel beams** used in building construction are generally either channels or I beams. When the word *beam* is used, it is understood that an I beam is meant. For instance, a 12-inch 40-pound beam means an I beam 12 inches in depth, weighing 40 pounds to the lineal foot; a channel of the same size is expressed as a 12-inch 40-pound channel. In designating rolled shapes on working drawings, various systems of abbreviations are used. A 12-inch 40-pound beam may be expressed as 12″ I 40 #, or a channel as 12″ [40 #. This is entirely a matter of judgment with the draftsman, or is governed by the practice used in the particular drafting room. As long as the size, character, and weight of the beam are given, it matters little how they are expressed, if intelligibly written.

TRANSVERSE STRENGTH

16. In calculating the strength of steel beams, it is first necessary to find the bending moment, using the methods and rules already given. Then the section modulus required in the beam may be obtained by dividing the bending moment, in inch-pounds, by the quotient obtained by dividing the modulus of rupture by the factor of safety. Assume, for example, the bending moment on a beam to be 50,000 foot-pounds. Reduce it to inch-pounds by multiplying it by 12, which gives 600,000 inch-pounds. The modulus of rupture for structural steel is 60,000 pounds. If a factor of safety of 4 is used, the safe working value of this material will be 60,000 ÷ 4 = 15,000 pounds per square inch. Then 600,000 ÷ 15,000 = 40, the section modulus required.

The approximate section modulus of an I beam, or a channel, may be found by the following rules:

Rule I.—*To obtain the approximate section modulus of an I beam, multiply the sectional area of the beam, in square inches, by the depth, in inches, and divide by the constant 3.2.*

Rule II.—*To obtain the approximate section modulus of a channel, multiply the sectional area of the channel, in square inches, by the depth, in inches, and divide by the constant 3.67.*

Letting A equal the sectional area, in square inches, of an I beam or a channel, and d its depth, in inches, the approximate section moduli S_i and S_c of an I beam and channel, respectively, may be found from the formulas

$$S_i = \frac{A\,d}{3.2} \qquad (3)$$

and

$$S_c = \frac{A\,d}{3.67} \qquad (4)$$

EXAMPLE.—What is the section modulus of a 12-inch I beam, the sectional area of which is 9.01 square inches?

SOLUTION.—Applying the formula,

$$S_i = \frac{A\,d}{3.2} = \frac{9.01 \times 12}{3.2} = 33.8. \quad \text{Ans.}$$

17. Finding the Dimensions of a Steel Beam.—To illustrate the method of calculating the dimensions of a steel beam, let it be required to find what size steel I beams are necessary to support the floor of an office building; this floor rests on brick arches sprung between the beams, and weighs complete 110 pounds per square foot. The building is designed to carry a live load of 40 pounds per square foot. The span of the beams is 20 feet and they are spaced 5 feet on centers. The owner requires that the building have a large factor of safety, and suggests that for the floorbeams a safety factor of 5 be used. The total dead and live load on the floor is 110 pounds + 40 pounds = 150 pounds per square foot. The floor area supported by one beam is $20 \times 5 = 100$ square feet. Then the total load on one beam is $100 \times 150 = 15{,}000$ pounds. The load being uniformly distributed, the formula for the bending moment is $M = \frac{WL}{8}$; substituting the values for W and L, $M = \frac{15{,}000 \times 20}{8} = 37{,}500$, the bending moment in foot-pounds, which, being multiplied by 12, equals 450,000 inch-pounds.

The modulus of rupture for structural steel is 60,000 pounds per square inch, and, since a factor of safety of 5 is required, the safe working value will be $60{,}000 \div 5 = 12{,}000$ pounds per square inch. The bending moment in inch-pounds is 450,000, which, divided by 12,000, gives a section modulus of 37.5.

From the table Properties of Standard I Beams in *Properties of Sections*, it is found that the section modulus of a 12-inch 35-pound beam is 38; hence this size beam should be used.

In selecting beams from the table, care should be taken to obtain the deepest beam of the least weight with the required section modulus. Thus, from the table, it is found that the section modulus of a 10-inch beam, weighing 40 pounds, is 31.7, while a 12-inch beam of 40 pounds, or the same weight as the 10-inch beam, has a section modulus of 41, and, in consequence, possesses nearly one-third more strength, making it, therefore, the more economical beam to use.

18. By way of general review of the subject of beams, the following practical example will be considered: Fig. 23

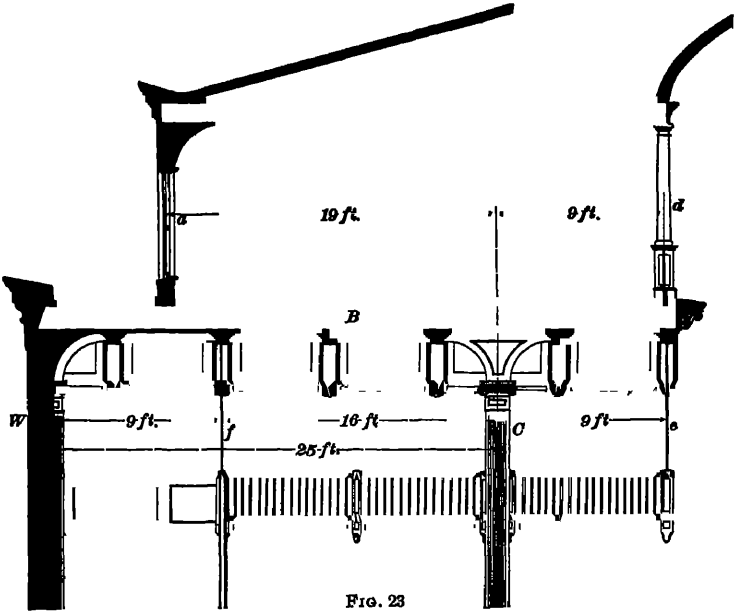


FIG. 23

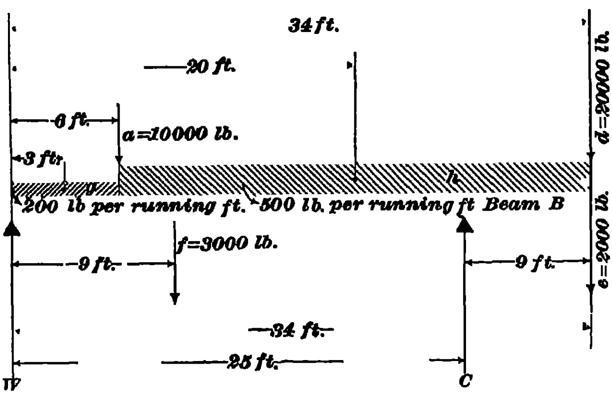


FIG 24

shows the transverse sectional elevation of a large department store, in which the girder *B* is made up of two I beams. What is the size and weight of these steel beams?

Before commencing the calculations, draw the outline diagram, as shown in Fig. 24; this is called a frame diagram. The two supports for the girder are the wall *W* and the column *C*. The loads *g*, *h* on the girder are uniform; the load *h* is due to the weight of the floor, girder, and the ceiling, together with the live load on the floor due to the people, furniture, etc. This load has been assumed to amount to 500 pounds per running foot of the girder. The load *g*, being due only to the ceiling and a portion of the roof, and there being no floor load on it, has been considered as amounting to 200 pounds per running foot.

The girder is also loaded with four concentrated loads: *a* of 10,000 pounds, due to the weight of the light wall and a portion of the roof; *d* of 20,000 pounds, due to the load coming down the small column from a portion of the roof; and two hanging loads *f* and *e*, of 3,000 and 2,000 pounds, respectively, from the weight of the stair landing or hall.

The reactions may now be calculated. The moments about *W* are as follows:

DUE TO		FOOT-POUNDS
Load *g* ($200 \times 6 = 1{,}200$ lb.) .	$1{,}200 \times 3 =$	3600
Load *h* ($500 \times 28 = 14{,}000$ lb.) .	$14{,}000 \times 20 =$	280000
Load *a*	$10{,}000 \times 6 =$	60000
Load *f*	$3{,}000 \times 9 =$	27000
Load *d*	$20{,}000 \times 34 =$	680000
Load *e*	$2{,}000 \times 34 =$	68000
Total moments	=	1118600

This, divided by the distance between the supports, or the span, 25 feet, will give 44,744, the load, in pounds, coming on the column *C*; or, in other words, the reaction at *C*. The loads are as follows:

Load *g* =	1200	pounds
Load *h* =	14000	pounds
Load *a* =	10000	pounds
Load *f* =	3000	pounds
Load *d* =	20000	pounds
Load *e* =	2000	pounds
Total load =	50200	pounds

Then, the reaction at *W* is 50,200 − 44,744 = 5,456 pounds.

Find the point between the two supports *W* and *C*, where the shear changes sign.

Working out on the beam from *W*, the first load encountered and to be deducted from the reaction *W* is the uniform load *g*, equal to 200 × 6 = 1,200 pounds. Then, 5,456 (reaction at *W*) − 1,200 (load *g*) = 4,256 pounds. The next load on the beam is the concentrated load *a* of 10,000 pounds, which is much more than the remaining portion of the reaction *W*. The greatest bending moment occurring between the column and the wall is, therefore, at the point *a*, and is equal to 5,456 (reaction at *W*) × 6 = 32,736 foot-pounds, from which is to be taken the moment of the load *g* of 1,200 × 3 = 3,600 foot-pounds. Then, 32,736 − 3,600 = 29,136 foot-pounds, the greatest bending moment between the supports *C* and *W*.

Again referring to the diagram, Fig. 24, it is seen that there is quite a bending moment directly over the column *C*, due to the two concentrated loads *d*, *e* on the end of the beam and the portion of the uniform load *h* overhanging the support *C*. This portion of the beam may be considered as a cantilever; the bending moment at *C* is equal to the sum of the moments of all the loads on the overhanging portion of the beam, which are:

Load		Foot-Pounds
d	20,000 × 9.0 =	180000
e	2,000 × 9.0 =	18000
h (500 × 9 = 4,500 lb.) .	4,500 × 4.5 =	20250
Total moments	=	218250

218,250 × 12 = 2,619,000 inch-pounds. This, divided by 20,000, the safe working value of structural steel (using the modulus of rupture of 60,000 pounds ÷ 3, the safety factor used in this case) = 131, the required section modulus in the two beams. Then, 131 ÷ 2 = 65.5, section modulus required in one of the beams.

From the table in *Properties of Sections*, it is found that the section modulus of a 15-inch 55-pound beam is 68.1. The 18-inch beam of the same weight has a section modulus

of 88.4, and while this is in excess of the required amount, it is the preferable beam to use, two of the kind being required.

19. Relation Between Span and Depth of Beam.—In order to select beams that will not deflect too much under the load that they are required to sustain, the depth of the beam, in inches, should never be less than half the span of the beam, in feet. Thus, if the span of the beam be 20 feet, a beam not less than 10 inches in depth should be used to avoid excessive deflection.

20. Separators for I Beams.—In building construction, it frequently happens that a single I beam is insufficient to carry the imposed load. Where heavy loads, such as brick walls, vaults, etc., are to be supported, a single I beam is inadequate; therefore, two or more beams are placed side by side and bolted together, with steel **separators** between, as shown in Fig. 25, or with cast-iron separators, as shown in Fig. 26 (*a*) and (*b*). In (*a*) is shown a type of cast-iron separator so formed that the bolts pass through a hole in the

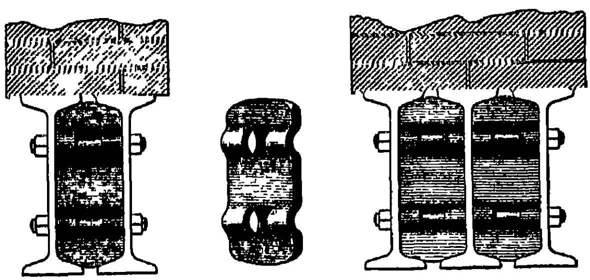
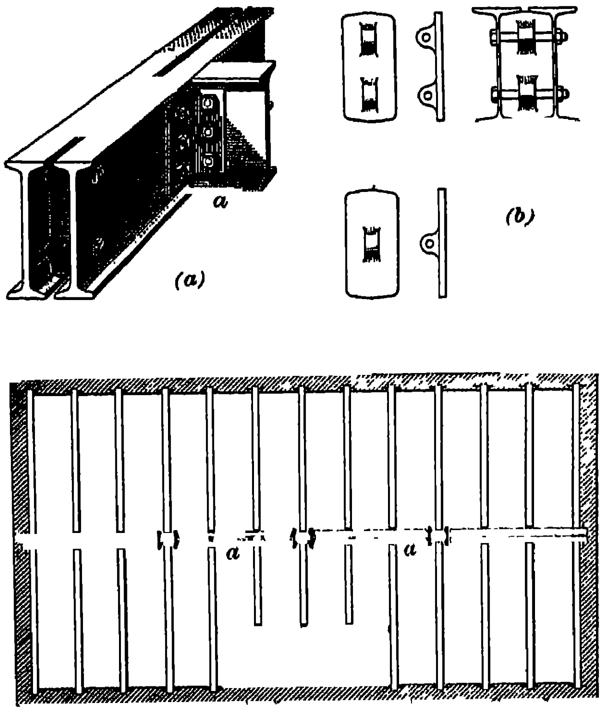


FIG. 25

swollen portion of the casting. Another pattern of cast-iron separator is shown in (*b*), in which the connecting bolt passes through a lug cast on the separator. A common use for a double beam secured together by bolts and separators is shown at *a*, *a*, Fig. 26 (*c*). These separators hold the compression flanges of the beams in position, preventing deflection sideways, and also, in a measure, cause the beams to act together, distributing the load uniformly on both. Separators should be spaced from 6 to 7 feet apart

throughout the length of the beam; they should also be provided at the supports and at the points where heavy loads are concentrated.

Standard separators may be obtained in such widths that the inner edges of the flanges of the two beams connected are about ¼ inch apart. For beams 10 inches in depth and under, one ¾-inch bolt is used through the separator, while



(c)
FIG. 26

two ¾-inch bolts are used for connecting beams over 10 inches in depth. Where two bolts are used, the distance between centers is usually made 10 inches for beams 18 and 20 inches in depth, 7 inches for 15-inch beams, and 6 inches for 12-inch beams. Separators for 18-inch and 20-inch beams weigh about 20 pounds each; for 15-inch beams, about 12 pounds;

for 12-inch beams the weight is from 8 to 10 pounds; for 8-, 9-, and 10-inch beams, from 5 to 7 pounds; and for 4-, 5-, and 6-inch beams, from 1 to 4 pounds.

21. I Beam Girders.—In designing floors of buildings it is desirable to have a minimum number of interior supporting columns, consistent with economy. A beam girder consisting of a pair of I beams is frequently advantageous for supporting the steel floorbeams, as shown at *a* in Fig. 26 (*a*).

Girders composed of two or more I beams are commonly used to span openings in brick walls. If the wall to be supported is thoroughly seasoned and without openings, the

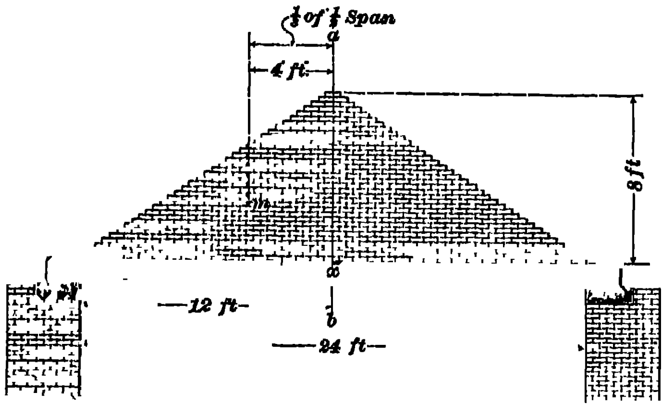


FIG. 27

weight carried by the girder can safely be assumed as the weight of a triangular piece of brickwork, whose altitude is one-third of the span of the girder. If the wall is newly built, or has openings for windows or other purposes, the girder must be designed to carry the entire wall above it between the supports.

EXAMPLE.—Required, the size of a steel I-beam girder to carry a wall 12 inches thick, made of hard brick laid in lime mortar; there are no openings in the wall above the girder, nor does the wall support floor joists or roof beams, while the span of the opening is 24 feet.

SOLUTION—Draw the diagram as shown in Fig. 27 The area of the triangular piece of brickwork is 24 × 4 = 96 sq. ft., since the area of a triangle is equal to the base, multiplied by one-half the altitude.

As the wall is 1 ft. thick, there are 96 cu. ft. in this triangular piece. The weight of brickwork in lime mortar per cubic foot is 120 lb. Then, 96 × 120 = 11,520 lb., the load on the girder

The bending moment may be determined by the formula or rule given in *Beams and Girders*, Part 1, for a beam carrying a triangular load, or it may be determined by calculating the moments, as follows: The reactions at the two supports are each equal to half the load, or 11,520 ÷ 2 = 5,760 lb. The greatest bending moment is at the center of the beam. Then, the moment of the reaction about the point x is 5,760 × 12 = 69,120 ft.-lb. But, counteracting this, and to be deducted from it, is the moment of the load at the left of x, equal to half of the triangular piece of brickwork. The moment of this load about the point x is equal to the product of its weight, multiplied by the horizontal distance from a vertical line through its center of gravity to the point x. Take the line $a\,b$ as the base of a triangle, remembering that a line drawn parallel to the base line of a triangle, at a distance of one-third of the altitude from it, always passes through its center of gravity. Now, the distance from the point x to the vertical line through the center of gravity m of the triangle is 4 ft., and the moment due to the triangular piece of brickwork to the left of the center is 5,760 × 4 = 23,040 ft.-lb. Deducting this from the moment of the reaction already found, the calculation is. 69,120 − 23,040 = 46,080 ft.-lb., the bending moment on this beam or girder, or, 46,080 × 12 = 552,960 in.-lb. This calculation may be checked by applying the formula $\frac{WL}{6}$. The bending moment, in inch-pounds, being 552,960, using a safe working value, or fiber stress, of 15,000 lb., the section modulus required would be 552,960 ÷ 15,000 = 36.86. From the table in *Properties of Sections*, it is seen that the section modulus of a 12-in. 35 lb. beam is 38, which gives the required strength in this case. It may be found to be better practice to use two channels instead of one I beam, for the top flange of the I beam may be too narrow to properly support the brick wall, while the two channels placed side by side, with separators between, could be made of the same thickness as the wall.

22. Connections.—The standard connections for the principal sizes and weights of steel I beams are illustrated in Fig. 28. These connections are based on an allowable shearing stress of 10,000 pounds per square inch, a bearing stress of 20,000 pounds per square inch on the bolts, and an extreme fiber stress of 16,000 pounds. They may be used for beams whose spans are not less than those given in Table III.

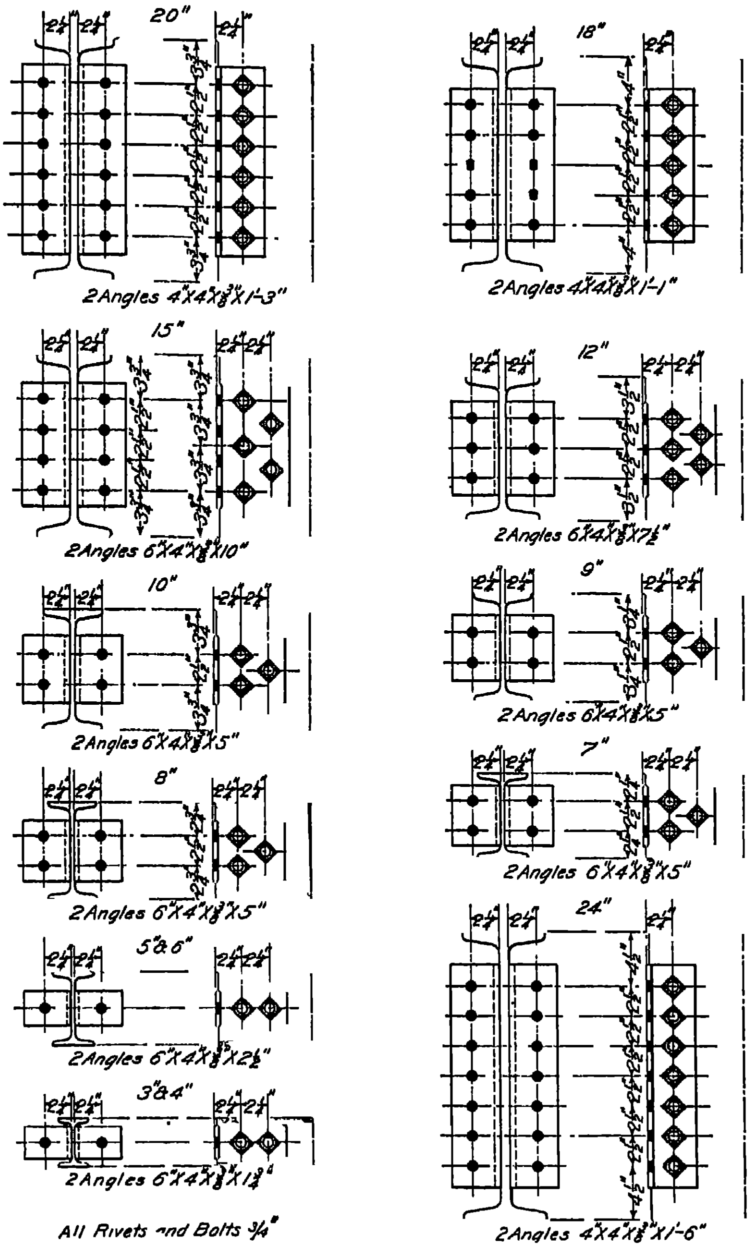

20"
2Angles 4"X4"X3/8"X1'-3"
18"
2Angles 4"X4"X3/8"X1'-1"
15"
2Angles 6"X4"X3/8"X10"
12"
2Angles 6"X4"X3/8"X7½"
10"
2Angles 6"X4"X3/8"X5"
9"
2Angles 6"X4"X3/8"X5"
8"
2Angles 6"X4"X3/8"X5"
7"
2Angles 6"X4"X3/8"X5"
5"&6"
2Angles 6"X4"X3/8"X2½"
24"
3"&4"
2Angles 6"X4"X3/8"X1¾"
2Angles 4"X4"X3/8"X1'-6"
All Rivets and Bolts 3/4"

Fig 28

TABLE III

MINIMUM SPANS FOR I BEAMS, WITH STANDARD CONNECTIONS, THAT ARE UNIFORMLY LOADED TO REALIZE A UNIT STRESS OF 16,000 POUNDS

Depth of Beam	Weight per Foot	Minimum Safe Span	Depth of Beam	Weight per Foot	Minimum Safe Span	Depth of Beam	Weight per Foot	Minimum Safe Span
Inches	Pounds	Feet	Inches	Pounds	Feet	Inches	Pounds	Feet
3	5.50	1.7	9	30.0	6.8	15	80.0	15.9
3	6.50	1.2	9	35.0	7.5	15	85.0	16.4
3	7.50	1.2	10	25.0	9.3	15	90.0	17.0
4	7.50	2.8	10	30.0	8.1	15	95.0	17.5
4	8.50	2.2	10	35.0	8.8	15	100.0	18.1
4	9.50	2.0	10	40.0	9.6	18	55.0	13.7
4	10.50	2.2	12	31.5	7.3	18	60.0	11.9
5	9.75	4.1	12	35.0	7.7	18	65.0	11.8
5	12.25	3.3	12	40.0	8.2	18	70.0	12.4
5	14.75	3.7	12	45.0	9.6	20	65 0	13.9
6	12.25	5.6	12	50.0	10.2	20	70.0	12.5
6	14.75	4.8	12	55.0	10.8	20	75.0	12.8
6	17.25	5.3	15	42.0	10.2	20	80.0	14.8
7	15.00	4.9	15	45.0	9.4	20	85.0	15.2
7	17.50	3.8	15	50.0	9.7	20	90.0	15.7
7	20.00	3.6	15	55.0	10.3	20	95.0	16.2
8	18.00	6.2	15	60.0	10.8	20	100.0	16.7
8	20.25	5.1	15	65.0	12.8	24	80.0	17.7
8	22.75	4.8	15	70.0	13.4	24	85.0	16.1
8	25.25	5.1	15	75.0	13.9	24	90.0	16.1
9	21.00	7.7	15	80.0	14.5	24	95.0	16.6
9	25.00	6.2				24	100.0	17.1

Beams that are shorter than this are, as a consequence, able to support greater loads without exceeding the unit stress of 16,000 pounds on which the standard connections are based. The joints illustrated would, in such cases, be exposed to a stress greater than that for which they are

intended. To avoid such excessive stresses the minimum safe spans have been specified in Table III for the various beams. The larger beams to which these connections are made are supposed to have webs not less than $\frac{9}{16}$ inch in thickness. If these beams or girders are framed opposite one another, into another beam or girder whose web is less in thickness than $\frac{9}{16}$ inch, the length of the minimum spans should be increased. The reason for this is that a thinner web cannot carry so large a load. To prevent the overloading of the connections it is necessary to increase the length of the spans of the adjoining beams in the ratio of the $\frac{9}{16}$-inch web to the thickness of the thinner web.

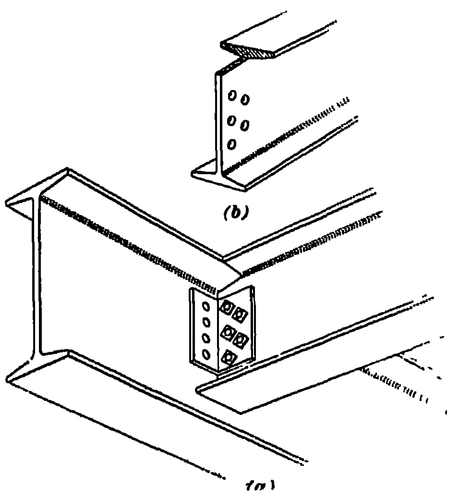


FIG 29

For instance, a beam 7 inches deep, weighing 20 pounds per foot should, according to Table III, have a minimum span of 3.6 feet when connected to a beam with a $\frac{9}{16}$-inch web. In case the latter web is only $\frac{3}{8}$ inch thick, the following proportion will give the length x of the longer beam required:

$$x : 3.6 = \frac{9}{16} : \frac{6}{16}, \text{ or } x = \frac{3.6 \times \frac{9}{16}}{\frac{6}{16}} = 5.4 \text{ feet}$$

The standard connections given are designed for $\frac{13}{16}$-inch holes and $\frac{3}{4}$-inch diameter rivets or bolts. Connection angles may, if so specified, be riveted instead of bolted to the beams; but, unless otherwise ordered, bolted connections are generally used.

When beams having very short spans are loaded to their full capacity, the shear at the end, which must be transmitted through the connections, becomes so great that connections

stronger than the standard must be used. Table III gives the limits of lengths below which the standard connections cannot be used and below which special designs should be made.

Where floorbeams of different depths are to be joined to a larger girder and the tops or bottoms of the beams are to be level, a special construction is required. An example of this is given in Fig. 29 (*a*), in which two beams are joined with their tops flush. It was necessary to cut the flange of one enough to receive half the flange of the other, as shown in (*b*). This method of cutting the beam is called *coping*.

LATERAL STRENGTH OF BEAMS

23. Obtaining the Fiber Stress of Beams.—Where beams are subjected to a horizontal force in addition to the vertical load, some means must be provided to resist the lateral flexure. The beams in a floor composed of arches receive a horizontal thrust in addition to the vertical load. It is assumed that, for the interior beams, the thrusts on opposite sides, due to the dead load, balance each other; therefore, the horizontal thrust is calculated for the live load only. However, in beams which receive a thrust on one side only, as end beams, and beams around shafts and wells, the total load is used in figuring the horizontal thrust. The total stress, due to both the horizontal and vertical loads, should not exceed the allowable fiber stress. The stress due to the lateral force is obtained by the formula

$$s_h = \frac{T c x^2}{I'} \qquad (5)$$

in which s_h = stress, in pounds per square inch, due to lateral forces;

T = thrust of arch, in pounds per lineal foot;

c = distance of extreme fiber from neutral axis, in inches;

x = distance between tie-rods or lateral supports, in feet;

I' = moment of inertia about vertical axis of section, or axis at right angles to line of application of lateral forces.

When the web of the I beam is placed vertically, as usual, c is equal to $\frac{b}{2}$, b representing the width of the flange in inches. Then the preceding formula becomes

$$s_h = \frac{T b x^2}{2 I'} \qquad (6)$$

The formula is derived from the equation, bending moment M = resisting moment M_1.

The portion of the beam between two adjacent tie-rods may be regarded as a beam with fixed ends; the bending moment for a uniform load will then be $\frac{WL}{12}$. The distance between supports is represented by x in formula 5; therefore, x will be used in this formula instead of L, while instead of W, the thrust per lineal foot, or T, will be used. The load is equal to Tx and the bending moment created by it is $\frac{Tx \times 12x}{12}$, or Tx^2, considering the span in inches, as usual.

The resisting moment is equal to the section modulus multiplied by the safe unit fiber stress, or Ss. As explained in *Properties of Sections*, the section modulus is equal to the moment of inertia divided by the distance from the neutral axis to the farthest edge of the section, or $S = \frac{I}{c}$. In this case, the moment of inertia is taken with respect to the neutral axis parallel to the web; consequently, c equals half the width of the flange, or $\frac{b}{2}$, and $S = \frac{2I}{b}$. Hence, substituting the values thus obtained for M and M_1 gives $Tx^2 = \frac{2Is}{b}$. Then, $s = \frac{Tbx^2}{2I}$.

24. Obtaining the Horizontal Thrust.—The horizontal thrust is found by the formula

$$T = \frac{3 w L^2}{2 r} \qquad (7)$$

in which T = pressure or thrust, in pounds per lineal foot of arch;
w = load on arch, in pounds per square foot, uniformly distributed;
L = span of arch, in feet;
r = rise of arch, in inches.

The thrust of an arch having a concentrated load at the center equal to W, is found from the formula

$$T = \frac{3WL}{r} \qquad (8)$$

If the floor arch is flat, the value of r may be taken as the effective depth of the arch, as will be explained later.

25. Obtaining the Resultant Stress.—In a simple beam a vertical load produces a maximum compressive stress of s_v at the extreme top fibers and a maximum tensile stress of s_v in the extreme bottom fibers. A horizontal load, as the thrust of an arch, will produce a compressive stress of s_h in the extreme fibers on the side toward the thrust and a tensile stress of s_h on the fibers farthest from the thrust. Therefore, the compressive stress due to both vertical and horizontal loads, on the upper surface of the beam at the corner toward the horizontal thrust, is $s_v + s_h$. On the corner of the beam diagonally opposite, there is a tensile stress of $s_v + s_h$. On the other corners the resultant stresses are $s_v - s_h$ and $s_h - s_v$.

26. Therefore, $s_v + s_h$ should equal the safe working stress per square inch of the material in the beam. It is customary when combining stresses due to vertical loading and horizontal arch thrust, to use a little higher safe fiber stress than is usual under other circumstances, because the mortar in the joints of the arch takes up a considerable part of the horizontal thrust. For interior beams, the safe stress is taken at 20,000 pounds per square inch and for exterior beams, 18,000 pounds is allowed; hence, the formula for interior beams may be written

$$s_v + s_h = 20{,}000 \text{ pounds} \qquad (9)$$

and for exterior beams,

$$s_v + s_h = 18{,}000 \text{ pounds} \qquad (10)$$

In each of the above formulas, however, s_v must not exceed 16,000 pounds, which is the same quantity that is usually taken for the safe working unit tensile stress.

EXAMPLE.—Determine whether 15-inch 42-pound I beams have sufficient strength to support a brick-arch floor over a span of 15 feet. The rise of the arches is assumed to be 6 inches, the span is taken at 4 feet, and it is decided to place the tie-rods 5 feet apart. The dead load is 150 pounds per square foot and the live load 200 pounds per square foot.

SOLUTION.—The thrust of the arch, due to the live load, is obtained from formula 7, $T = \frac{3\,w\,L^2}{2\,r}$ where w = 200 pounds per square foot, L = 4 feet, and r = 6 inches; therefore,

$$T = \frac{3 \times 200 \times 4 \times 4}{2 \times 6} = 800 \text{ lb. per ft.}$$

From the table giving the properties of standard I beams in the paper on *Properties of Sections*, the moment of inertia with respect to an axis parallel to the web of a 15-in. 42-lb. I beam is 14.62 and the width of the flange b is 5.50 Then substituting in formula 6 for the live load only, $s_h = \frac{800 \times 5.50 \times 5 \times 5}{2 \times 14.62} = 3{,}762$ lb., and for the total load, $s_h = \frac{1{,}400 \times 5.50 \times 5 \times 5}{2 \times 14.62} = 6{,}583$ lb.

The section modulus of this beam, obtained from the same table in *Properties of Sections*, is 58.9; therefore,

$$s_v = \frac{W\,l}{8\,S} = \frac{350 \times 4 \times 15 \times 15 \times 12}{8 \times 58.9} = 8{,}022 \text{ lb.}$$

Now for the inside beams $s_v = 8{,}022$, which is less than the allowable stress of 16,000, and $s_v + s_h = 8{,}022 + 3{,}762 = 11{,}784$, which is less than the allowable stress of 20,000; therefore, the inside beams are sufficiently strong. For the outside beams $s_v + s_h = 4{,}011 + 6{,}583 = 10{,}594$, which is less than the allowable stress of 18,000; hence, these beams are also amply strong.

SPACING OF TIE-RODS

27. When an arch is placed between two beams, tie-rods should be supplied to resist its thrust. The conditions are shown in Fig. 30, in which $a\,b\,c\,d$ represents a flat tile arch sprung between the I beams e and f. In this case the nominal depth of the arch is considered as 10 inches. The dotted line

dgc represents the theoretical line of pressure in the flat arch while *gh* may be considered as the theoretical rise, or as it is called, the *effective depth* of the arch. The point *g* may be

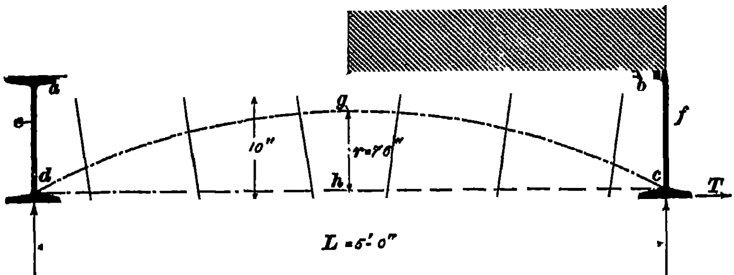


FIG. 30

taken as a center of moments, and the thrust of the arch to be resisted by the tie-rods may be represented by the force T. For a section 1 foot in width, the load on one-half of the arch is equal to $\frac{wL}{2}$ when w equals the load per square foot of surface and L the span of the arch, in feet. The moment of the force T about the point g must equal the moment of the load about the same point; and as the lever arm r of the force T is in inches, the lever arm of the load must also be in inches in order to equate the expressions; hence, the lever arm of the load will be $\frac{12L}{4}$ and its moment $\frac{wL}{2} \times \frac{12L}{4} = \frac{3wL^2}{2}$. The moment of the force T about the point g is Tr. Then, as these moments must be equal, $Tr = \frac{3wL^2}{2}$. The strength of one rod is equal to its area A multiplied by its safe unit fiber stress, and this value divided by the resistance T required in 1 foot of length of beam will give the distance between rods. Hence, assuming the safe unit fiber stress to be 15,000 pounds, the spacing of the rods is obtained from the formula

$$x = \frac{A \times 15{,}000}{T} \qquad (11)$$

From the formula $Tr = \frac{3wL^2}{2}$, the value of T is found

to be $\frac{3\,w\,L^2}{2\,r}$. Substituting this value for T in formula **11** gives $x = \frac{A \times 15{,}000}{\frac{3\,w\,L^2}{2\,r}}$, or $x = \frac{A \times 15{,}000 \times 2\,r}{3\,w\,L^2}$. By cancelation the formula becomes

$$x = \frac{10{,}000\,A\,r}{w\,L^2} \qquad \textbf{(12)}$$

In the formulas just given, the load w includes the weight of the arch as well as the load on it. Ordinarily, $\frac{3}{4}$-inch tie-rods are used in floor construction; these should not be placed farther apart than 6 feet. In calculating the spacing of the rods the area should be taken at the root of the thread.

EXAMPLE.—In the example in Art. **26**, what size rods will be required if they are placed 5 feet apart?

SOLUTION.—Transposing formula **12**, $A = \frac{x\,w\,L^2}{10{,}000\,r}$; then substituting the values obtained from the example in Art. **26** in this formula gives

$$A = \frac{5 \times 350 \times 16}{10{,}000 \times 6} = .466 \text{ sq. in.}$$

A 1-in. rod, which has an area of .550 sq. in. at the root of the thread is the nearest commercial bar that can be used, for a $\frac{7}{8}$-in. rod has an area at root of thread of only .420 sq in. Ans.

28. The effective depth and nominal depth of flat tile arches, in inches, are given below:

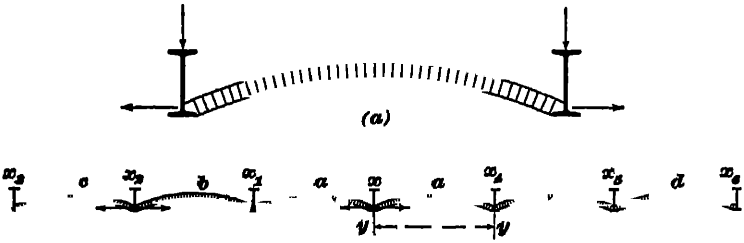
Nominal depth . .	6.0	7.0	8.0	9.0	10.0	12.0
Effective depth . .	3.6	4.6	5.6	6.6	7.6	9.6

EXAMPLE.—What should be the greatest distance between tie-rods that resist the thrust of an 8-inch tile arch having a span of 4 feet? The diameter of the rods is $\frac{3}{4}$ inch, and the arch carries a load of 120 pounds per square foot.

SOLUTION.—The area of the rod at the root of the thread is found to be .302 sq. in., and the weight of the arch, including the filling, flooring, and ceiling is found to be 63 lb. per sq. ft.; therefore, the total load per square foot is 120 + 63 = 183 lb. From the above table, the effective depth of the arch is found to be 5.6 in. Substituting these values in formula **12** gives

$$x = \frac{.302 \times 5.6 \times 10{,}000}{183 \times 4 \times 4} = 5.77, \text{ or 5 ft. 9 in. Ans.}$$

29. In designing a floor system for the support of fireproof arches of brick, it is advisable to provide tie-rods throughout the system. Though the beams supporting many portions of the floor will not be subjected to a material thrust, from the fact that adjacent arches react against each other, nevertheless, in actual construction it is often convenient to lay the arches in alternate panels, and the wooden centers are frequently removed from one panel to the adjacent one. Where a single arch is supported on two I beams, as shown in Fig. 31 (*a*), each beam is subjected to a lateral force equal to the thrust of the arch, and in proportioning such beams for the floor load, this lateral thrust should be considered in conjunction with the vertical load on the beam, in order to determine the ultimate unit stress.



(*b*)

FIG. 31

Considering the system of arch construction shown in Fig. 31 (*b*) from the center beam x outwards each way, it will be noticed that the arches a, a react on the beam x and their thrusts are balanced. Likewise, the thrust of the arches a and b are balanced on the beam x_1, and in a similar manner b and c react against each other. The horizontal thrust of the arch c must be provided for in the beam x_3 and sufficient transverse lateral resistance must be obtained either by the use of a larger beam, or tie-rods must be placed at frequent intervals. If tie-rods connect all the beams in the series, the effect is the same as though a continuous tie-rod extended from the beam x_3 to the beam x_6, and the outward thrusts of the system due to the arches c and d are resisted by the series of tie-rods. If the tie-rods in the bay $y y$

toward the center of the system are omitted for reasons of economy or design, the thrust of the arch *a* is resisted by the transverse strength of the three beams x_4, x_5, and x_6. In large floor systems, however, if it is possible to contruct all the arches at once, the tie-rods may be omitted in central portions of the system and provided only for the exterior beams and three or four adjacent panels.

30. It is interesting to observe the effect of the horizontal force on the bending moment in a beam supporting

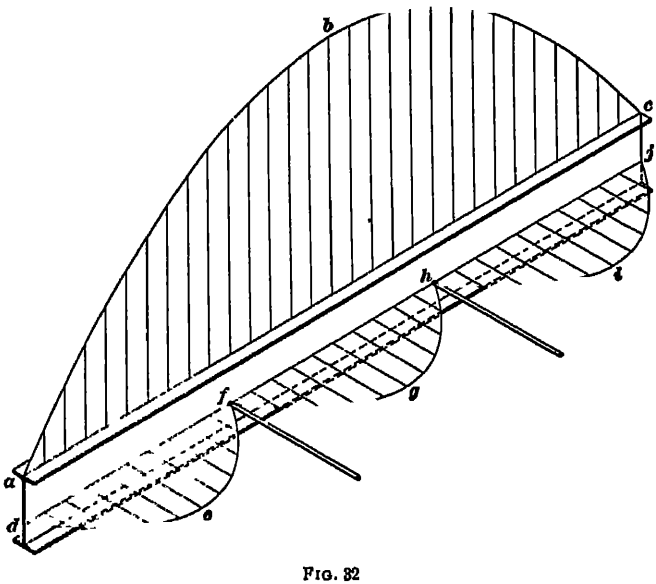


FIG. 32

arches and tied to the adjacent beam by tie-rods. Such a beam is subjected to two bending moments—one vertical and the other horizontal. In each case the load is uniform and the bending moment may be represented by a parabola. Fig. 32 shows the conditions that exist, the parabola *a b c* representing the bending moment due to the vertical load, and *d e f*, *f g h*, and *h i j* the bending moments between tie-rods or support and tie-rod. The resultant bending moment is found as shown in Fig. 33, in which *a b* represents the

length of the beam and ac the bending moment for one-half of the beam. The tie-rods are located at the points d and e, and af shows the bending moment for one-half of the distance between the end support and the first tie-rod.

31. The bending moment at any point on the beam is equal to the resultant of the vertical and horizontal bending moments; hence, the bending moment at g is represented by the resultant of gh and gi or the line hj. This is obtained by laying off gj at right angles to hg and equal to gi, and joining h and j. The divisions from x to b are made equal to those from a to x and the line hj is laid off at $j'h'$. Likewise, the bending moment at k is equal to the

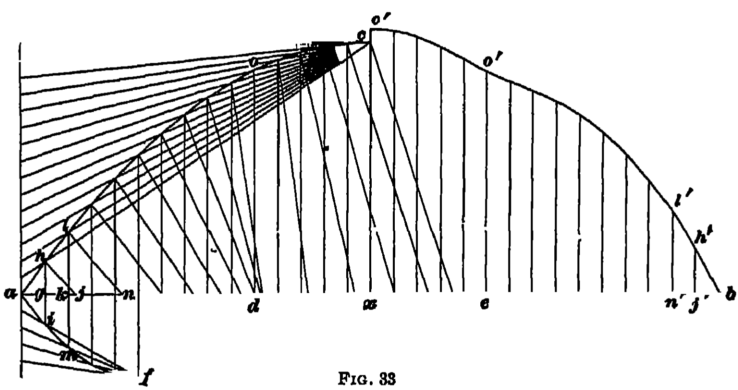


FIG. 33

resultant of kl and km, or ln, and this distance is laid off at $n'l'$. The bending moment at the succeeding points is found in a similar manner until the first tie-rod is reached. The horizontal thrust is counteracted at this point and hence no horizontal bending moment is produced. Therefore, the distance do is laid off at eo' and the resultant bending moments at the remaining points are laid off as explained. A curve drawn through the points thus determined, as $bh'l'o'c'$, represents the resultant bending moment on one-half of the beam.

EXAMPLE 1.—What size I beam is required to support a tile arch on one side for a distance of 20 feet, the conditions being as follows. The span of the arch is 5 feet; its nominal depth, 10 inches; and the

weight, including filling, flooring, etc., is 69 pounds per square foot; the live load is 150 pounds per square foot. The tie-rods are $\frac{3}{4}$ inch in diameter and the allowable unit fiber stress of the beam is 18,000 pounds.

SOLUTION.—The spacing of the rods is found from formula **12**; thus, $x = \frac{.302 \times 7.6 \times 10{,}000}{219 \times 5 \times 5} = 4.19$ ft.

The lateral thrust of the arch is determined from formula **7**,

$$T = \frac{3 \times 219 \times 5 \times 5}{2 \times 7.6} = 1{,}080.59, \text{ or, say, } 1{,}081 \text{ lb.}$$

Assuming a 12-in. 31.5-lb. beam, the values of b and I' are 5 and 9.5, respectively. Substituting these values in formula **5**, gives

$$s_h = \frac{1{,}081 \times 5 \times 4.19 \times 4.19}{2 \times 9.5} = 4{,}994 \text{ lb. per sq. in.}$$

Hence, 18,000 − 4,994 = 13,006 lb. per sq. in. Now the section modulus for this beam under a vertical load is found from *Properties of Sections* to be 36. Therefore,

$$s_v = \frac{Wl}{8S} = \frac{219 \times 5 \times 20 \times 20 \times 12}{2 \times 8 \times 36} = 9{,}125 < 13{,}006 \text{ lb. per sq. in.}$$

Hence, a 12-in. 31.5-lb. beam is sufficiently strong. A 10-in. 30-lb. beam by a similar calculation would be found too light, consequently, a 12-in. 31.5-lb. beam is the lightest one that is safe under the given conditions.

EXAMPLE 2.—What size I beam will be required to support a tile arch on each side, for a distance of 18 feet? The span of the arch is 5 feet; the nominal depth, 9 inches; and the spacing of the tie-rods is 4 feet. The live load is 160 pounds per square foot, and the dead load is 40 pounds per square foot.

SOLUTION.—From formula **7**, $T = \frac{3 \times 160 \times 5 \times 5}{2 \times 6.6} = 909$ lb. per ft.

Assuming a 12-in. 31.5-lb. I beam, the values of b and I' are 5 and 9.5, respectively. Substituting these values in formula **6**,

$$s_h = \frac{909 \times 5 \times 4 \times 4}{2 \times 9.5} = 3{,}827 \text{ lb. per sq. in.}$$

The section modulus of a 12-in., 31.5-lb. beam is 36. Then the vertical stress s_v is $\frac{W'l}{8S}$, or

$$s_v = \frac{(160 + 40) \times 5 \times 18 \times 18 \times 12}{8 \times 36} = 13{,}500 \text{ lb. per sq. in.}$$

Then, as $s_v = 13{,}500 < 16{,}000$, and $s_v + s_h = 13{,}500 + 3{,}827 = 17{,}327 < 20{,}000$, the beam assumed is sufficiently strong.

EXAMPLES FOR PRACTICE

1. What size I beam will be required to support a tile arch on one side over a distance of 18 feet? The nominal depth of the arch is 9 inches

and it has a span of 4 feet. The weight of the arch, including filling, flooring, and ceiling, is 65 pounds per square foot and the load to be supported by it is 135 pounds per square foot. The only lateral support is the usual tie-rods, in this case ¾-inch rods, spaced according to formula. Ans. 10-in. 25-lb. beam

2. The outside beam of a floor system supporting brick segmental arches is connected with the other beams by the usual ¾-inch tie-rods, and it is to have a safe unit fiber stress of 18,000 pounds. The span of the arch is 4 feet, the rise or effective depth 6 inches, and its weight 130 pounds per square foot. The live load is 200 pounds per square foot and the span of the beam is 22 feet. What size I beam will be required? Ans. 12-in. 31½-lb. I beam

BEARING PLATES

32. Beams resting on masonry walls or piers will usually require bearing plates to distribute the load over an area such that the safe bearing value of the masonry will not be exceeded. These bearing plates are usually of steel or cast iron, though stone templets are sometimes used. The method of computing the size required is explained in *Statics of Masonry*, Part 1.

Table IV gives the bearing value, in pounds, for plates of various sizes bearing on the different kinds of masonry. As the thickness of the plate is not stated, it must be computed for each case. It depends on the allowable load and unit stress and the width of the beam or channel resting on it, and may be determined by the following formula:

$$t = .866\,(l - b)\sqrt{\frac{R}{s_a b' l}} \qquad (13)$$

in which t = thickness of plate, in inches;
l = length of plate perpendicular to axis of beam, in inches;
b = width of flange of beam or channel, in inches;
R = reaction at point of support, in pounds;
b' = width of plate, in inches, in direction of axis of beam or channel;
s_a = allowable stress, in pounds per square inch, on extreme fiber of plate

TABLE IV

BEARING PLATES FOR I BEAMS AND CHANNELS

Bearing on Wall Inches	Size of Plate Inches	Safe Bearing Value of Plates, in Pounds				
		Ordinary Stone Masonry	Good Stone Masonry	Brick in Lime Mortar	Brick in Rosendale Cement Mortar	Brick in Portland Cement Mortar
4	4 × 4	2,880	4,800	1,600	2,400	3,200
4	4 × 6	4,320	7,200	2,400	3,600	4,800
4	4 × 8	5,760	9,600	3,200	4,800	6,400
6	6 × 6	6,480	10,800	3,600	5,400	7,200
6	6 × 8	8,640	14,400	4,800	7,200	9,600
6	6 × 10	10,800	18,000	6,000	9,000	12,000
8	8 × 8	11,520	19,200	6,400	9,600	12,800
8	8 × 10	14,400	24,000	8,000	12,000	16,000
8	8 × 12	17,280	28,800	9,600	14,400	19,200
10	10 × 10	18,000	30,000	10,000	15,000	20,000
10	10 × 12	21,600	36,000	12,000	18,000	24,000
10	10 × 14	25,200	42,000	14,000	21,000	28,000
12	12 × 12	25,920	43,200	14,400	21,600	28,800
12	12 × 14	30,240	50,400	16,800	25,200	33,600
12	12 × 16	34,560	57,600	19,200	28,800	38,400
12	12 × 18	38,880	64,800	21,600	32,400	43,200
14	14 × 14	35,280	58,800	19,600	29,400	39,200
14	14 × 16	40,320	67,200	22,400	33,600	44,800
14	14 × 18	45,360	75,600	25,200	37,800	50,400
14	14 × 20	50,400	84,000	28,000	42,000	56,000
16	16 × 16	46,080	76,800	25,600	38,400	51,200
16	16 × 18	51,840	86,400	28,800	43,200	57,600
16	16 × 20	57,600	96,000	32,000	48,000	64,000
16	16 × 22	63,360	105,600	35,200	52,800	70,400
18	18 × 18	58,320	97,200	32,400	48,600	64,800
18	18 × 20	64,800	108,000	36,000	54,000	72,000
18	18 × 22	71,280	118,800	39,600	59,400	79,200
18	18 × 24	77,760	129,600	43,200	64,800	86,400
20	20 × 20	72,000	120,000	40,000	60,000	80,000
20	20 × 22	79,200	132,000	44,000	66,000	80,000
20	20 × 24	86,000	144,000	48,000	72,000	96,000
20	20 × 26	93,600	156,000	52,000	78,000	104,000

If the allowable unit stress in the extreme fiber for steel is considered as 16,000 pounds, the formula may be written:

$$t = .00685(l - b)\sqrt{\frac{R}{b'l}} \qquad (14)$$

EXAMPLE.—What size steel bearing plate is required in a brick wall laid in lime mortar, to support the end of a 12-inch 31.5-pound standard I beam having a flange width of 5 inches, which carries a uniformly distributed load of 600 pounds per lineal foot over a span of 25 feet? The allowable unit stress in the extreme fiber of the plate may be considered as 16,000 pounds.

SOLUTION.—The bearing value required is equal to the reaction, or $\frac{15,000}{2} = 7,500$ lb. From Table IV, it is seen that an 8″ × 10″ plate will give the required bearing value for a plate on brickwork laid in lime mortar. Substituting the values in the formula $t = .00685(l - b)\sqrt{\frac{R}{b'l}}$ gives

$$t = .00685(10 - 5)\sqrt{\frac{7,500}{8 \times 10}} = .332 \text{ in.}$$

The nearest size above this is $\frac{3}{8}$ in.; hence, an 8″ × 10″ × $\frac{3}{8}$″ bearing plate will be required. Ans.

EXAMPLES FOR PRACTICE

1. An 18-inch 65-pound I beam having a flange width of 6.18 inches supports a uniformly distributed load of 1,000 pounds per lineal foot. The span of the beam is 32 feet and its ends rest on piers built of ordinary stone masonry. Provided that the allowable unit stress in the extreme fiber of the plate is 16,000 pounds, what size bearing plate will be required?

Ans. $\begin{cases} .515 \text{ in. thick} \\ 8'' \times 12'' \times \frac{9}{16}'' \text{ plate} \end{cases}$

2. A steel lintel composed of two 15-inch 35-pound channels placed flange edge to flange edge supports the wall above and also a floor load, the combined weight amounting to 4,000 pounds per lineal foot. The span is 15 feet, and the over-all width of flange is 8 inches. Assuming an allowable unit stress of 16,000 pounds in the extreme fiber of the plate, what size bearing plate will be required to support the end of the lintel on a brick wall laid in Portland cement mortar?

Ans. 12″ × 14″ × $\frac{9}{16}$″ plate

BEAMS AND GIRDERS

(PART 3)

BUILT-UP GIRDERS

GENERAL CONSTRUCTION

1. Definitions.—A **plate girder** is one built up of a number of plates and angles securely riveted together. The names given to its different parts may be understood by referring to Fig. 1, in which *a* is the **flange plate**, of which there may be one or more on each flange, depending on the strength required. These plates are the principal elements that resist the bending stresses in the girder. The **flange angles** *b*, *b* are the means of connecting the flange plates to the **web-plate** *c*. When the load on the girder is small, the flange plates may be omitted, in which case the flange angles are the members that chiefly offer resistance to the bending stresses.

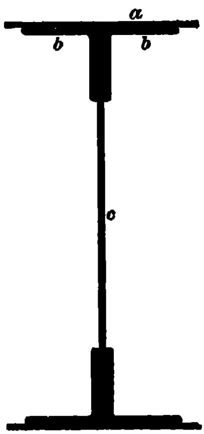


FIG 1

On account of the construction of a plate girder, there is very little stiffness in the web-plate; consequently, there is always a strong tendency for it to fail by buckling and twisting under the load imposed on the girder. This tendency to buckle is greatest at the supports or abutments of the girder and at the points where concentrated loads are applied. Because of this buckling tendency it becomes

necessary to reenforce the girder by riveting to it at stated intervals **stiffeners** generally made of angles.

The most common and cheapest form of stiffener is shown in Fig. 2 (*a*). This is simply a straight piece of angle riveted to the web-plate and flange angles. The space between the stiffeners and web-plate, due to the thickness of the flange angles, is filled with a piece of bar iron or plate, as shown at *d*; this is called a **filler** or **packing piece**.

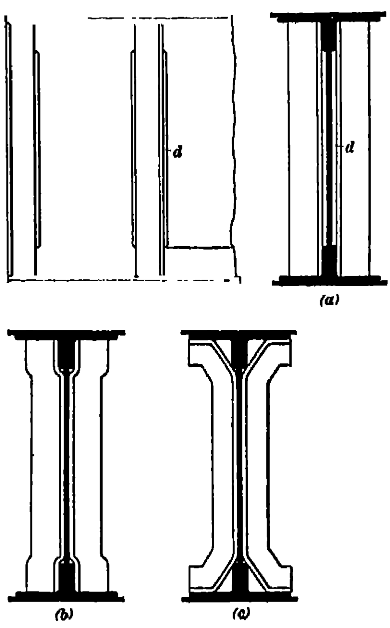


FIG. 2

Another form of stiffener for plate girders is shown in Fig. 2 (*b*). The angle is swaged out, to allow it to fit over the flange angles, and is riveted directly to the web-plate, thus doing away with the filler or packing piece. This construction does not require as much material as that shown in (*a*), but, unless there are a large number of girders of the same dimensions to be built, in which case dies, in connection with a power or hydraulic press, may be used for swaging the ends, the labor required is so much greater that it makes the girder more expensive.

The stiffeners shown at (*c*) are sometimes used, but are subject to the same general criticism in regard to cost of manufacture as those shown at (*b*). Stiffeners of this shape possess a possible advantage in the fact that they stiffen the flanges considerably more than either of the other styles.

2. Usual Forms of Sections.—The four principal sections used in plate-girder construction are shown in Fig. 3.

A simple plate girder with a web-plate and two flange angles, but with no flange plate, is shown in (*a*); this section is used for short spans or light loads. In (*b*) is shown a similar girder with one flange plate; this girder is used to support heavier loads and to clear longer spans. The girder in (*c*), which may have two or more flange plates at each flange, may, if the conditions require, be made as heavy as is necessary in order to carry great loads over long spans. In fact, the strength of a girder of this character may be increased almost indefinitely by the addition of flange plates. The section, Fig. 3 (*d*), is a plate girder of box section. It is stiffer laterally than the other forms shown, but the difficulty of reaching the interior for painting and inspecting and

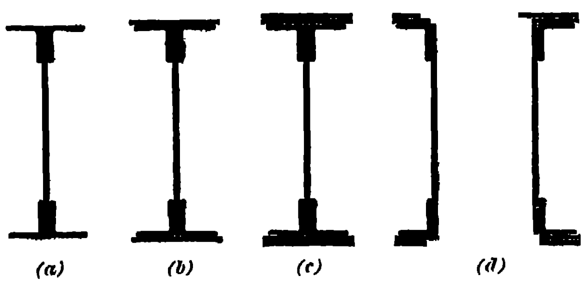


FIG. 3

the excessive amount of labor required in its construction, are such serious objections that it is much less used than the sections shown at (*a*), (*b*), and (*c*). On account of their open construction, the latter are especially good forms to use in buildings where the objection in regard to lateral stiffness does not hold good, as when the girder is used in the position in which it is usually found, it is generally prevented from deflecting laterally by the floorbeams; any lack of stiffness in comparison with the box girder is more than compensated for by the simplicity of construction and easy access on all sides for painting and inspecting.

PRINCIPLES OF DESIGN

STRESSES

3. The external forces, loads, and reactions produce the same kind of **stresses** in a plate girder as in an ordinary beam, but, on account of its special construction, the distribution of these stresses in the girder is assumed to be somewhat different from that in a beam made of a single piece. In the girder, the shear is generally assumed to be borne wholly by the web-plate, while the bending moment is assumed to be resisted by the stresses in the flange members. The method of calculating the magnitude of the shear and bending moment is the same as that for beams; owing, however, to the different assumption in regard to the distribution of these forces, a different method of calculation is used in determining the relations between them and the stresses in the girder.

4. Shearing Stresses in Web-Plate.—In discussing the methods of calculating the dimensions of a plate girder for a given purpose, we will first consider the shear, which is the principal factor that determines the thickness of the web-plate and the number and size of stiffeners required. The greatest shear in a beam occurs at the point of support at which the reaction is greatest, and the magnitude of the shear is equal to the reaction at that point; consequently, in a simple plate girder, the greatest shear occurs at a point of support, and is equal in amount to the reaction at that point.

WEB-PLATES AND STIFFENERS

5. Depth of Girder.—Having calculated the shear, the depth of the girder is assumed in accordance with practical rules that fix the relation between the depth and span. In accordance with the best practice, the depth should not be less than one-fifteenth of the span, though some authorities consider one-twentieth ample. The latter proportion, however, gives an exceedingly shallow girder, and cannot be

recommended except where the loads are very light and the span short, or where it is absolutely necessary that an extremely shallow girder be used, on account of decorative features, or lack of space in regard to headroom, in which case the girder should be so proportioned that when fully loaded its deflection will not be excessive.

6. Thickness of Web-Plate.—Knowing the depth of the girder, and the shear at the points of support, the thickness of the web-plate is proportioned so as to give it sufficient area to resist the maximum shear. It is always necessary to stiffen the plates over the supports, as shown in Fig. 5; these stiffeners are riveted to the plate and transfer the shearing stress from it to the supports.

A considerable portion of the plate is cut away by the holes for the rivets by which it is fastened to the stiffeners; hence, the least strength of the plate is along the line of the rivet holes. It can be seen, by referring to Fig. 4, which shows the end of a plate with the holes punched for riveting to the stiffener, that the net or efficient depth of the plate is equal to the actual depth minus the sum of the diameters of the rivet holes.

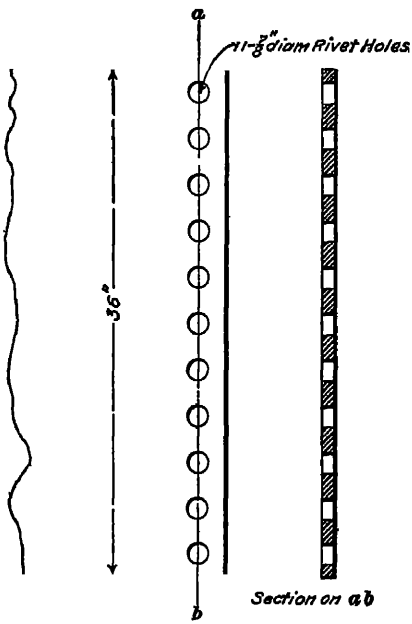


FIG. 4

The following rule may be used for calculating the thickness of a web-plate so that it will have sufficient strength to resist the shearing stress:

Rule.—*From the total depth of the web-plate, deduct the sum of the diameters of the rivet holes, which will give the net*

or efficient depth of the web-plate; multiply the net depth by the safe resistance of the material to shear, and divide the maximum shear, in pounds, by the product; the quotient will be the required thickness of the metal in the web of the girder.

This rule may be expressed by the formula

$$t = \frac{R}{d \times s} \qquad (1)$$

in which t = thickness of web-plate;
R = greatest reaction or maximum shear;
s = safe shearing resistance of material per square inch;
d = net depth of web-plate after all rivet holes have been deducted.

7. The safe resistance of the material to shear is of course governed by the factor of safety required in the girder. For example, the ultimate shearing strength of structural steel being 52,000 pounds per square inch, if a factor of safety of 4 is required, the safe resistance of the metal will be 52,000 ÷ 4 = 13,000 pounds, while if a factor of safety of 5 is desired, the safe strength will be 52,000 ÷ 5 = 10,400 pounds.

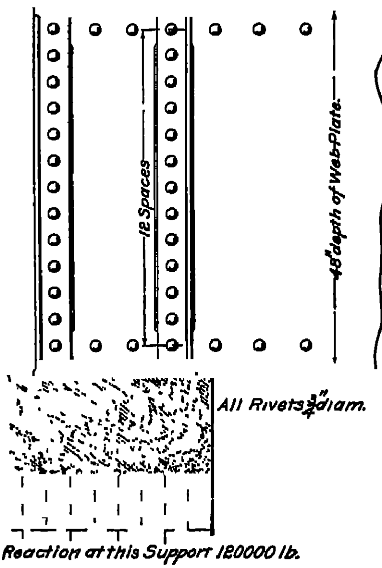


FIG. 5

In deducting the metal for the rivet holes in order to ascertain the net depth of the web-plate, the holes should always be considered as being $\frac{1}{8}$ inch larger than the nominal diameter of the rivet; this allowance is made because the holes are always made $\frac{1}{16}$ inch larger in diameter than the rivet so

that the rivet may be inserted easily, and another $\frac{1}{16}$ inch should also be allowed in the diameter of the hole to compensate for any injury that the metal immediately around it may suffer from the punch.

It will often be found that the calculated thickness of the web-plate is less than is allowable for practical reasons. The thinnest plate that should be used for any case is $\frac{5}{16}$ inch.

EXAMPLE.—Fig. 5 shows the end of a plate girder in which the greatest reaction is 120,000 pounds The girder is made of structural steel, the safe fiber stress of which is assumed to be 11,000 pounds per square inch for shear. What should be the thickness of the web-plate?

SOLUTION.—The width of the plate is 48 in., and there are thirteen holes punched for $\frac{3}{4}$-in. rivets. The deduction to be made for each rivet hole is $\frac{1}{8}$ in. + $\frac{3}{4}$ in. = $\frac{7}{8}$ in.; therefore, the net depth of the plate is $48 - 13 \times \frac{7}{8} = 48 - 11\frac{3}{8} = 36\frac{5}{8}$ in. Applying formula **1**, the thickness of the plate is $t = \frac{120,000}{36\frac{5}{8} \times 11,000} = .297$ in. In no case, however, should the web-plate of a girder be less than $\frac{5}{16}$ in. in thickness. Hence, as .297 is less than $\frac{5}{16}$, the thickness of the web-plate in this girder should be $\frac{5}{16}$ in. Ans.

8. Buckling of Web-Plate and Distribution of Stiffeners.—The shearing stresses in a web-plate, in addition to their tendency to shear the plate, are liable to cause it to fail by buckling; therefore, in order to properly resist the vertical shearing stresses and prevent them from buckling the web-plate before its full shearing strength is realized, it is necessary to provide the stiffeners.

The shearing stresses in a simple beam are always greatest at the points of support, and diminish toward the center of the beam until a point is reached such that the sum of the loads between it and the support is equal to the reaction; at such a point the shear is said to change sign. Therefore, the stiffeners should be most numerous at the points of greatest vertical shear, and should decrease in number as the shear decreases. Theoretically, this would be a correct method of locating the stiffeners, but practically they are spaced at equal distances along the length of the girder,

except at the points of support, where several are placed near together in order to give the end of the girder more nearly the character of a column and enable it to successfully resist the great vertical shear, due to the reaction at this point. In no case should the stiffeners at the end of a plate girder be omitted, even if the conditions make the intermediate ones unnecessary.

It is good practice to place stiffeners directly under any concentrated load that may be placed on the girder. These not only stiffen the web-plate at the point of application of the load, and thus prevent buckling, but also, through the medium of the rivets, assist in distributing the load on the web-plate and other members of the girder.

The end stiffeners of a plate girder may be considered as columns subjected to a compressive stress equal to the reaction, and calculated by the rules and formulas for columns. For safety, the stress on the end stiffeners should never exceed 15,000 pounds per square inch of section.

9. Practice in regard to the placing of stiffeners on plate girders varies considerably, being more a matter of judgment and experience than of calculation. Some engineers determine the resistance of the web-plate to buckling by the formula

$$b = \frac{11,000}{1 + \frac{d^2}{3,000\,t^2}} \qquad (2)$$

in which b = safe resistance of web to buckling, in pounds per square inch;

d = depth of web-plate, in inches;

t = thickness of web-plate, in inches.

If the value of b given by this formula is less than the unit shearing stress, the girder should be stiffened.

EXAMPLE.—The shearing stress on the web of a plate girder is 9,000 pounds per square inch The stiffeners at the end supports are riveted by nine $\frac{7}{8}$-inch rivets to the web-plate, which is 36 inches wide. The end reaction on the girder is 80,000 pounds. (*a*) What should be the thickness of the web-plate? (*b*) Will it be sufficiently strong without the addition of stiffeners?

SOLUTION.—(*a*) Since $\frac{7}{8}$-in. rivets are used, the allowance to be made for one rivet hole is $\frac{7}{8}+\frac{1}{8}=1$ in., and the effective depth of the plate along the line of rivets is $36-9\times 1=27$ in. Applying formula **1**, the thickness of the plate is found to be $t=\frac{80{,}000}{27\times 9{,}000}=.33$ in. The thickness of the nearest standard-size plate above this is $\frac{3}{8}$ in, which will be the thickness used. Ans.

(*b*) By formula **2**, the safe unit resistance of the plate to buckling is $b=\dfrac{11{,}000}{1+\dfrac{36^2}{3{,}000\times(\frac{3}{8})^2}}=2{,}700$ lb. per sq. in; which, since it is much less than the unit shearing stress on the web-plate, shows that stiffeners are required. Ans.

10. Practical Rule for Providing Stiffeners.—It is not the general practice to make the above calculations to determine whether stiffeners are required; according to the best engineering practice, stiffeners should be provided, unless the thickness of the web-plate is at least one-fiftieth of the clear distance between the vertical legs of the flange angles.

EXAMPLE 1.—Assume the girder in the previous problem to be provided with $6''\times 6''$ flange angles; the depth and thickness of the plate, as shown, are 36 inches and $\frac{3}{8}$ inch, respectively. According to the above rule, does this girder require stiffeners?

SOLUTION.—The unsupported depth of the plate between the flange angles is $36-2\times 6=24$ in.; $\frac{1}{50}$ of 24 in. $=.48$, say, $\frac{1}{2}$ in. As the thickness of the web-plate is only $\frac{3}{8}$ in., the girder must be provided with stiffeners. Ans.

Another rule, which gives nearly the same result, is as follows:

Rule.—*Provide stiffeners whenever the thickness of the web-plate is less than one-sixtieth of its total depth.*

This rule is modified by some authorities so as to allow a thickness of one-eightieth of the total depth of the plate as being amply safe without stiffeners. The more conservative rule, however, which requires the thickness to be at least one-fiftieth of the unsupported depth of the web or the distance between the flange angles, is the one to be recommended, and will be used in this Section.

The spacing and size of stiffeners to be used on a plate girder is almost entirely a matter of experience and judgment. As a general rule, it may be said that stiffeners should be provided at the ends of all plate girders over the supports or abutments, and they should be so proportioned that they will take care of the entire reaction at these points. The stiffeners between the abutments or supports should be of such size that they will best suit the general requirements of the design of the girder. The practice in spacing intermediate stiffeners is to make the distance between their center lines equal to the depth of the girder, thus dividing the girder into equal square panels. Under no conditions, however, should stiffeners be placed more than 5 feet apart from center to center of line of rivets.

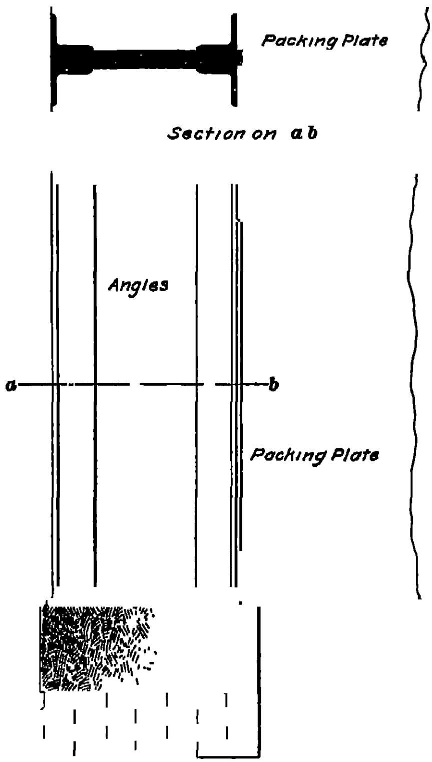


FIG. 6

Having proportioned the stiffeners at the abutments to take the entire reaction, it is good practice, when possible, to make the intermediate stiffeners of the same size as the end ones. In general, the angles used for stiffeners should not be less than 3 inches by 3 inches by $\frac{5}{16}$ inch, though on shallow girders, with extremely light loads, it might be economical to use angles as light as $2\frac{1}{2}$ inches by $2\frac{1}{2}$ inches by $\frac{5}{16}$ inch. Sizes smaller than this should certainly never be used for this purpose.

Stiffeners should always extend over the vertical legs of the flange angles; they should always be either swaged out to fit over the flange angles, or be provided with a filling piece, as illustrated in Fig. 2.

EXAMPLE 2.—The end reaction on a plate girder is 300,000 pounds. If a compressive fiber stress of 13,000 pounds per square inch is allowed on the stiffeners, and four stiffeners are used, as shown in Fig. 6, what should be the size of the angles?

SOLUTION.—The reaction or greatest shear being 300,000 lb., and the allowable stress 13,000 lb. per sq. in., the area of the stiffeners required must be $300{,}000 \div 13{,}000 = 23$ sq. in.; this sectional area, divided among four angles, gives $23 \div 4 = 5.75$ sq. in. as the area required for each angle.

By referring to the list of Angles With Equal Legs, in the table in *Properties of Sections*, it is seen that a $5'' \times 5'' \times \frac{5}{8}''$ angle has a sectional area of 5.86, while in the list of areas of the angles with unequal legs, a $5'' \times 4'' \times \frac{11}{16}''$ is shown to have an area of 5.72 sq. in.; therefore, either of these angles may be used. Ans.

FLANGES

11. The **flanges** of a riveted girder include all the metal at the top and bottom of the girder, and are sometimes called the top and bottom chords, though this term is more frequently applied to lattice or open girders, such as are more often used for railroad and highway bridges.

In building construction, it is customary to include in the flange the two flange angles, the flange plates, and one-sixth or one-eighth of the web-plate included between the flange angles.

The resisting moment, or the resistance to bending, of the section may be calculated as follows:

Taking the center of gravity of one flange section as a center, the moment of resistance of the other flange is equal to the area of the flange section multiplied by the distance between the centers of gravity of the flange areas and the safe stress per square inch of section. The resistance of the web is equal to its section modulus multiplied by the safe stress per square inch. Assuming d as the depth of the web which is considered here as being the distance between the

centers of gravity of the two flanges, the resisting moment of the entire section may be expressed by the formula

$$M_1 = A\,d\,s + \frac{s\,b\,d^2}{6} \qquad (3)$$

in which A = area of flange section;
d = height of girder between centers of gravity of flanges;
s = allowable stress per square inch;
b = thickness of web.

Considering $b\,d$ as equal to the area of the web, or A', we have $M_1 = A\,d\,s + \frac{A'\,s\,d}{6}$, or $M_1 = d\,s\left(A + \frac{A'}{6}\right)$. Thus, it may be seen that the entire resistance of the section includes about one-sixth of the area of the web, and consequently, one-sixth of the web may be considered as assisting to resist the bending moment on the girder.

The reduction of the moment of resistance of the web by rivet holes must be taken into consideration, however, and consequently it would be more nearly correct to assume one-eighth of the depth of the web as acting with the flange to resist the bending moment.

The building ordinances of some of the large cities in the United States, however, will not allow any portion of the web-plate to be included as part of the flange. In this Section, unless otherwise stated, the web-plate will not be considered in calculating the flange area.

If, because of economic considerations, one-sixth or one-eighth of the web-plate must be included as part of the flange, it should be remembered that the plate should never be spliced near the center, when the girder is uniformly or symmetrically loaded, or directly under the point of greatest bending moment, when the load on the girder is unsymmetrically placed. Especial care must also be taken to insure that any splice made on the length of such a web-plate is so designed as to furnish the greatest possible percentage of strength of the solid plate included within one-sixth or one-eighth of the depth of the web.

The best practice dictates that where flange plates are used, the sectional area of the flange angles should equal the sectional area of the flange plates. This, however, is not possible in heavy work, where the best that can be done is to use the heaviest sections obtainable for the flange angles.

12. **Flange Stresses.**—In a simple girder, the top flange is subjected to compression and the bottom flange to tension. Nevertheless, it is customary in practice to make the two flanges equal and composed of the same size of rolled plates and angles.

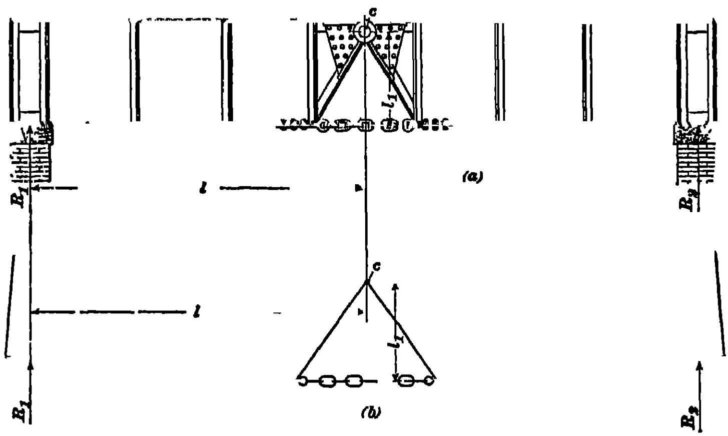


FIG. 7

In proportioning the flanges of a plate girder, the lower flange is calculated for tension; the areas of the rivet holes cut out of the flanges are deducted from the total area, so as to give the net or actual area of the flange at the point of least strength.

The stresses in the flanges are assumed to be produced wholly by the bending moment on the girder, and the moments of these stresses are assumed to be equal to the moments of the external forces.

The principles on which the flange stresses of a plate girder are calculated will be made clear by reference to Fig. 7, which shows a girder in two sections joined by a

hinge pin c at the upper flange, and a chain at the lower flange. The resultant moment of the loads and reactions tends to produce rotation about the center c, which, however, is taken at a point in the upper flange instead of on the neutral axis, as was done in the case of the beam composed of a single section; in reality, owing to the fact that the web is entirely neglected in calculating the resistance of the girder to the bending stresses, there is no neutral axis, in the sense in which that term was used in connection with ordinary beams. Strictly speaking, there may be said to exist a neutral axis in the web, similar to that in an ordinary beam, but, on account of the thinness of the web, the additional tensile and compressive resistance offered by it is so small that it may be neglected, as far as the longitudinal stresses in the flanges are concerned. The centers of moments of the tensile and compressive stresses are therefore supposed to be shifted to the upper and lower flanges, respectively. The vertical stresses have still to be transmitted by the web to the points of support, the compressive stresses being taken up by the stiffeners and the tensile ones by the web.

The stress in the chain, which represents the lower chord or flange of the girder, resists the tendency to rotation about the center of moments c, with a lever arm l_1, which is the perpendicular distance from the chain to the point c. It is evident, then, that the strength of the girder depends on two factors: the tensile strength of the lower chord, and its distance from the center of the hinge c, the latter of which represents the depth of the girder.

If the center of moments is taken on the center line of the chain, directly under the point c, it is evident that the resultant moment of the external forces, with respect to this center, is the same as when the center was taken at c; it is also evident that the force in the beam whose moment, with respect to this center, balances the resultant moment of the external forces, is the compression on the pin c. Since the moment and the lever arm of the compressive stress on the pin are respectively equal to the moment and the lever

arm of the tensile stress in the chain, it follows that these two stresses are equal; in other words, the compressive stress in the top flange of the girder is equal to the tensile stress in the bottom flange.

13. Proportioning the Flanges.—Having determined the principles on which the bending strength of a plate girder is calculated, it remains to show a method for proportioning the metal in the flanges. The usual process is as follows:

First calculate the maximum bending moment on the girder; in calculating the bending moment on a plate girder it is customary to express the moment in foot-pounds, the depth of a girder being generally given in feet and not in inches, as in solid beams of shallow depth. If, however, the depth of the girder is expressed in inches, the bending moment must be calculated in inch-pounds.

Having found the maximum bending moment on the girder, it is necessary to assume an allowable fiber stress for the material of which the flanges are composed. The following rule may be used to calculate the sectional area of either flange:

Rule.—*Divide the bending moment on the girder, in foot-pounds, by the product obtained by multiplying the depth of the girder, in feet, by the safe fiber stress.*

The safe fiber stress for a given case is obtained by dividing the ultimate fiber stress per square inch of the material by the factor of safety required in the girder.

The rule may be expressed by the formula

$$A = \frac{M}{D \times s_a} \qquad (4)$$

in which A = net area of one flange, in square inches;
D = depth of girder, in feet;
s_a = safe fiber stress per square inch of material;
M = bending moment on girder, in foot-pounds.

EXAMPLE.—The depth of a plate girder is 6 feet, the span is 80 feet, and the load on the girder is 3,000 pounds per lineal foot: (*a*) What

will be the required net flange area for structural steel, if a factor of safety of 4 is used? (*b*) Of what size rolled sections should the flange be composed?

SOLUTION.—(*a*) The span being 80 ft. and the load 3,000 lb. per lineal ft., the entire load on the girder will be 80 × 3,000 = 240,000 lb. Substituting in the formula $M = \frac{W L}{8}$ for the bending moment on a simple beam, gives $M = \frac{240,000 \times 80}{8} = 2,400,000$ ft.-lb. The net area of the flange, from formula 4, is

$$A = \frac{2,400,000}{6 \times 15,000} = 26.7 \text{ sq. in. Ans.}$$

(*b*) In order to determine the size of the flange plates and angles, it is useful to assume some particular size of angles and plates, and make a detail sketch of the flange, as shown in Fig. 8, marking on it the size of the respective plates and angles that have been assumed. The rivets should also be shown, so that the metal cut out of the rivet holes may be deducted from the sectional area of the flange in order to determine that area.

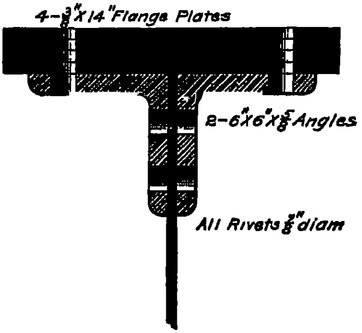


FIG. 8

It is assumed in the section under consideration that there are two rows of rivets through the vertical legs of the angles, each pair of these rivets being placed in the same vertical plane, in consequence of which the amount to be deducted from the net section is double the area cut out for one rivet. The rivets in the two rows through the horizontal legs of each angle are staggered, and consequently only one rivet hole in each horizontal leg affects the area of the flange section.

According to the table Properties of Standard Angles, in *Properties of Sections*, the area of a 6″ × 6″ × ⅝″ angle is 7.11 sq. in.; therefore, the total area of the metal in the flange is

Two 6″ × 6″ × ⅝″ angles,	7.11 × 2 =	14.22 sq. in.
Four 14″ × ⅜″ plates,	4 × 14 × ⅜ =	21.00 sq. in.
	Total,	35.22 sq. in.

From the total area of the flange it is necessary to deduct the metal cut out for the rivet holes. As ⅞-in. rivets are used, the rivet holes are considered to be ⅛ in. larger, or 1 in. in diameter.

There are four 1-in. holes through $\frac{5}{8}$ in of metal to be deducted from the vertical legs of the angles, and in the plates and the horizontal legs of the angles there are two 1-in. holes through $2\frac{1}{4}$ in. of metal. The areas to be deducted for the rivet holes are, therefore,

Four 1-in. holes through $\frac{5}{8}$ in. of metal	=	$2\frac{1}{2}$ sq. in.
Two 1-in. holes through $2\frac{1}{8}$ in. of metal	=	$4\frac{1}{4}$ sq. in.
Total,		$6\frac{3}{4}$ sq. in.

and the net area of the flange is 35 22 − 6.75 = 28.47 sq. in. Since the calculations showed that the net area required in this flange is 26.6 sq in, it is evident that the assumed flange is amply strong. Ans.

While plate girders are more economical than box girders, the latter are stiffer in a lateral direction and hence should be used where a wide top flange is required in order to furnish the necessary lateral stiffness for a long span. If the girder is not held in place laterally the top flange should have a width equal to at least $\frac{1}{20}$ of the span. Otherwise, the gross area of the top flange may be found by the following formula:

$$A' = A\left(1 + \frac{e^2}{5{,}000}\right) \qquad (5)$$

in which A = gross area required in top flange, with girder supported laterally;

A' = gross area in top flange, girder unsupported laterally;

e = span ÷ width of flange, both in inches.

14. Lengths of Flange Plates.—Since the bending moment in a simple beam varies along the entire length of the beam, the location of the maximum bending moment depending on the distribution of the load, it would seem that in order to design an economical girder, the area of the flange should vary with the bending moment. Where flange plates are used, this condition may be partially fulfilled by the use of plates of different lengths, each extending only as far as may be required in order to provide the flange section demanded by the bending moment.

Reference to Fig. 9 will make this construction more clear. In this figure, it is seen that the top plate of the top flange is the shortest, and extends over a small portion of

the girder only, each successive plate under this one being longer than the one above it. The third plate from the top is the longest and extends nearly the full length of the girder, while the angles extend from end to end.

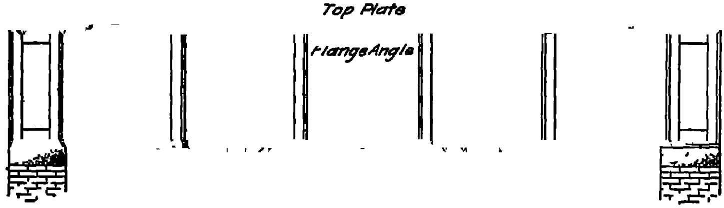


FIG 9

Where the beam is uniformly loaded, the following method may be used to obtain the theoretical length of each of the flange plates:

Commencing with the outside plate of the flange, find the sum of all the net areas, in square inches, of the plates to and including the plate in question. Thus, in Fig. 10, if it be required to obtain the length of the third plate from the top, find the sum of the areas of the first, second, and third plates. If the length of the second plate is required, then the sum of the areas of the first and second plates is to is taken. Divide the area so obtained by the net area of the whole flange, in square inches, and multiply the square root of this quotient by the length of the girder, in feet; the product will be the theoretical length of the plate in feet.

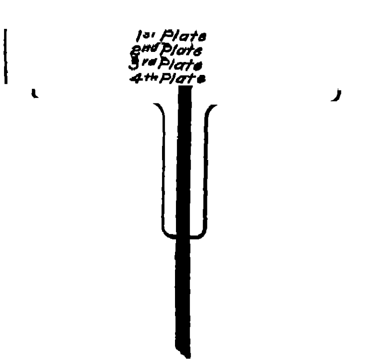


FIG. 10

Having obtained the theoretical length of the plate, it is necessary to add from 12 to 16 inches to each end, in order that the plate in question may be carried sufficiently past the point of bending moment that governs the area of the flange at its ends to be securely riveted to

the plates and angles making up the flange from there on to the abutment.

The method for determining the length of flange plates where the beam is uniformly loaded may be expressed by the formula

$$L_p = L\sqrt{\frac{a}{A}} \qquad (6)$$

in which L_p = theoretical length, in feet, of plate in question;

L = length of girder, in feet;

a = net area of all plates to and including plate in question, beginning with outside plate;

A = total net area of entire flange.

EXAMPLE.—In Fig. 11 is shown a section through the flange of a plate girder the span of which is 60 feet. What is the theoretical length of each of the three flange plates?

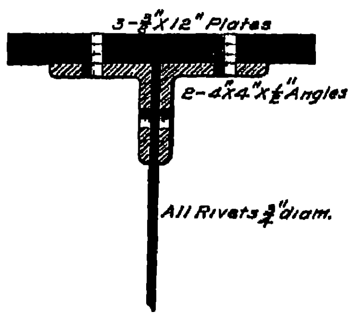


FIG. 11

SOLUTION.—The area of a $4'' \times 4'' \times \frac{1}{2}''$ angle, according to the table Properties of Standard Angles, in *Properties of Sections*, is 3.75 sq. in. The area of each plate is $\frac{3}{8} \times 12 = 4.5$ sq. in. The diameter to be deducted for the rivet holes is $\frac{3}{4} + \frac{1}{8} = \frac{7}{8}$ in.

The area cut out by a $\frac{7}{8}$-in. hole through a $\frac{3}{8}$ in. plate is $.875 \times .375 = .328$ sq. in. Then, as there are two rivet holes in each plate, its net area is 4.5 sq. in. − (.328 sq. in. × 2) = 3.844 sq. in.

The net area of the angles is (3.75 sq. in. × 2) − (.4375 sq. in. × 4) = 5.75 sq in.

The net area of the flange section is, therefore,

Three plates 3.844 × 3	=	11.532 sq. in.
Two angles	=	5.750 sq. in.
Total,		17.282 sq. in.

Now calculate the length of the outside plate. Substituting in the formula gives

$$L_p = 60\sqrt{\frac{3.844}{17.282}} = 28.29 \text{ ft.}$$

By substituting the proper values, the theoretical length of the second or middle plate is

$$L_p = 60\sqrt{\frac{7.688}{17.282}} = 40.0 \text{ ft.}$$

The length of the third or last plate in the flange, that is, the one next to the flange angles, is next to be calculated, though some engineers prefer to run this the entire length of the girder, as it stiffens the girder laterally and assists in preventing any tendency toward side deflection. The theoretical length of this plate is

$$L_p = 60\sqrt{\frac{11.532}{17.282}} = 49\ 12 \text{ ft.}$$

15. Graphic Method of Determining Length of Flange Plates.—The graphic method for determining the theoretical length of flange plates in built-up girders is more convenient than the analytic method previously given. In order to explain this method, a section through the flange of a plate girder will be assumed and the lengths of the several flange plates determined.

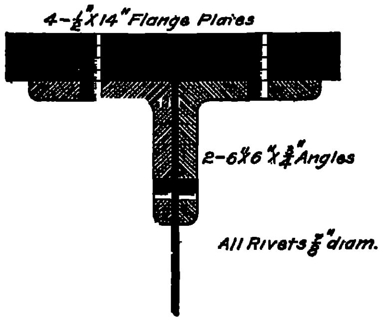


FIG. 12

Fig. 12 shows a section through the flange of a girder, built up of four ½″ × 14″ flange plates, the span of the girder being 90 feet. It will be noticed that there are two rows of rivets in the flange, and two rows in the vertical leg of the angles, but as the latter are staggered, there will be but one rivet hole to be deducted from the vertical leg of each angle.

The sectional area of a 6″ × 6″ × ¾″ angle is found from the table Properties of Angles, in *Properties of Sections*, to be 8.44 square inches; from this deduct 1½ square inches, the area cut out by the two rivet holes, making the net area of each flange angle 6.94 square inches.

The sectional area of a ½″ × 14″ flange plate is 7 square inches, from which there is to be deducted 1 square inch for

the sectional area cut out by the two rivet holes. Hence, the net area of one flange plate is 7 square inches − 1 square inch = 6 square inches.

The net area of the entire flange will, therefore, be

Two $6'' \times 6'' \times \frac{7}{8}''$ angles	=	13.88 square inches
Four $\frac{1}{2}'' \times 14''$ flange plates	=	24.00 square inches
Total,		37.88 square inches

Since the load is uniformly distributed, the flange plates extend equally on each side of the center; consequently, the diagram for only one-half of the girder will be drawn. Draw, to any scale, a horizontal line $a\,b$, Fig. 13, equal to one-half of the span; divide this line into any number of equal parts (in the figure, twelve parts have been used). Upwards from the points of division, draw indefinite perpendicular lines. On the perpendicular from b, lay off to some scale a distance that represents the entire net section of the flange, thus locating the point r. For example, the net area of the flange in this case is 37.88 square inches; letting $\frac{1}{16}$ inch represent 1 square inch, the distance $b\,r$ must be $37.88 \times \frac{1}{16} = 2.37$ inches, nearly.

Lay off to the same scale on the line $b\,r$ a distance $b\,n$, which represents 13.88 square inches, the net area of the two flange angles, also the distances $n\,o$, $o\,p$, $p\,q$, and $q\,r$, each representing 6 square inches, the net area of each of the flange plates. From the point r, draw a horizontal line cutting the vertical line erected at a, thus locating the point b'. Divide the vertical line $a\,b'$ into the same number of equal parts as the line $a\,b$, thus locating the points c', d', e', f', etc.; and from these points, draw the lines $c'\,r$, $d'\,r$, etc. Draw the curve $a\,s\,v\,r$ through the points where the vertical lines from c, d, e, etc. intersect the corresponding lines $c'\,r$, $d'\,r$, $e'\,r$, etc. Now from the points q, p, o, and n draw horizontal lines as shown, cutting the curve in the points s, t, u, and v, and from each of these points of intersection, draw a perpendicular, extending it until it intersects the horizontal line next above. The rectangles $v'\,r\,q\,v$, $u'\,q\,p\,u$, $t'\,p\,o\,t$, etc. thus formed, represent the flange plates and angles.

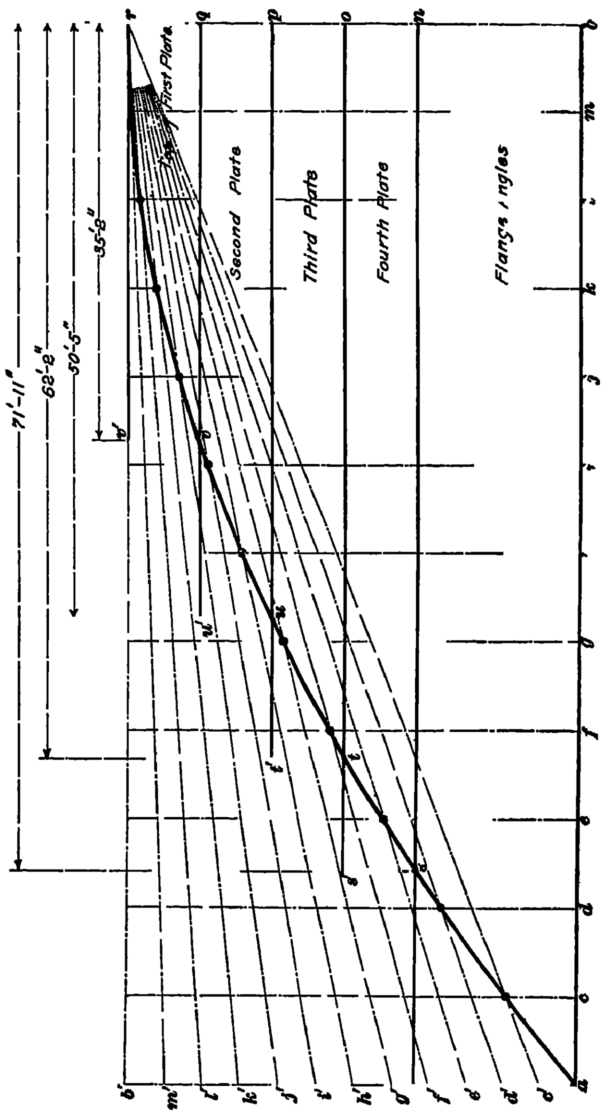

First Plate
Second Plate
Third Plate
Fourth Plate
71'-11"
62'-2"
50'-5"
35'-8"

FIG. 13

In order to obtain the theoretical length of any flange plate, measure the length of the corresponding rectangle by the scale to which the half span was laid out on the line *a b*; this length multiplied by 2 gives the length of the plate in question. For example, if it is desired to obtain the length of the first or top plate with the scale to which the half span was laid out, measure the length of the line *v′ r*; as only one-half of the diagram is drawn, this gives one-half of the length of the top or first plate, and by doubling this the entire theoretical length of the plate in question is obtained.

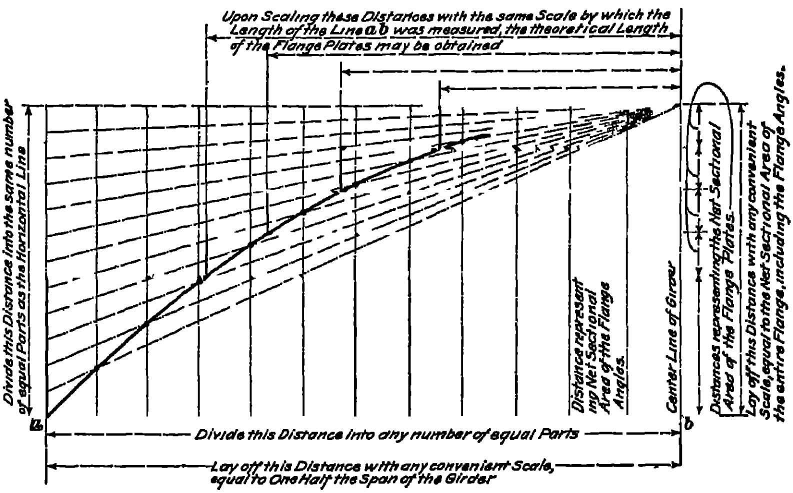


FIG 14

The length of the other plates may be determined in like manner. It is to be borne in mind that to the theoretical length given by the diagram it is necessary to add a length of about 1 foot at each end of the plate. From the diagram, the theoretical lengths of the flange plates of the girder shown in Fig. 13 are found to be 35 feet 2 inches, 50 feet 5 inches, 62 feet 2 inches, and 71 feet 11 inches, respectively.

The student will find on checking these lengths by formula 6 that they are approximately correct. Fig. 14, in which all the different steps are indicated, is presented in

order that the student may always have a guide for laying out this diagram.

16. Application of Graphic Method to Girders With Concentrated Loads.—The graphic method for determining the theoretical length of flange plates when the girder is loaded with concentrated loads, which is similar to that given for a uniformly loaded girder, will be illustrated by constructing a diagram for the lengths of the four flange plates required for a girder with a span of 80 feet and a depth of 6 feet, carrying a concentrated load of 185,000 pounds at 30 feet from one end.

The bending moment on the girder may be calculated by the formula

$$M = W \times \frac{xy}{L} \qquad (7)$$

in which M = bending moment;
W = load on girder;
L = span, in feet;
x = distance that load is located from one abutment;
y = distance that load is located from other abutment.

In Fig. 15, the load is located at the point f, 30 feet from R_1 and 50 feet from R_2. Substituting these values in the formula, the bending moment is found to be $M = 185{,}000 \times \frac{30 \times 50}{80} = 3{,}468{,}750$ foot-pounds.

From this bending moment, the required net flange area, assuming a safe unit stress of 15,000 pounds per square inch, is found to be, approximately, 38 square inches, which is provided by the use of four $\frac{1}{2}'' \times 14''$ plates and two $6'' \times 6'' \times \frac{3}{4}''$ angles. The flange therefore has the section shown in Fig. 12.

To construct the diagram, which is shown in Fig. 15, draw the base line cd, to any scale, equal to the span of the girder, in feet. (In this case, owing to the fact that the load is not symmetrically placed, the center line of the girder will not

divide the lengths of the plates in halves, and it will be necessary to draw the entire diagram.) Locate the point of application of the concentrated load at 30 feet from R_1,

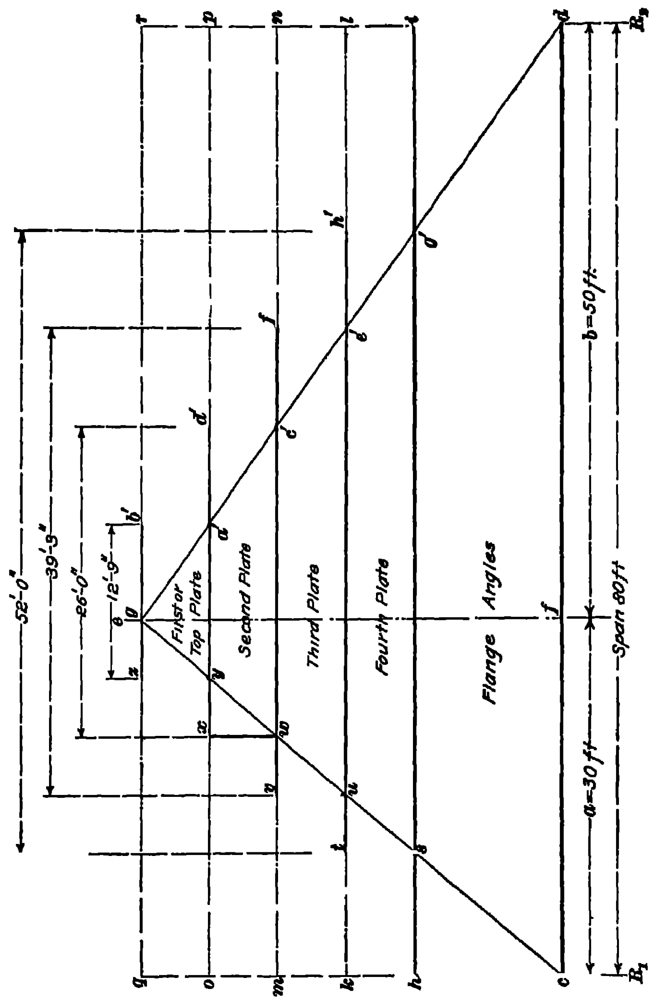


FIG. 15

and draw the perpendicular line ef. On this line, lay off, to any scale, a distance that represents the bending moment. For example, in this case, a scale has been used on which

50,000 foot-pounds is represented by $\frac{1}{32}$ inch; the distance to be laid off is, therefore, $\frac{3468750}{50000} \times \frac{1}{32} = 2.168$ inches. This locates the point *g*, which is then connected by straight lines to the points *c* and *d*. Draw vertical lines from the points *c* and *d* until they meet a horizontal line through the point *g*, at *q* and *r*.

In the flange section, the angles have a combined net area of 13 88 square inches, and each flange plate a net area of 6 square inches. The combined net sectional area of the flange is 37.88 square inches.

Place the zero mark of any convenient scale at the point *f*, and slant the scale until the marking on the scale that represents 37.88 square inches, the net area of the flange, falls on the horizontal line *q r*. Thus, in this case, a $\frac{1}{16}$-inch scale has been used, and the zero mark is placed at *f*, while the mark on the scale representing the division 37.88 is placed on the horizontal line *q r*. With the scale still in this position begin at the zero mark and lay off a distance 13.88 to represent the net sectional area of the two angles; then lay off four distances, each equal to 6, to represent the net sectional area of each of the flange plates.

Through the points thus found, draw the horizontal lines *h i*, *k l*, *m n*, and *o p*. At the points *s*, *u*, *w*, *y*, etc., where these horizontal lines cut the oblique lines *g c* and *g d*, draw the perpendicular lines *s t*, *u v*, *w x*, *y z*, $a' b'$, etc., extending each until it meets the next horizontal line above. Then the rectangles enclosed by the horizontal and vertical lines, shown heavy in the diagram, represent the cover-plates and flange angles. By measuring the lengths of these rectangles with the scale to which the span was laid off on the line *c d*, the theoretical length of the plates may be determined.

In this case the length of the angles is equal to the span, 80 feet, as marked on the diagram. The length of the first, or top, plate measures 12 feet 9 inches; of the second plate, 26 feet 0 inches; of the third plate, 39 feet 3 inches; and of the fourth, or last, plate, 52 feet 0 inches. It may be that when the student lays out the diagram for himself, he will obtain results that will vary slightly from those given.

However, a variation of a few inches in the length of a flange plate on a girder need not be considered.

17. Graphic Diagram for Several Concentrated Loads.—In order to illustrate this method still further, a more complicated problem, in which there are three concentrated loads, will now be presented.

Assume that a girder having a span of 80 feet with a depth of 6 feet is loaded as shown in Fig. 16. What flange area is required, provided that a safe unit fiber stress of 16,000 pounds per square inch is used, and of what should the flange be constructed; also, what will be the length of the several flange plates?

The reactions at R_1 and R_2 are found to be 142,500 and 117,500 pounds, respectively. It will first be necessary to

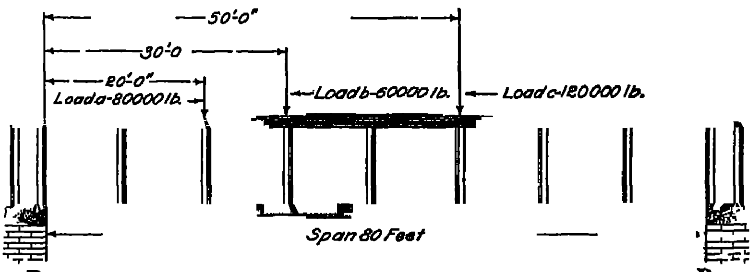


FIG 16

calculate the bending moment, in foot-pounds, at the points where the loads *a*, *b*, and *c*, are concentrated. The bending moment under load *a* is 142,500 × 20 = 2,850,000 foot-pounds; under load *b*, the bending moment is (142,500 × 30) − (80,000 × 10) = 3,475,000 foot-pounds; and the bending moment under load *c* is (142,500 × 50) − [(80,000 × 30) + (60,000 × 20)] = 3,525,000 foot-pounds.

From this it is seen that the greatest bending moment is under the load *c*, and its magnitude is 3,525,000 foot-pounds.

From this the flange area required may be calculated by applying formula **4**, as follows:

$$A = \frac{3{,}525{,}000}{6 \times 16{,}000} = 36.71 \text{ square inches}$$

We will now select a flange section that will have the required net sectional area; referring to the section shown in Fig. 12, the area is found to be 37.88 square inches; this flange will therefore satisfy the requirements.

18. Having determined the bending moments and the greatest net flange area, begin the diagram shown in Fig. 17 by drawing to any scale the horizontal line *d e* equal in length to the span of the girder; with the same scale locate the points of application *a*, *b*, and *c* of the concentrated loads. Upwards from the points *d*, *a*, *b*, *c*, and *e* draw indefinite perpendicular lines, and on the perpendiculars from *a*, *b*, and *c* lay off to some convenient scale distances *a f*, *b g*, and *c h*, which represent the respective bending moments at these points.

For example, the bending moment at *a* is 2,850,000 foot-pounds; at *b*, it is 3,475,000 foot-pounds; and at *c*, it is 3,525,000 foot-pounds; therefore, assuming a scale on which each $\frac{1}{32}$ inch represents 50,000 foot-pounds of bending moment, the respective bending moments at the points *a*, *b*, and *c* are represented by lengths *a f* of 57 thirty-seconds, *b g* of 69½ thirty-seconds, and *c h* of 70½ thirty-seconds.

Draw straight lines connecting the points *d*, *f*, *g*, *h*, and *e*. Through the highest point *h*, representing the greatest bending moment, draw the horizontal line *j k*.

The net area of the flange being 37.88 square inches, place the zero mark of any convenient scale on the line *d e*, and slant the scale until the mark that represents 37.88 falls on the line *j k*.

Starting from the zero mark on the scale, lay off a distance that represents the net area of the two flange angles, in this case 13.88 square inches, then divide the remaining distance into equal parts, each of which represents 6 square inches, the net sectional area of the several flange plates.

Through the points just found, draw the horizontal lines *l m*, *n o*, *p q*, and *r s*. Where these horizontal lines intersect the oblique lines at the points *t*, *u*, *v*, *w*, *x*, etc., draw vertical lines until they intersect the next horizontal line above.

Then draw in, with heavy lines, the rectangles representing the flange plates and flange angles.

The theoretical length of the flange plates may now be

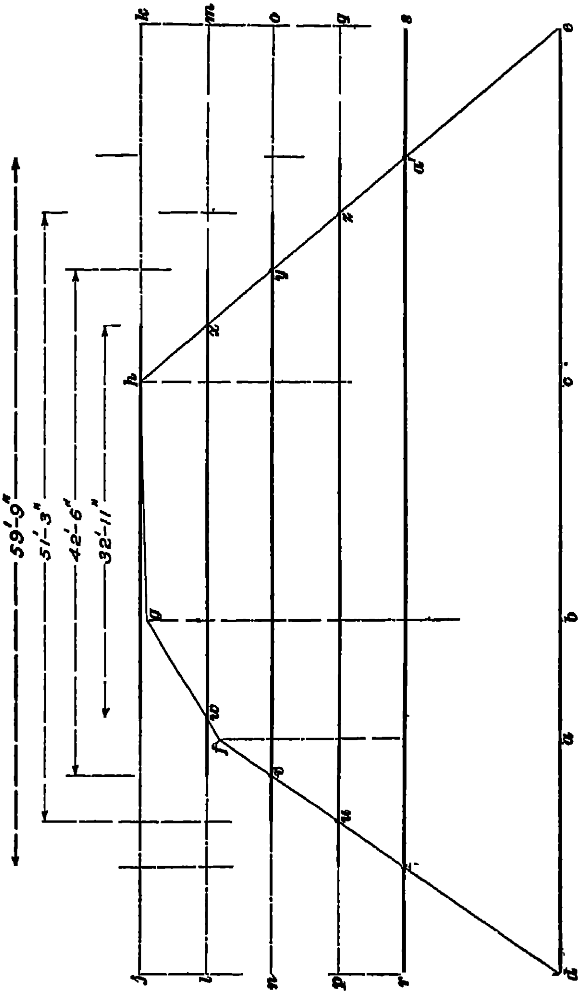


FIG. 17

determined by measuring with the scale to which the span was laid off on the line *de*. The length of the top, or first, plate in this case is found to be 32 feet 11 inches; of the

second plate, 42 feet 6 inches; of the third plate, 51 feet 3 inches; and of the fourth, or last, plate, 59 feet 9 inches.

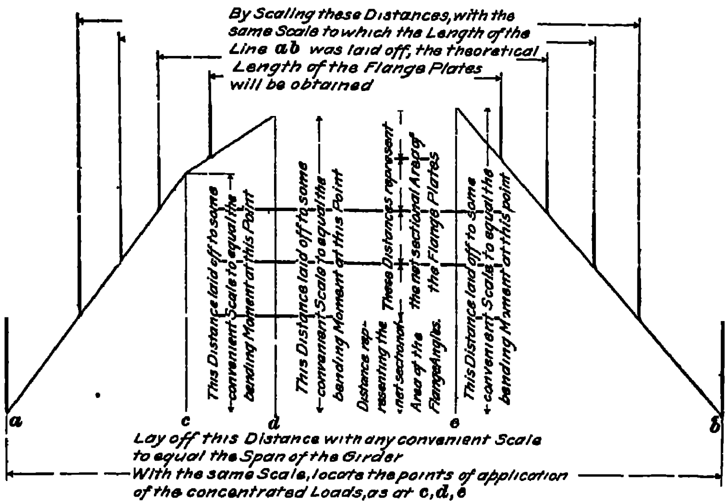


FIG 18

In Fig. 18 is shown a diagram that will serve as a general rule for determining the length of the several flange plates of a girder loaded with several concentrated loads.

19. Diagram for a Combination of Concentrated Loads With a Uniformly Distributed Load.—There is another condition of girder loading that is frequently

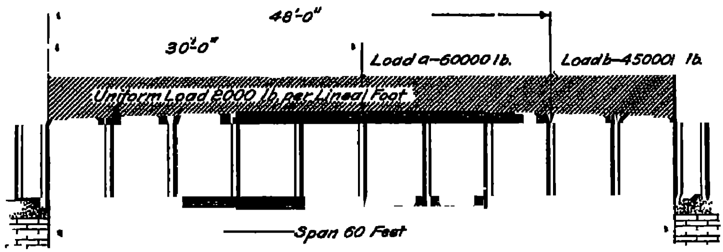


FIG 19

encountered in practical work in which it is necessary to determine the length of the several flange plates by the graphic method; this condition is produced by a combination

of a uniformly distributed load with several concentrated loads located at different points along the girder.

In order to explain the method for obtaining the length of the flange plates in a girder loaded in this manner, the following problem will be assumed and the diagram will be constructed as was done in previous cases.

Assume the girder to be loaded, as shown in Fig. 19, with a uniformly distributed load and the two concentrated loads. The flange section shown in Fig. 20 is sufficient to resist the bending moments due to these loads. It is required to determine by the graphic method the theoretical lengths of the several cover-plates making up the flange section.

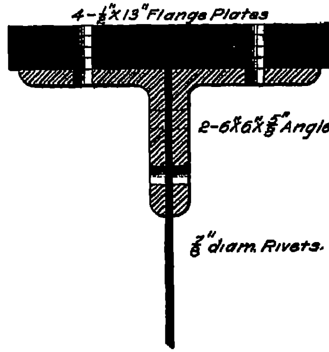


FIG 20

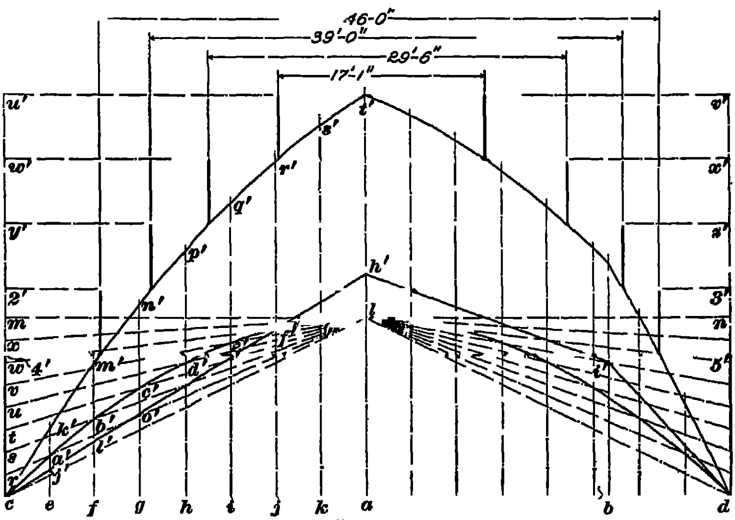


FIG. 21

Before starting to draw the diagram shown in Fig. 21, it is necessary to make the calculations for the following: The greatest bending moment; the maximum bending

moment due to the uniformly distributed load; and the bending moment under each of the concentrated loads, neglecting the uniformly distributed load. These bending moments should be expressed in foot-pounds. The flange area required to resist successfully each of these bending moments should also be calculated.

The calculations in this case have been made in the usual manner, and the results are as follows:

	Foot-Pounds
Greatest bending moment	= 2,070,000
Bending moment due to a uniform load . . .	= 900,000
Bending moment under concentrated load *a*, considering the concentrated loads only . .	= 1,170,000
Bending moment under concentrated load *b*, considering the concentrated loads only . .	= 792,000

Since the depth of the girder is 4 feet, if a unit fiber stress of 15,000 pounds is used, the flange area required to resist the greatest bending moment is $A = \frac{2,070,000}{4 \times 15,000} = 34.5$ square inches.

The flange area required to resist the bending moment due to the uniform load is $A = \frac{900,000}{4 \times 15,000} = 15$ square inches.

The flange area required to resist the bending moment at the point on the girder where the concentrated load *a* is situated, considering the concentrated loads only, is $A = \frac{1,170,000}{4 \times 15,000} = 19\frac{1}{2}$ square inches.

The flange area required to resist the bending moment at the point on the girder under the concentrated load *b* is $A = \frac{792,000}{4 \times 15,000} = 13.2$ square inches, considering, as before, only the concentrated loads.

20. As the loads on the girder are not symmetrically placed with regard to the center, it will be necessary to draw the complete diagram. Begin the diagram by drawing, to any convenient scale, the horizontal line *c d*, Fig. 21, equal

in length to the span of the girder, and locate the points of application of the concentrated loads at *a* and *b*; upwards from the points *c*, *a*, *b*, and *d* draw indefinite vertical lines.

Now, in accordance with the method explained in Art. **15**, make the construction to determine the curved line representing the bending moment due to the uniform load as follows: At the center of the girder draw a vertical line (in this case the center of the girder is found to be at the point where the load *a* is concentrated); divide half the span into any number of equal parts, as at *e*, *f*, *g*, *h*, *i*, etc., and from the points so obtained draw perpendiculars. Lay off on the vertical line passing through the center a distance *a l*, which may represent either the greatest bending moment at this point due to the uniform load, or the flange area required to resist this bending moment, as they are proportional. In this case the net area required in the flange will be used; hence, as the area of the flange required for the uniform load is 15 square inches, if $\frac{1}{16}$ inch is assumed to represent 1 square inch of flange area, the distance *a l* will be $\frac{15}{16}$ inch.

Through the point *l*, draw the horizontal line *m n*. Divide the distance *m c* into the number of equal parts into which the half of the span was divided, and from the points *r*, *s*, *t*, *u*, *v*, etc. thus obtained, draw converging lines to the point *l*; where these oblique lines intersect the vertical lines, mark the points *a′*, *b′*, *c′*, *d′*, etc., and through these points draw the curve *c l*. Draw the other half of the curve *l d* in the same manner, thus completing the diagram for the uniform load.

21. Now draw the diagram for the concentrated loads. On the vertical line *a l*, extended, lay off the distance *a h′*, equal to the net flange area required to support the concentrated load *a*, and on the perpendicular line erected at *b* lay off the distance *b i′* equal to the flange area required at the point *b* to support the concentrated loads.

It must be remembered that the same scale is to be used as that with which the flange area required for the uniform load diagram was laid off; also, that if the vertical distances are laid off to represent the bending moment in the one

case, the bending moment should be used in the other, while if the flange area required is used in the one case, it is evident that it should also be used in the other. The student will understand the importance of this fact when he proceeds further with the diagram. Having located the points h' and i', complete the diagram by connecting, with straight lines, the points c, h', i', and d, as was done in the previous diagrams of concentrated loads.

The next step in the process is to measure the distance $e\,j'$ with a pair of dividers, and from the point a' on the curve representing the uniform load, lay off on the vertical line the distance $a'\,k'$ equal to $e\,j'$; also lay off from the point b' the distance $b'\,m'$ equal to $f\,l'$, and from the point c' lay off on the vertical line the distance $c'\,n'$ equal to $g\,o'$; continue in this manner through the entire diagram. Having determined the points k', m', n', p', etc. through the entire diagram, draw in the curve $c\,k'\,m'\,n'$, etc. The point t' is the highest point in the diagram, and its distance from the horizontal line $c\,d$ represents the entire flange area required in the girder to resist the greatest bending due to both the uniform and the concentrated loads.

Through the point t', draw the horizontal line $u'\,v'$, and lay off between the horizontal lines $u'\,v'$ and $c\,d$ the several distances representing the net area of the flange plates and flange angles. Through the points of these divisions, draw the horizontal lines w'–x', y'–z', $2'$–$3'$, and $4'$–$5'$. Where these horizontal lines intersect the curved line representing the net area required for the combined uniform and concentrated loads, draw short vertical lines to the next horizontal line above; draw, with heavy lines, the rectangles representing the flange plates and flange angles; scale the length of the flange plates with the scale to which the span $c\,d$ was laid off, and the theoretical length of the flange plates will be found.

In the girder under consideration, the theoretical length of the top, or first, plate is found to be 17 feet 1 inch; the length of the second plate, 29 feet 6 inches; the length of the third plate, 39 feet 0 inches; and the length of the last, or fourth, plate is 46 feet 0 inches.

RIVET SPACING

22. Rivets in End Angles or Stiffeners Over Abutments.—First, the allowable safe load on the rivet should be determined. Whether the double shear of the rivet or the bearing value of the plate around the rivet hole is the greater, should be found as previously explained. Having obtained the safe allowable load for each rivet, a sufficient number should be placed in the end angles or stiffeners to take care of the entire shear at that point. Assume the reaction at the end of a girder to be 100,000 pounds; $\frac{7}{8}$-inch rivets are used and the web is $\frac{3}{8}$ inch thick. Using an allowable double shearing stress of 12,000 pounds per square inch, the value of the rivet in double shear is 7,216 pounds, while the web-bearing value is 5,251 pounds; as the latter is the smaller, it is the allowable load on the rivet. The number of rivets required in the two pair of end angles is, therefore, $100{,}000 \div 5{,}251 = 19.04$, say 20, or 10 rivets in each pair.

23. Rivets in Stiffeners Between Abutments.—If possible, the rivets in the intermediate stiffeners are usually spaced the same as in the end stiffeners. It is hardly possible to make any calculation of practical value in regard to the number and spacing of these rivets, and in fact no calculation is required; a practical rule is that the pitch of these rivets should never exceed 6 inches, nor should it exceed sixteen times the thickness of the leg of the angle.

24. Rivets Connecting Flange Angles With Web. When a plate girder is loaded, the tendency of the flanges and angles is to slide horizontally past the web; this tendency to slide induces a horizontal flange stress. The rivets connecting the angles to the web resist this tendency, and there must be a sufficient number of rivets to do it safely.

The stress that is transmitted horizontally from the web to the flange at any point is equal to the increment of the flange stress at that point. When the web is not considered

as resisting any portion of the bending moment, this increment is found by the formula

$$f_i = \frac{S}{h} \qquad (8)$$

in which S = maximum shear at point considered;
h = height, in inches, between center lines of rivets;
f_i = increment of stress per inch of run.

The increment of stress divided into the resistance of one rivet, gives the distance between centers of rivets, or their pitch. Hence,

$$p = \frac{r}{\frac{S}{h}} = \frac{r\,h}{S} \qquad (9)$$

in which p = pitch of rivets;
r = resistance of one rivet;
h and S = same as in formula **8**.

Where the web is assumed to help in resisting bending moment, the pitch of the rivets is increased by multiplying the value obtained in formula **9** by the ratio of the net area of the flange plus one-eighth of the gross area of the web to the net area of the flange alone. Therefore, when a portion of the web is included in the area resisting the bending moment,

$$p = \frac{A + \frac{A'}{8}}{A} \times \frac{r\,h}{S} \qquad (10)$$

Here A is the net area of one flange, exclusive of the web; A' is the gross area of the web; while p, r, h, and S have the same meanings as in formula **9**. The application of formula **10** is shown in the following example:

EXAMPLE.—A plate girder having a span of 40 feet supports a uniform load of 5,000 pounds per lineal foot. The distance from the inside end stiffener to the first intermediate stiffener is 4 feet, while the depth of the girder from center to center of the flange rivets is 3 feet. The thickness of the web-plate is $\frac{5}{16}$ inch and its depth is 40 inches. Provided that

the safe unit flange stress is 15,000 pounds, what will be the theoretical pitch of the $\frac{3}{4}$-inch rivets in the vertical legs of the flange angles of the second panel?

SOLUTION.—The bending moment at the first panel point is equal to the moment of the reaction about this point minus the moment of the load on the first panel; hence, $M = (100{,}000\times 4) - (5{,}000\times 4\times 2) = 360{,}000$ ft.-lb. In this example, the depth of the girder will be considered as the distance between centers of rivets. Then by the principle of Art. **11**, $M = d\,s\left(A + \frac{A'}{8}\right)$, 6 being replaced by 8 in the formula for M_1. Hence, $A + \frac{A'}{8} = \frac{M}{d\,s} = \frac{360{,}000}{3\times 15{,}000} = 8.00$ sq. in. But $\frac{A'}{8} = \frac{1}{8}\times 40\times\frac{5}{16} = 1.56$ sq. in. Therefore, $A = 8.00 - 1.56 = 6.44$ sq. in.

The bearing value of a $\frac{5}{16}$-in. plate on a $\frac{3}{4}$-in. rivet at 15,000 lb. per sq. in. is 3,516 lb. For web bearing, it is $3{,}516\times 1\frac{1}{3} = 4{,}688$ lb., which is less than the value of the rivet in double shear, or 6,627 lb. The maximum shear at the point under consideration is $100{,}000 - (5{,}000\times 4) = 80{,}000$ lb. Then, from formula **10**,

$$p = \frac{A + \frac{A'}{8}}{A}\times\frac{r\,h}{S} = \frac{8.00}{6.44}\times\frac{4{,}688\times 36}{80{,}000} = 2.62 \text{ in., or } 2\tfrac{5}{8} \text{ in. Ans.}$$

25. The example given below illustrates the method of finding the pitch of the rivets when no portion of the web-plate is considered as resisting the bending moment, or when formula **9** is applied.

Assume a girder of 40 feet span, as shown in Fig. 22, with a depth of 4 feet, and a uniformly distributed load of

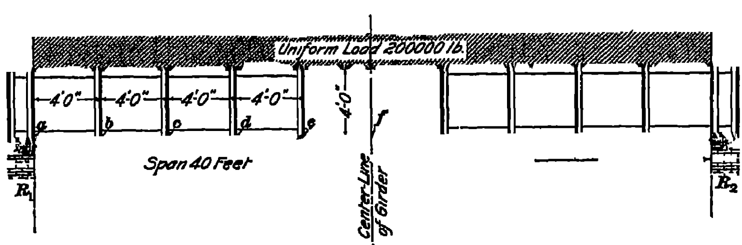


FIG. 22

200,000 pounds. The shearing stress in the girder at the left reaction, or point a, is equal to R_1, in this case 100,000 pounds. At b, 4 feet from R_1, the vertical shear in the girder is $100{,}000 - (5{,}000 \times 4) = 80{,}000$ pounds; at c, 8 feet

from R_1, the vertical shear is $100{,}000-(5{,}000\times 8)=60{,}000$ pounds; at d, it is $100{,}000-(5{,}000\times 12)=40{,}000$ pounds; at e, the shear is 20,000 pounds, and at f it is zero.

By substituting the above results in formula 8, the rate of increase, per inch of length, or the increment of the horizontal stress in the flange at the several points a, b, c, d, e, and f may be obtained. Thus, at the end a of the girder the increase in the horizontal flange stress is $\frac{100{,}000}{4\times 12}=2{,}083$ pounds per inch of run; at b, $\frac{80{,}000}{4\times 12}=1{,}667$ pounds per inch of run; at c, $\frac{60{,}000}{4\times 12}=1{,}250$ pounds per inch of run; at d, $\frac{40{,}000}{4\times 12}=833$ pounds per inch of run; and at e, $\frac{20{,}000}{4\times 12}=417$ pounds per inch of run

If $\frac{7}{8}$-inch rivets are used and the safe stress is 12,000 pounds per square inch, the bearing strength of one rivet in a $\frac{3}{8}$-inch plate is 3,938 pounds, from the table of Bearing Values of Riveted Plates in *Details of Construction* In web bearing, the strength of one rivet is $3{,}938\times 1\frac{1}{3}=5{,}251$ pounds. At the end, where the increase in stress is 2,083 pounds per inch of run the pitch of the rivets should be $5{,}251\div 2{,}083=2\ 52$ inches, from center to center. At b, the maximum allowable pitch of the rivets is $5{,}251\div 1{,}667=3.15$ inches; at c, the pitch may be $5{,}251\div 1{,}250=4\ 20$ inches; and at d, $5{,}251\div 833=6\ 30$ inches. Since, for practical reasons, the rivets in the vertical leg of the flange are spaced the same in both the upper and lower chords, and since the greatest allowable pitch of rivets in a compression member is 6 inches, it is needless to carry the calculation further

Hence, the pitch of the rivets between a and b should be $2\frac{1}{2}$ inches; between b and c, $3\frac{1}{4}$ inches; between c and d, $4\frac{1}{4}$ inches; since the theoretical pitch between d and e is more than 6 inches, which for practical reasons is not allowable, all the rivets between d and the center of the girder should be spaced 6 inches from center to center.

26. Effect of Vertical Stress.—Sometimes the vertical as well as the horizontal stress in the flange is taken into account in spacing the rivets, in which case the resultant of the two stresses is the stress that must be provided for. The vertical stress is due directly to the load resting on the flange of the girder, which, through the rivets, is transmitted to the web-plate.

In the plate girder shown in Fig. 22, the increase in the horizontal flange stress at the end is, as previously calculated, 2,083 pounds per inch of run; the load on the girder being uniformly distributed, the vertical stress on the flange, per lineal inch, is equal to the entire load on the girder divided by the span of the girder in inches; it is, therefore, $200{,}000 \div (40 \times 12) = 416$ pounds per inch of run.

The total stress to be resisted by the rivets is, therefore, equal to the resultant of 2,083 pounds—due to the increase in the horizontal stress on the flange—and the vertical stress of 416 pounds; this resultant is $\sqrt{2{,}083^2 + 416^2} = 2{,}124$ pounds per inch of run. The pitch of the rivets at the end of the girder would then be $5{,}251 \div 2{,}124 = 2.47$, approximately, $2\frac{1}{2}$ inches.

At *b*, 4 feet from the end of the girder, the horizontal increment of stress on the flange, as previously calculated, is 1,667 pounds per inch of run, while the vertical stress remains the same; the combined action of these two forces produces a resultant stress on the rivets of $\sqrt{1{,}667^2 + 416^2} = 1{,}717$ pounds per inch of run, and this divided into the value of one rivet gives $5{,}251 \div 1{,}717 = 3.06$, or about $3\frac{1}{4}$ inches. Similar calculations may be made for each panel point to the center of the girder, or until the pitch exceeds the allowable limit of 6 inches.

The above results show that the values of the pitch in which the vertical stress due to the load is taken into account, are nearly the same as those first obtained; the effect of the vertical stress has, therefore, little influence on the pitch of the rivets, and it is hardly necessary to go into such refinement in the design of an ordinary plate girder.

27. Rivets Spaced According to Stress Produced by Bending Moment.—The rivets that connect the flange angles with the web-plate may also be spaced according to the stresses produced on the flanges by the bending moment.

The horizontal stress on the flanges diminishes either way from the point of greatest bending moment toward the end reactions, where it becomes zero, and for any point this stress may be calculated by the application of the principle of moments.

If the bending moment is obtained at any panel point and is divided by the depth of the girder, the stress on the flange at that point will be obtained; and, if this stress is divided by the allowable load on one rivet, the number of rivets required between that point and the end reaction will be obtained.

For example, in the girder used in the previous illustration, Fig. 22, the span being 40 feet and the load 200,000 pounds, the bending moment at the center is equal to $\frac{WL}{8} = \frac{200{,}000 \times 40}{8} = 1{,}000{,}000$ foot-pounds; then the depth of the girder being 4 feet, the flange stress at this point is $1{,}000{,}000 \div 4 = 250{,}000$ pounds. The allowable load on each rivet being 5,251 pounds, the number of rivets between the center and the end reaction is $250{,}000 \div 5{,}251 = 48$ rivets, approximately.

Now, although the number of rivets between the end reaction and the center of the girder has been obtained, the pitch of these rivets is still unknown. Since the increase of horizontal stress in the flange varies, being greatest at the ends and least under the position of maximum bending moment, it follows that the rivets should be spaced nearer together at the ends, with an increase in the spacing toward the point of greatest bending moment.

In practical work, the rivet spacing is seldom varied in any one panel; if, however, the flange stress is obtained at each of the stiffeners b, c, d, e, and f, Fig. 22, the number of rivets required between each of these points and the end reaction may be obtained; by finding the difference between

these numbers for any two consecutive stiffeners, the number of rivets required in the panel between those stiffeners is arrived at. For example, the stresses on the flange at each of the stiffeners of the girder shown in Fig. 22 are as follows:

	Bending Moment Foot-Pounds		Depth of Girder Feet		Flange Stress Pounds
At *b*,	360,000	÷	4	=	90,000
At *c*,	640,000	÷	4	=	160,000
At *d*,	840,000	÷	4	=	210,000
At *e*,	960,000	÷	4	=	240,000
At *f*,	1,000,000	÷	4	=	250,000

The approximate number of rivets between each stiffener and the reaction R_1 is as follows:

Between *b* and R_1, $90{,}000 \div 5{,}251 = 18$ rivets
Between *c* and R_1, $160{,}000 \div 5{,}251 = 31$ rivets
Between *d* and R_1, $210{,}000 \div 5{,}251 = 40$ rivets
Between *e* and R_1, $240{,}000 \div 5{,}251 = 46$ rivets
Between *f* and R_1, $250{,}000 \div 5{,}251 = 48$ rivets

Then the number of rivets required is:

Between *b* and *a*, $18 - 0 = 18$ rivets
Between *c* and *b*, $31 - 18 = 13$ rivets
Between *d* and *c*, $40 - 31 = 9$ rivets
Between *e* and *d*, $46 - 40 = 6$ rivets
Between *f* and *e*, $48 - 46 = 2$ rivets

Consequently, the pitch between the stiffeners will be as follows:

Between *b* and *a*, $48 \div 18 = 2.67$ inches
Between *c* and *b*, $48 \div 13 = 3.69$ inches
Between *d* and *c*, $48 \div 9 = 5.33$ inches
Between *d* and *e*, $48 \div 6 = 8.00$ inches

Between *d* and *e* the theoretical pitch exceeds 6 inches, the limit allowable for a compression member.

By the first method, the pitch at each stiffener or panel point is determined, while by the second the average pitch between two consecutive panel points is obtained; in order

to compare the two, we will reduce the results obtained by the first to the basis of the second. In the first method the pitch at the several stiffeners or panel points was found to be:

At $a = 2.52$ inches At $c = 4$ 20 inches
At $b = 3.15$ inches At $d = 6$ 30 inches

From these the average pitch between the several points would be:

Between a and b, $(2\ 52+3\ 15) \div 2 = 2\ 84$ inches
Between b and c, $(3.15+4.20) \div 2 = 3.68$ inches
Between c and d, $(4.20+6.30) \div 2 = 5.25$ inches

These, on comparison, are found to correspond approximately with the values 2.67, 3.69, and 5.33 inches obtained by the second method.

28. Rivets Spaced According to Direct Vertical Shear.—This is the method much used in practical work, and will be found to give safe results, corresponding favorably with those obtained by the previous methods. The method is based on the assumption that at any point the horizontal shear between the flange angles and the web-plate is equal to the vertical shear on the girder; for example, the vertical shear at the end stiffener or point a, Fig. 22, is 100,000 pounds; then, according to this method, the shearing stress between the flange angles and the web-plate is 100,000 pounds, distributed over the space between the panel points a and b, and sufficient rivets should be placed between these points to safely sustain this shear.

The allowable web-bearing load on a $\frac{7}{8}$-inch rivet in a $\frac{3}{8}$-inch plate being 5,251 pounds, the number of rivets required between a and b is $100{,}000 \div 5{,}251 = 20$, approximately; the vertical shear at b is 80,000 pounds, and $80{,}000 \div 5{,}251 = 16$, approximately, the number of rivets to be used between b and c; the shear at c is 60,000 pounds, and $60{,}000 \div 5{,}251 = 12$, approximately, the number of rivets to be used between c and d; similarly, the number of rivets required between d and e is found to be 8. According to these results, the pitch of the rivets between a and b should be $48 \div 20$

= 2.4 inches; between *b* and *c*, 48 ÷ 16 = 3 inches; between *c* and *d*, 4 inches; and from there on, 6 inches.

Rivet spacing in plate girders is governed so largely by practical considerations, that this method is to be recommended on account of its convenience. It gives safe results that agree closely with those obtained by the more cumbersome methods.

29. Graphic Method of Determining Number of Rivets in Vertical Leg of Flange Angles.—Besides the several analytical methods of determining the number of

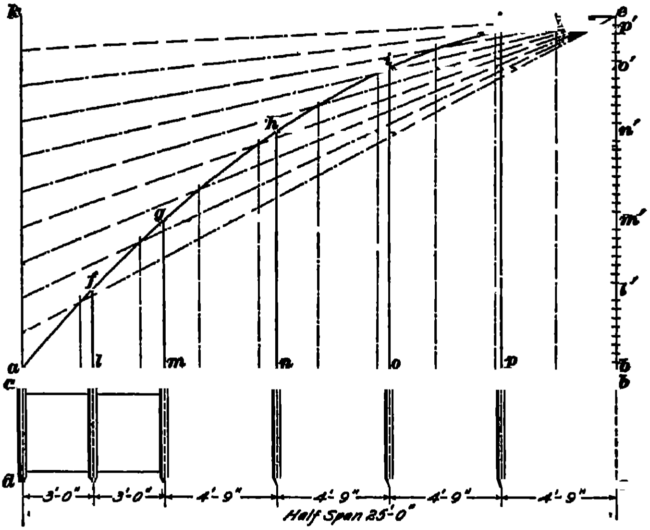


FIG 23

rivets through the vertical legs of the flange angles in the several panels of the plate girder, a convenient graphic method that gives approximate results sufficiently accurate for all practical purposes is illustrated in Fig. 23. In this method, a diagrammatic drawing of one-half of the plate girder with the stiffeners properly placed, is made to scale, as shown at *a b c d*. The number of rivets required through the vertical leg of the top flange of the plate girder, from the center line of the girder to the abutment, is next calculated

by the method described in Art. **27**, which consists in first calculating the bending moment, in foot-pounds, due to the uniformly distributed load and dividing by the depth of the girder, in feet, the result being the horizontal flange stress. This stress divided by the allowable resistance of one rivet will give the number of rivets between the center of the plate girder and the abutment, or between a and b in the sketch. Having determined the number of rivets required between these points, lay off any vertical distance, as $b\,e$, and through the points a and e describe a parabola by the method explained in Art. **15.** Having drawn in the parabola as shown in the figure, extend upwards the center lines of the angle stiffeners on the plate girder until they intersect the parabola, as at f, g, h, i, and j. Through the points of intersection, extend horizontal lines until they intersect the line $b\,e$, the greatest ordinate of the parabola, which is coincident with the center line of the plate girder. Having proceeded thus far, divide the length of the greatest ordinate of the parabola, or the distance $b\,e$, into the same number of equal parts as there are rivets required between a and b. This may conveniently be done by applying the scale obliquely between the horizontal lines $k\,e$ and $a\,b$, as explained in connection with the method for determining the length of the flange plates in compound riveted girders.

Assuming that in the diagram forty rivets are required between a and b, and that in consequence there are forty divisions on the scale, the number of rivets between a and l will equal the number of spaces between b and l', or nine. The number of rivets between l and m will equal the spaces between l' and m', or eight, while between m and n, from the portion included between m' and n', there would be required ten rivets. In the panel $n\,o$, the theoretical number of rivets will equal eight, while between o and p, the theoretical requirements will be fulfilled by placing four rivets through the vertical leg of the flange angles. In all cases where there is a fraction of a rivet in a panel, one additional rivet should be used. Since between a and l nine rivets are required, and the distance from the center of the end stiffener to the

center of the stiffener l is 36 inches, the theoretical pitch between a and l will equal $36 \div 9 = 4$ inches.

In the panel $l\,m$, since there are eight rivets required, the pitch will be $36 \div 8 = 4.5$ inches. Between m and n, there are ten rivets required and the theoretical pitch will be $57 \div 10 = 5.70$ inches. The number of rivets required between n and o is eight, and consequently their pitch will be $57 \div 8 = 7.13$ inches. This last distance is greater than the maximum allowable pitch for rivet spacing in plate girders; therefore, it is needless to calculate the theoretical pitches in the remaining panels. From these calculations it is probable that in designing the plate girder a pitch of 4 inches will be adopted for the panels $a\,l$ and $l\,m$, provided that a single row of rivets is used, while throughout the remaining panels of the girder the rivets will be spaced at the maximum pitch of 6 inches.

30. The student will observe in this method the application of the principles involved in the graphic method for determining the length of the flange plates of the girder supporting the uniformly distributed load. The parabola always represents graphically the bending moment created in a simple beam by a uniformly distributed load. Since the horizontal flange stress varies directly with the amount of the bending moment throughout the girder, it is evident that the parabola likewise truly represents the flange stress between the abutments and the center line of the girder. Likewise, if the middle or greatest ordinate of the parabola represents the horizontal flange stress, to scale, at the center of the girder, the ordinates of the parabola at any point will equal the horizontal flange stress at that point. If the heights of the ordinates projected from the center line of each stiffener are laid off on the center ordinate and the center ordinate is reduced from horizontal flange stress to the number of rivets whose resistance will equal the amount of flange stress, then the ratio between the several ordinates and the greatest ordinate will be the same as the ratio of the number of rivets required between the ordinate and

the abutment of the girder and the number of rivets required between the center of the girder and the abutments; that is, the length mg is to the length eb as the number of rivets between a and m is to the number of rivets between a and b; thus, if the length of the ordinate mg were equal to $1\frac{19}{32}$ inches and the length of the ordinate eb were $3\frac{3}{4}$ inches, while the number of rivets required between a and b is 40, the number of rivets required between m and a would equal $\frac{mg \times 40}{be}$; or, substituting the values, $\frac{1.59375 \times 40}{3.75} = 16.92$, or 17. Figuring in the same way, if lf were $\frac{27}{32}$ inch, the number of rivets between l and a would equal $\frac{.84375 \times 40}{3.75} = 9$. The difference between the number of rivets required between the points a and m and the points a and l equals $17 - 9 = 8$, which accurately expresses the number of rivets called for by the space $l'm'$.

EXAMPLE 1.—A plate girder 60 feet long and 4 feet in depth is loaded with a uniformly distributed load of 2,000 pounds per lineal foot. Required, by the graphic method, the theoretical pitch of rivets through the vertical leg of the flange angle in the several panels, assuming that the allowable resistance of each rivet is 4,800 pounds. The first two panels are 3 feet 9 inches, and the remainder 5 feet in length.

SOLUTION.—The maximum bending moment, which occurs at the center of the girder, is expressed by the formula $M = \frac{WL}{8}$. The value of W, according to the problem, is $2,000 \times 60 = 120,000$, while L or the length of the girder equals 60 ft. By substitution, the bending moment, $M = \frac{120,000 \times 60}{8} = 900,000$ ft.-lb. Since the depth of the girder is 4 ft, the horizontal flange stress at the center of the girder is $900,000 \div 4 = 225,000$ lb. The strength of each rivet equals 4,800 lb.; hence, the number of rivets required between the center and the end is $225,000 \div 4,800 = 46\ 875$, or approximately 47. Having obtained this result, lay out a diagrammatic elevation of one-half of the plate girder to scale, as shown in Fig. 24, locating the stiffeners where required. On this plate girder draw one-half of the parabola, the center ordinate of the parabola coinciding with the center line of the girder. By placing a convenient scale obliquely between the horizontal lines ab and cd, divide the greatest ordinate into 47 equal

parts. Extend upwards from the points k, l, m, n, o, and p the several ordinates, as shown, and project the points e, f, g, h, i, and j horizontally to the greatest ordinate of the parabola.

From the diagram, Fig. 24, the number of rivets required between the several panel points is as follows:

Between a and k, 11 rivets
Between k and l, 10 rivets
Between l and m, 11 rivets
Between m and n, 8 rivets
Between n and o, 5 rivets
Between o and p, 3 rivets
Between p and b, 1 rivet

The distance between the panel points or the center line of the stiffeners for the first two panels is equal to 45 in., while for the

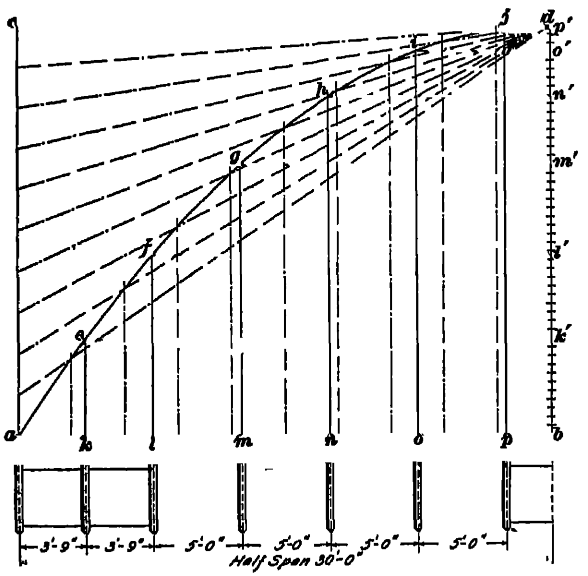


FIG 24

remaining panels the stiffeners are spaced 60 in. on center lines; hence, the pitch between the several theoretical panel points from the abutment toward the center is as follows.

Between a and k = 45 ÷ 11 = 4.09 in.
Between k and l = 45 ÷ 10 = 4.5 in.
Between l and m = 60 ÷ 11 = 5.45 in.
Between m and n = 60 ÷ 8 = 7.5 in.

The calculations need not be carried further, for the theoretical pitch of the rivets in the next panel beyond lm exceeds the maximum allowable pitch for rivets in plate girders. Ans.

EXAMPLE 2.—Find the pitch of the rivets, by the graphic method, in the several panels of a plate girder having a span of 75 feet and a depth of 5 feet. The length of the panels throughout the girder is 5 feet and the load supported is 2,500 pounds per lineal foot. The strength of one rivet may be taken at 5,000 pounds, and the rivets should be placed in two rows.

SOLUTION.—As in the previous examples, the diagram should be laid out and the parabola representing the bending moment drawn, as shown in Fig. 25. The number of rivets required between the

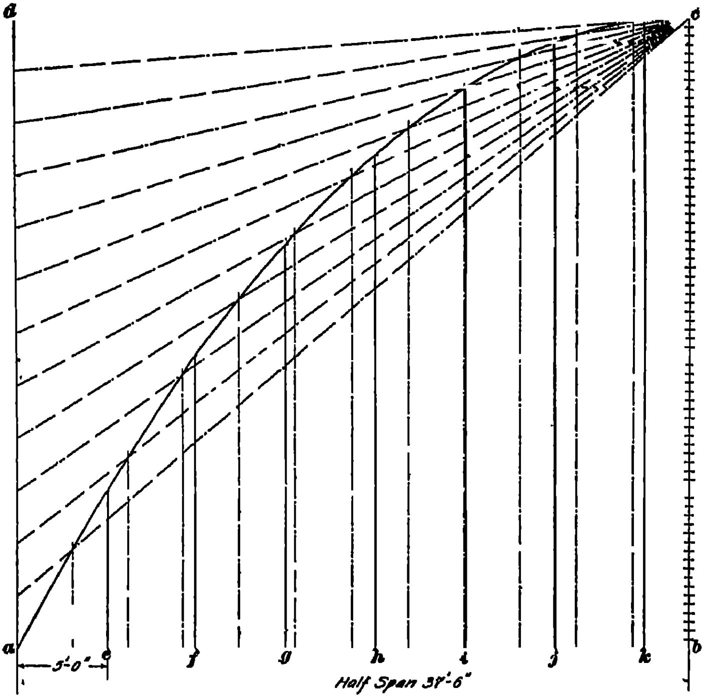


FIG. 25

center and the end is found by dividing the strength of one rivet into the quotient obtained by dividing the bending moment by the depth of the girder. Thus, $\frac{2,500 \times 75 \times 75}{8 \times 5 \times 5,000} = 70.3$, or 71 rivets.

Divide the greater ordinate bc into seventy-one equal spaces and project horizontal lines from the points where the lines drawn from the panel points e, f, g, h, etc. intersect the parabola, thus determining the number of rivets required in each panel. The first requires

eighteen; the second, fifteen; the third, thirteen; etc. Then the spacing in the first panel is $60 \div 18 = 3.3$ in.; for the second panel the pitch is $60 \div 15 = 4$ in.; for the third and fourth panels the pitch is found to be 4.6 in. and 6 in., respectively. As 6-in. angles would undoubtedly be used in this girder, it is advisable to place the rivets in two rows; in the first two panels they may be spaced 6 in. in each row, while in the remaining panels the pitch will be 8 in., which is the maximum pitch for rivets placed in a double row. Ans.

31. Pitch of Rivets in Girders Supporting Concentrated Loads.—In order to illustrate the method of finding the pitch of the rivets in a girder that supports concentrated loads, the conditions shown in Fig. 26 (*a*) will be

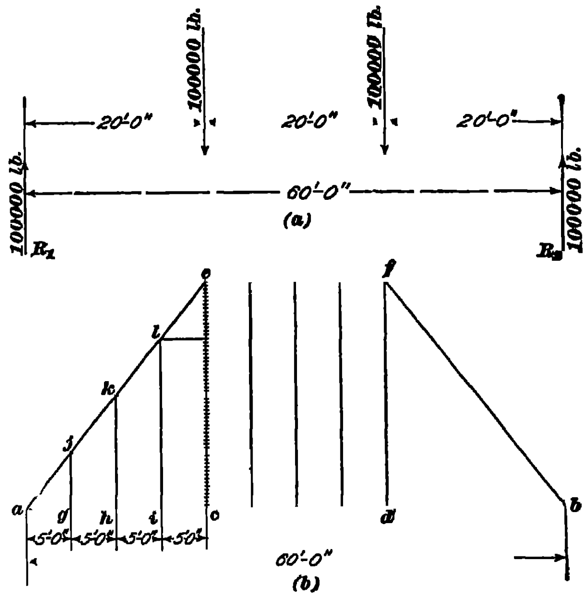


FIG. 26

assumed. The greatest bending moment occurs under the loads and along the length of the girder between them. Lay off $a\,b$, in (*b*), equal to the span of the girder, and mark the points c and d where the concentrated loads occur. At these points erect perpendiculars and lay off any convenient distance on each, as $c\,e$ and $d\,f$, to represent the bending moment. Draw $a\,e$, $e\,f$, and $f\,b$, and from the panel points

erect perpendiculars intersecting these lines at *j*, *k*, and *l*. The maximum bending moment, occurring at *c*, is equal to the moment of the reaction about that point, or 100,000 × 20 = 2,000,000 foot-pounds. The flange stress equals 2,000,000 ÷ 5 = 400,000 pounds. Assuming the strength of one rivet to be 6,750 pounds, the number required is 400,000 ÷ 6,750 = 59.26, or 60. Divide *ce* into sixty equal parts and from

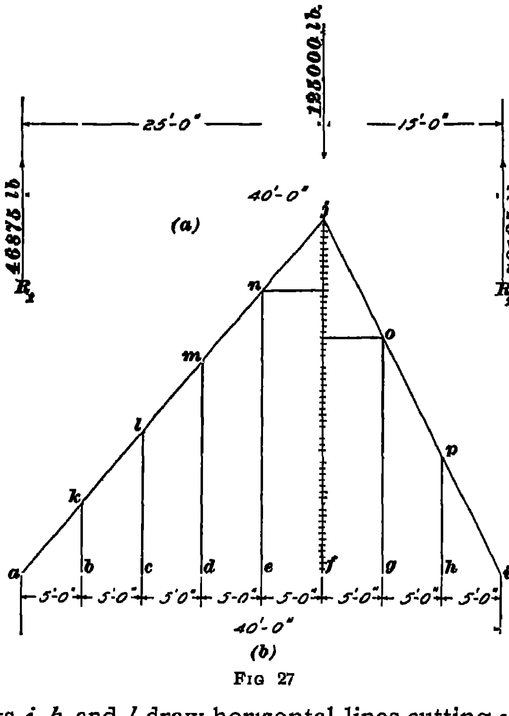


FIG 27

the points *j*, *k*, and *l* draw horizontal lines cutting *ce*. These lines show that fifteen rivets are required for each panel up to the point *c* and from *c* to *d* no rivets are necessary, theoretically. The pitch of the rivets in the first four panels is 60 ÷ 15 = 4 inches, while for the panels from *c* to *d*, they will be spaced at the maximum pitch, or 6 inches.

EXAMPLE.—In Fig. 27 (*a*) is shown a girder having a span of 40 feet and carrying a concentrated load of 125,000 pounds at a distance of 15 feet from one end. If the girder is 4 feet deep and the

panels are 5 feet in length, what will be the pitch of the rivets, assuming the strength of 1 rivet to be 5,000 pounds?

SOLUTION.—Lay off the span of the girder at *a i* in view (*b*), marking the panel points, as at *b*, *c*, *d*, etc., and at the location of the concentrated load erect a perpendicular, as *f j*, of any convenient height. Draw *a j* and *j i*, and from the panel points draw perpendiculars to *a i*, intersecting these lines at *k*, *l*, *m*, *n*, *o*, and *p*. The maximum bending moment is under the load and is equal to 46,875 × 25 = 1,171,875 ft.-lb.; 1,171,875 ÷ 4 = 292,968 75 lb. flange stress; 292,968.75 ÷ 5,000 = 58.59+, or, say, sixty rivets are required between the concentrated load and each end of the girder Divide *f j* into 60 equal parts and draw horizontal lines from the points *k*, *l*, *m*, etc. until they intersect *f j*. This shows that twelve rivets are required in each panel to the left of the load, while twenty are needed in each panel to the right. The pitch will then be 60 ÷ 12 = 5 in. in the first five panels from the left, and 60 ÷ 20 = 3 in. in the remaining panels.

32. Rivet Spacing in Flange Plates.—In spacing the rivets that bind the several flange plates together, a sufficient number of rivets, spaced from 2¾ to 3 inches on centers, should be used at the ends of each plate to transmit the allowable stress in it to the members below. For the remainder of the plate, the rivets should have the greatest allowable pitch for a compression member; that is, sixteen times the thickness of the thinnest outside plate, provided that such a distance does not exceed 6 inches. To illustrate:

An intermediate flange plate in a certain girder is ⅜ inch by 12 inches, the sectional area thus being 4½ square inches. From this area is to be deducted the section cut out by two ⅞-inch rivet holes, (1 × ⅜) × 2 = ¾ square inch; then the net area of the cover-plate is 4½ − ¾ = 3¾ square inches. Assuming that a safe fiber stress of 15,000 pounds was used in calculating the strength of the girder, the safe strength of the cover-plate is 3¾ × 15,000 = 56,250 pounds. Now the safe load on a ⅞-inch rivet depends, in this position, on the ordinary bearing value of a ⅜-inch plate, which, calculated on the basis of a fiber stress of 12,000 pounds, is 3,938 pounds. Hence, the number of rivets required in the end of this cover-plate is 56,250 ÷ 3,938 = 14.3, say, 15; but in order to have them symmetrical, there should be eight on each side

of the web, and they should be spaced about 3 inches from center to center. The remaining rivets in this plate may have the greatest allowable pitch until the next cover-plate is reached.

EXAMPLES FOR PRACTICE

1. Determine, by the graphic method, the theoretical pitch for the rivets in the vertical legs of the flange angles in a plate girder 45 feet long from center to center of bearing plate. The depth of the girder is 36 inches and the load per lineal foot is 1,400 pounds. The safe resistance of the rivets is determined to be 4,200 pounds; the distance between the stiffeners or panel points is 5 feet throughout the girder.

Ans. { First panel, 5 in.
Second panel, 6.67 in.
Third panel, 10 in., etc.

2. Theoretically, how many rivets will be required in the vertical legs of the flange angles between the several panel points or stiffeners of a plate girder having a span of 80 feet and a depth of 5 feet, assuming that the load on the girder is uniformly distributed and is equal to 3,000 pounds per lineal foot; also, that the resistance one rivet offers is 5,200 pounds. The panel lengths are each 5′ 0″.

Ans. { First panel, 22 rivets
Second panel, 19 rivets
Third panel, 16 rivets
Fourth panel, 13 rivets
Fifth panel, 10 rivets

3. Find the pitch of the rivets in the vertical legs of the flange angles in a girder 4 feet in depth and having a span of 50 feet, which supports a concentrated load of 90,000 pounds located at a distance of 20 feet from one end of the girder. The value of one rivet is 4,500 pounds and the length of each panel is 5 feet.

Ans. { 4-in. pitch in four panels at end nearest load
6-in. pitch in remainder

PRACTICAL DESIGN

33. In order to illustrate the application of the rules and formulas previously given, the following practical problem will be assumed and worked out:

The floor of a building used for light manufacturing purposes is to be supported by three plate girders, as shown at *a*, *a*, *a*, Fig. 28. The floor is composed of 1-inch yellow-pine flooring laid on 3″ × 12″ hemlock joists, spaced on 16-inch centers; these joists are to carry a plastered ceiling on the under side. The live load on the floor will be

80 pounds per square foot. The girder itself is to extend below the surface of the ceiling and is to be painted. A detail of the construction is shown in Fig. 29.

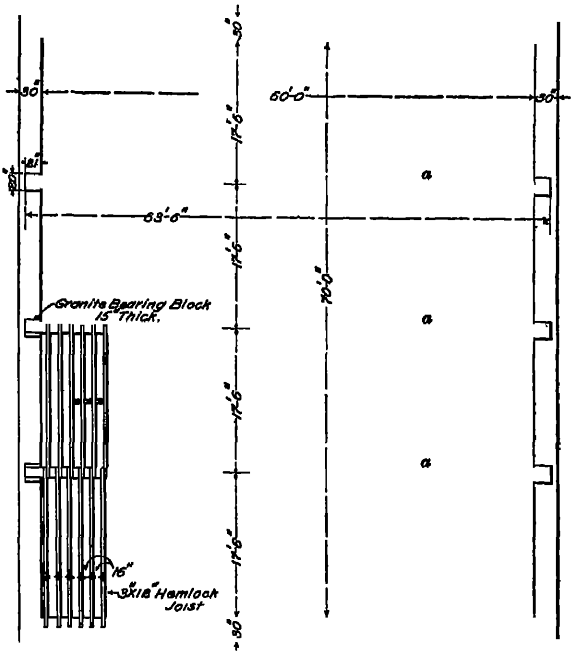


FIG. 28

The total load on each square foot of floor surface is as follows:

Live load, per square foot of floor surface .	80 pounds
Lath and plaster, per square foot of floor surface	8 pounds
1-inch yellow-pine flooring, per square foot of floor surface	4 pounds
Hemlock joist flooring, per square foot of floor surface	6 pounds
Girder (assumed), per square foot of floor surface	8 pounds
Total	106 pounds

The floor area to be supported by one girder is $60 \times 17.5 = 1{,}050$ square feet; and the total uniformly distributed load on the girder is $1{,}050 \times 106 = 111{,}300$ pounds.

The greatest bending moment on the girder is

$$M = \frac{WL}{8} = \frac{111{,}300 \times 60}{8} = 834{,}750 \text{ foot-pounds}$$

The depth of the girder is 4 feet, and the allowable unit fiber stress to be used is 15,000 pounds; therefore, the

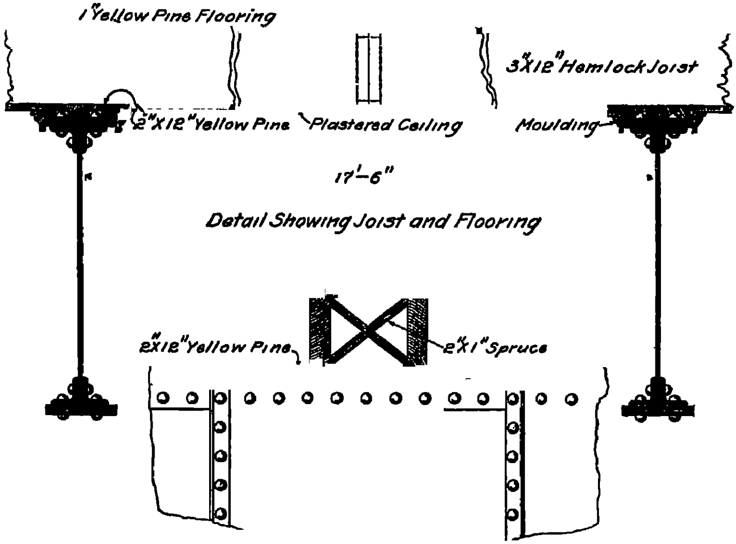


FIG. 29

required flange area may be determined by formula **4.** Substituting the proper values in the formula gives

$$A = \frac{834{,}750}{4 \times 15{,}000} = 13.91 \text{ square inches}$$

Assume a flange composed of two $5'' \times 5'' \times \frac{7}{16}''$ angles and two $\frac{3}{8}'' \times 12''$ flange plates; a sketch of the section with the location of the rivets is shown in Fig. 30. The entire area of the flange is:

Two $\frac{3}{8}'' \times 12''$ plates	=	9	square inches
Two $5'' \times 5'' \times \frac{7}{16}''$ angles	=	8.36	square inches
Total,		17.36	square inches

The sectional areas cut out for rivet holes are:

Four $\frac{7}{8}$-inch holes through $\frac{3}{8}$-inch plate	=	1.3 1 2 sq. in.
Four $\frac{7}{8}$-inch holes through $\frac{7}{16}$-inch angles	=	1.5 3 1 sq. in.
	Total,	2.8 4 3 sq. in.

The net area of the flange is, therefore, 17.36 − 2.84 = 14.52 square inches, which, since the required area is 13.91 square inches, is ample, and this section will be adopted.

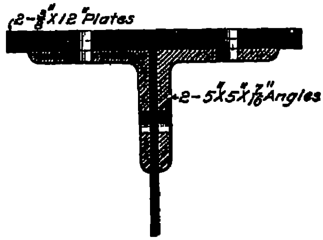


FIG. 80

We will now determine the thickness of the web-plate. The reaction at either end is equal to one-half of the load, or 55,650 pounds. Assuming that there are eleven $\frac{7}{8}$-inch holes cut in line through the web-plate, the net depth of the plate will be $48 - 11 \times .875 = 38.375$ inches. Using an allowable unit shearing stress of 11,000 pounds, the theoretical thickness of the web-plate, from formula **1**, is $t = \frac{55{,}650}{38.375 \times 11{,}000} = .132$ inch.

However, it is not practicable to use this thickness of metal for a web-plate, since it would not provide sufficient bearing value for the rivets. As it is never good practice to use a web-plate less than $\frac{5}{16}$ inch in thickness, this size will be adopted.

34. The lengths of the flange plates are now required; they may be determined either by the graphic method or by formula **6**; using the latter method, the theoretical length of the outside plate is found to be $l = 60\sqrt{\frac{3.844}{14.52}} = 30.87$ feet, or about 30 feet 10 inches, to which is to be added 1 foot at each end to allow for riveting. The total length of the plate is therefore 32 feet 10 inches, say, 33 feet.

Applying the formula again, the length of the second flange plate is $l = 60\sqrt{\frac{7.688}{14.52}} = 43\ 66$ feet, or about 43 feet 8 inches; adding a foot at each end gives us 45 feet 8 inches.

Consider now the size of the four stiffeners at the end of the girder. The reaction at the end of the girder is 55,650 pounds, and the allowable compressive strength of the material in the girder will be taken at 13,000 pounds. Then the sectional area required in the four angles composing the stiffeners on the plate girder over the abutments is $55,650 \div 13,000 = 4.28$ square inches. Since it would not be advisable to use smaller than a $4'' \times 4'' \times \frac{5}{16}''$ angle in this position, the sectional area of which is 2.4 square inches (see table Properties of Angles, in *Properties of Sections*), it is evident that there will be ample strength in the four stiffeners. The other stiffeners may be made of $3'' \times 3'' \times \frac{5}{16}''$ angles, which is the smallest size that should be used for any girder requiring intermediate stiffeners.

35. The rivet spacing, etc. needs no explanation; it would be well, however, for the student to calculate the number of rivets for the several parts and compare the results with the number actually used, as shown by the detail drawing, Fig. 31. He will undoubtedly find that more rivets are used than are actually required, but he must bear in mind that there are always practical considerations that influence more or less the design of structural work.

In Fig. 31 it will be noticed that the web-plate is spliced at the point a. The shear at this point is equal to 55,650 pounds, the reaction at R_1 minus the load on the girder between R_1 and the point a under consideration, that is, $55,650 - 37,150 = 18,500$ pounds. A sufficient number of rivets must be placed on the two sides of the joint to take care of this shear safely.

Fig. 32 shows the design of a heavily loaded girder with a long span; this girder was designed to carry a uniformly distributed load of 2,400 pounds per lineal foot and two concentrated loads of 60,000 pounds, placed one on each side of the center of the girder and 12 feet 6 inches therefrom. The unit fiber stress allowable in calculating the flange section was taken at 15,000 pounds.

The student should note particularly the splices on this

girder. In this case it was necessary to splice the flange angles; when this is done, care must be taken to weaken the sectional area of the angle as little as possible by the punching of the rivet holes, and a sufficient number of rivets must also be placed each side of the joint, so that the resistance of the rivet section may equal that of the net section of the flange angles. A careful study should be made of the various other details on this drawing, which represents excellent modern practice.

CAMBERED GIRDERS

ANALYTIC METHOD OF COMPUTING STRESSES

GIRDERS WITH ONE STRUT

36. When wooden girders of great span are heavily loaded, it becomes necessary to strengthen them with iron or steel camber rods, as in Figs. 33 and 34 which show a girder with one and with two supports, respectively. The span of the beam or girder may be considered, in each case, as the distance between the supports, the strength of the girder being thereby materially increased.

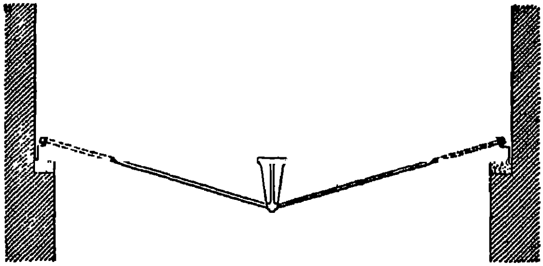


FIG. 33

37. In Fig. 35, let W represent the load concentrated at D. Then the stress in the member DC is equal to W. The stress in the other members may be found by applying the following rules:

Rule.—I. *To find the stress in $A\,C$ or $B\,C$, divide the length of the line $A\,C$ by the length of the line $D\,C$, and then multiply this result by one-half of the load W.*

II. *To find the stress in the beam $A\,B$, divide the length of the line $A\,D$ by the length of the line $D\,C$, and multiply this result by one-half of the load W.*

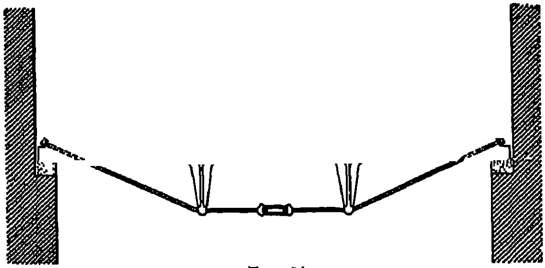


FIG. 34

In the diagram, Fig. 35, the members represented by the solid lines are in compression, and those shown dotted are in tension.

The length of the members in the above rules may be taken in feet or inches, but all lengths should be taken in the same

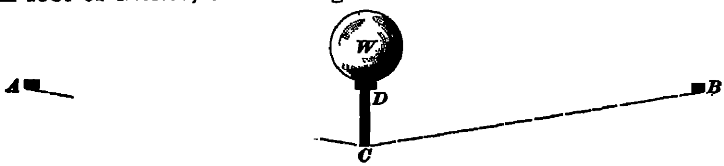


FIG. 35

unit of measurement. The rules may be expressed by the formulas:

$$\text{Stress } D\,C = +W \qquad (11)$$

$$\text{Stress } A\,C \text{ or } B\,C = -\frac{A\,C}{D\,C} \times \frac{W}{2} \qquad (12)$$

$$\text{Stress } A\,B = +\frac{A\,D}{D\,C} \times \frac{W}{2} \qquad (13)$$

The + and − signs in the formulas indicate compression and tension, respectively. The + sign denotes that the result obtained is a *compressive stress.* The − sign means that the result is a *tensile stress.*

EXAMPLE —What is: (*a*) the tension in the camber rod, (*b*) the compression on a trussed beam of the dimensions and loads shown in Fig 36?

SOLUTION.—The load *W* coming on the strut *D C* must first be computed. The load is, in this case, usually considered equal to one-half

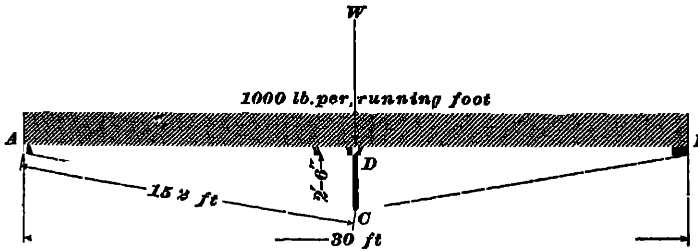


FIG. 36

the entire load on the beam. But as the beam is composed of one length of timber, and is not hinged at *D*, being, in effect, a continuous beam, it is more correct to consider the load on the center strut as being five-eighths of the entire load on the beam. The entire load on the beam is equal to 30 × 1,000 = 30,000 lb.; five-eighths of 30,000 = 18,750 lb., the load *W* acting on the beam directly over the strut *D C*.

(*a*) The tension in the camber rod *A C* is equal to the length *A C* ÷ *D C* multiplied by one-half of *W*, or, substituting the given dimensions, 15.2 ÷ 2.5 = 6.08; and 6.08 × (½ of 18,750) = 57,000 lb., the tensile stress in the rod *A C*. Ans.

(*b*) To determine the stress in beam *A B*, divide the length *A D* by *D C*, and multiply by one-half of *W* Thus, 15 ÷ 2.5 = 6; 6 × (½ of 18,750) = 56,250 lb., the compressive stress in the beam *A B*. Ans.

GIRDERS WITH TWO STRUTS

38. In Fig. 37, the calculations for the stresses in the various members are similar to those given for the trussed

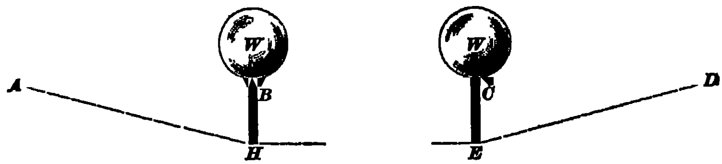


FIG 37

beam with one support. In the two trussed beams, the stress in *B H* or *C E* = *W*. The stresses in the other members may be expressed by rule, as follows:

Rule.—I. *To obtain the stress in AH or DE, divide the length of AH by the length of BH, and multiply this result by the amount of the load W.*

II. *To find the stress in AD or HE, divide the length of the line AB by the length of the line BH, and multiply this result by the amount of the load W.*

The above may be expressed in formulas:

$$\text{Stress } BH \text{ or } CE = +W \qquad \textbf{(14)}$$

$$\text{Stress } AH \text{ or } DE = -\frac{AH}{BH} \times W \qquad \textbf{(15)}$$

$$\text{Stress } HE = -\frac{AB}{BH} \times W \qquad \textbf{(16)}$$

$$\text{Stress } AD = +\frac{AB}{BH} \times W \qquad \textbf{(17)}$$

As previously noted, compression is indicated by the + sign, and tension by the − sign.

EXAMPLE.—A beam is trussed, as shown in Fig. 38. What is. (*a*) the stress in camber rod HE; (*b*) the compression in the beam AD?

SOLUTION.—The entire load on the beam AD is $30 \times 1{,}000 = 30{,}000$ lb. The loads W, W could be considered equal to one-third of the entire load on the beam. But the beam, being a con-

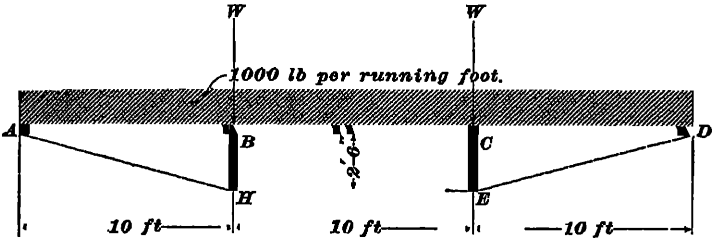


FIG 38

tinuous girder, as in the previous example, it is better practice to consider it equal to eleven-thirtieths of the entire load. Hence, W is equal to eleven-thirtieths of 30,000, or 11,000 lb.

(*a*) Applying formula **16**, we have

$$\text{Stress } HE = \frac{10}{2.5} \times 11{,}000 = 44{,}000 \text{ lb. Ans.}$$

(*b*) Applying formula **17**, we have

$$\text{Stress } AD = \frac{10}{2.5} \times 11{,}000 = 44{,}000 \text{ lb. Ans.}$$

GRAPHIC METHOD OF DETERMINING STRESSES

39. The stress in the various members of a trussed beam may be obtained by means of a graphic method that is simply an application of the principles of the resolution of forces. Although not as exact in its results as the mathematical method, it is probably more satisfactory, there being, under it, less chance of errors creeping into the calculation. This method is fully explained in the subjoined example:

A floor is to be supported by yellow-pine girders, each composed of two 4″ × 12″ beams, trussed with a wrought-iron rod, as shown in Fig. 39. The span of the girders is 24 feet, and they are spaced 8 feet from center to center. The load is light, amounting to only 40 pounds per square

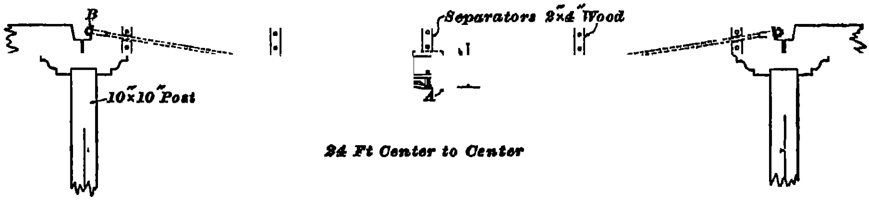


FIG 39

foot of floor surface. Required, to determine whether the two yellow-pine beams are sufficiently strong, and what should be the size of the wrought-iron camber rod; also, to design the detail construction for the parts A and B.

The floor area supported by each girder is 24 × 8 = 192 square feet; therefore, the total load on a girder is 192 × 40 = 7,680 pounds. To find the stress produced in the different members of the truss by this load, first draw to some convenient scale, as in Fig. 40, the lines ab, ac, bc, and dc corresponding, respectively, to the center lines of the girder, the wrought-iron camber rod, and the strut; thus, the line ab represents the center line of the pine beams, its length being equal to 24 feet on the assumed scale; while dc, drawn perpendicular to ab at its middle point, represents, on the same scale, 20 inches, the length of the strut.

In accordance with the principles stated in Art. **37**, the load carried by the strut may be taken as five-eighths of the total load on the girder; therefore, the force *f*, Fig. 40, acting downwards on the frame, and borne directly by the strut *dc*, is 7,680 × ⅝ = 4,800 pounds. This force is held in equilibrium by the stresses in the members of the truss, represented by the center lines *ad*, *db*, *ac*, and *cb*, one-half of it, or 2 400 pounds, being held by each of the pairs *ad*

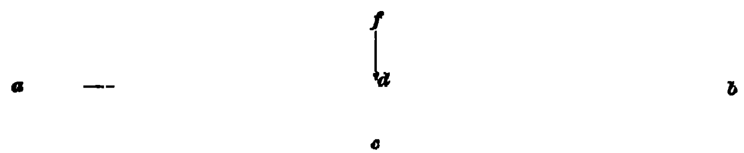


FIG 40

and *ac*, *db* and *bc*. Considering the half of the load carried by the pair *ad* and *ac*, we have a downward force of 2,400 pounds, which it is required to resolve into two components, one acting along the line *ac* and the other along *ad*. Assuming a scale of forces, one, for example, in which a line 1 inch long represents a force of 800 pounds, draw the line *dc*, Fig. 41, parallel to the center line *dc*, Fig. 40, of the strut, and make its length correspond to a force of 2,400 pounds, the part of the total load on the strut that is

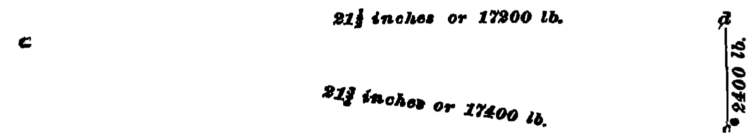


FIG. 41

borne by the members *ad* and *ac*. From the upper extremity of *dc*, Fig. 41, draw the line *da* parallel to the line *da* of Fig. 40, and from the lower extremity draw the line *ca* parallel to *ca* of Fig. 40, prolonging these two lines until they meet at the point *a*. The lines *da* and *ca* of Fig. 41 represent, on the scale of forces to which the line *dc* was drawn, the stresses in the corresponding members of the girder. With the assumed scale of 1 inch = 800 pounds,

the line dc must be 2,400 ÷ 800 = 3 inches long; by measurement, the lines da and ca are found to be $21\frac{1}{2}$ and $21\frac{3}{4}$ inches long, respectively; therefore, the stress represented by the line da is $21\frac{1}{2} \times 800 = 17,200$ pounds, and that represented by ca is $21\frac{3}{4} \times 800 = 17,400$ pounds.

DETAIL DESIGN

40. The stress of 17,200 pounds is the total compressive stress produced in the two yellow-pine beams through the action of the downward thrust on the strut. The ultimate resistance to compression of yellow pine per square inch is usually taken at about 4,400 pounds; and as wood is not so reliable as iron, it is considered advisable to use a factor of safety of 6, as against a factor of safety of 4 for the camber rods. Since the trussed girder is secured against lateral deflection by the floor joist, and as it is secured from deflection in an upward direction at the center by the load on the floor, and by the camber rod and strut, the length of the wooden girder, which may be considered as a column under compressive stress, is only one-half the span, or 12 feet. The sectional dimension of the girder is so great in comparison with its length, that it is not necessary to apply the column formula, and its strength may be considered as its resistance to direct compression. Hence, 4,400 ÷ 6 = 733 pounds, which is the allowable compressive strength of the girder per square inch of section. Then, 17,200 pounds (the compression) ÷ 733 pounds (the allowable unit stress) = 23 square inches required to take care of the compressive stress. As the girder is known to be 12 inches in depth, it is readily seen that this compressive stress will require a section of the timber girder equal to 2 inches by 12 inches.

There is, in addition to this, a transverse stress on one-half of the girder produced by the uniformly distributed load. To find the amount of this bending stress, consider the left-hand half of the girder as a simple beam sustaining a uniformly distributed load equal to one-half of the total load on the girder, that is, a load of 7,680 ÷ 2 = 3,840 pounds.

Applying the formula for uniformly loaded beams, the bending moment is $M = \frac{3{,}840 \times 12 \times 12}{8} = 69{,}120$ inch-pounds. The section modulus is obtained by the formula $S = \frac{M}{s_a}$, where S equals the section modulus, M the bending moment in inch-pounds, and s_a the allowable unit fiber stress of the material, which is equal, in this case, to 7,300 (the modulus of rupture of yellow pine) ÷ 6 (the factor of safety) = 1,216 pounds. Substituting the values in the above formula, $S = \frac{69{,}120}{1{,}216} = 56.8$, the section modulus required to resist the transverse stress. The bending moment might also be obtained by considering the beam as continuous, having three supports.

41. Since the section modulus of a rectangular beam may be obtained by the formula $S = \frac{b\,d^2}{6}$, b being the width of the beam in inches, and d the depth, and as S is already known to be 56.8 and the depth of the beam to be 12 inches, the width of the beam required to resist the transverse stress may be obtained by transposing the formula to $b = \frac{S \times 6}{d^2}$; the values substituted give $b = \frac{56.8 \times 6}{12 \times 12}$ = 2.4 inches, which is the width of the required beam. Then, adding the size of the timber required to resist compression and the size of timber required to resist the transverse stress, we have a timber 2 inches wide by 12 inches deep, added to a timber 2.4 or, say, $2\frac{1}{2}$ inches by 12 inches, which equals a piece $4\frac{1}{2}$ inches by 12 inches. In the girder, there are two $4'' \times 12''$ timbers, and, as only a single $4\frac{1}{2}'' \times 12''$ timber is required, it is evident that the girder is nearly twice as strong as is necessary. However, it must be borne in mind that the theoretical dimensions of members do not always agree with those required in practical rules; for instance, in the above case it would not be good practice to make the combined sectional area of the girder equivalent to that of a $4\frac{1}{2}'' \times 12''$ timber, as obtained by the calculation,

because this would make each timber a little larger than $2'' \times 12''$, and no timber or girder, especially where rafter or flooring is spiked to it, should be less than 3 inches wide.

42. The ultimate tensile strength of wrought iron is usually taken at 50,000 pounds per square inch; hence, if we use a factor of safety of 4, the safe working fiber stress in the rod must be $50,000 \div 4 = 12,500$ pounds per square inch.

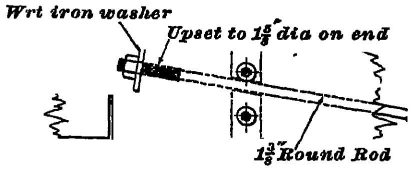


FIG. 42

According to the results given by the diagram, the total stress in the rod is 17,400 pounds; therefore, the rod must have a net section of $17,400 \div 12,500 = 1.39$ square inches. The area of a $1\frac{3}{8}$-inch round rod is 1.48 square inches, and as this is the nearest standard size having the required sectional area, it will be used. As the area at the bottom of the thread of a $1\frac{3}{8}$-inch bolt is, however, only 1.06 square inches, it will be necessary to upset or enlarge the ends of the rod to a diameter of $1\frac{5}{8}$ inches, in order to get the requisite strength in the threaded portion. The washer at B, Fig. 39, must be large enough to distribute the pressure due to the pull of the rod over a sufficient area of the end of the beams to prevent danger of crushing the wood. The allowable compressive strength of yellow pine, parallel to the grain, may be taken as 800 pounds per square inch; this requires a washer whose area is $17,400 \div 800 = 22$ square inches, nearly. Using a washer 6 inches wide, extending across the ends of the two beams, we get a bearing area of $2 \times 4 \times 6 = 48$ square inches. In order to

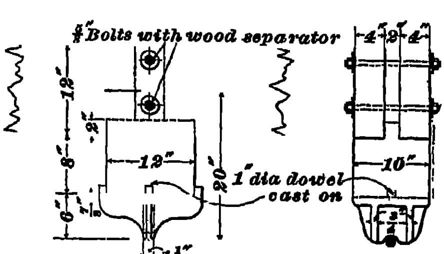


FIG. 43

resist the bending stress due to the pull of the rod, the washer should be from $\frac{3}{4}$ inch to 1 inch in thickness.

Figs. 39, 42, and 43, which are so clearly drawn as to require no further explanation, show excellent details for the different parts of the trussed stringer under consideration.

EXAMPLES FOR PRACTICE

1. It is found necessary to truss the yellow-pine purlins supporting a roof, with a wrought-iron camber rod on each side of the purlin; the length of the purlin is 20 feet, the depth of the truss from the center of the rods to the center of the purlin is 14 inches, and the load on the central strut is 3,200 pounds. What should be the diameter of the camber rods if the ends of the rods are upset, and a safety factor of 4 is desired? Ans. $\frac{7}{8}$ in diam.

2. A girder of 24-foot span is trussed at the center by a camber rod and strut; the depth of the truss from the center of the girder to the center of the rod is 2 feet; if the beam is loaded with a uniformly distributed load of 2,000 pounds per lineal foot: (*a*) what is the stress on the rod? (*b*) what is the compressive stress on the beam? (*c*) what is the stress on the central strut?

$$\text{Ans.}\begin{cases}(a)\ 91{,}200\text{ lb.}\\(b)\ 90{,}000\text{ lb.}\\(c)\ 30{,}000\text{ lb.}\end{cases}$$

TRUSSED GIRDERS

43. In some buildings it is necessary to use **trussed girders** to support the weight over a large space, such as a stock exchange, ballroom, etc. They are also sometimes required when it is desired to place a runway or bridge between two adjacent buildings.

In the first case, the girder may support only the weight of the floor above and its live load or it may carry the weight from several stories above, in which case the girder will probably have concentrated loads due to columns, in addition to the uniform load produced by the dead and live loads.

The trussed girders supporting a bridge between two buildings should be proportioned to carry the entire dead load of the bridge and also a moving or rolling load of considerable weight

A trussed girder may be constructed of wood with iron or steel tension members, or it may be entirely of steel.

THE HOWE TRUSS

44. A form of truss known as the **Howe** is shown in Fig. 44. The stresses in the different members may be determined analytically or graphically. As the latter method is explained in *Graphical Analysis of Stresses*, Parts 1 and 2, only the analytic method will be considered here.

Suppose the truss shown in Fig. 44 to be divided along the dotted line $d\,e$, thus cutting the members a, b, and c; it is evident that if the section at the left were considered as a separate piece, it would be held in equilibrium by applying to each bar that is cut, a force of the same direction and magnitude as the stress existing in the bar before the

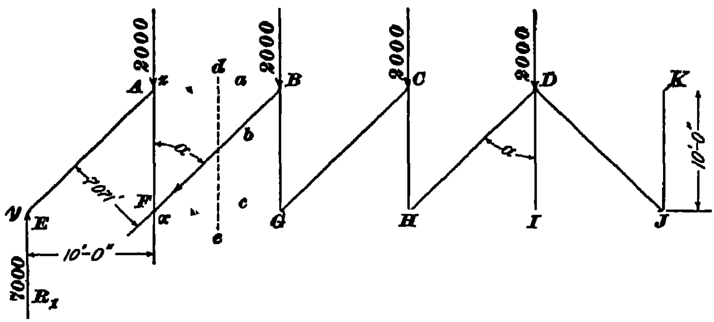


FIG. 44

section was made. Therefore, the forces R_1, A, a, b, and c acting on this portion of the truss are in equilibrium, and the sum of their moments is zero. To find the stress in a, moments will be taken about the point x. As the other unknown forces b and c and the vertical force A pass through this point, they have no effect on the equilibrium of the forces taken about this point, and consequently only the forces R_1 and a need be considered. The moment of R_1 is $7{,}000 \times 10 = 70{,}000$ foot-pounds, and the opposing force a is equal to this amount divided by its lever arm, or $70{,}000 \div 10 = 7{,}000$ pounds. Regarding the moment of the reaction as positive, the moment of the force a will be considered as negative, because it tends to revolve around the point x

in the opposite direction from the reaction R_1, or in the direction of the arrow. Since the arrow points toward the joint z, the stress in a is compressive.

The stress in b is found by taking the center of moments at the point y. As R_1 and c pass through this point, they are not taken into consideration, but only A, a, and b. The moment of a is $7,000 \times 10 = 70,000$ foot-pounds, while that of A is $2,000 \times 10 = 20,000$ foot-pounds. $70,000 - 20,000 = 50,000$ foot-pounds; then the value of b is $50,000 \div 7.071 = 7,071$ pounds.

Taking z as the center of moments, the forces to be considered are R_1, b, and c. The positive moments of R_1 and b are

$$
\begin{array}{lrl}
 & 7,000 \times 10 = & 70000 \text{ foot-pounds} \\
\text{and} & 7,071 \times 7.071 = & 50000 \text{ foot-pounds} \\
\hline
 & \text{Total,} & 120000 \text{ foot-pounds}
\end{array}
$$

Then the force c is equal to $120,000 \div 10 = 12,000$ pounds. The stress in b is compressive, as designated by the arrow, and, in this case, produces a positive moment, while that in c is tensile and produces a negative moment. In the same manner, a section may be taken at any point and thus the stress in any member may be obtained. This method is called the *method of sections.*

45. As stated above, the stress in the three members a, b, and c must be in equilibrium with the external forces on the left of the section; and therefore the algebraic sum of the vertical components of these forces equals zero the same as the horizontal components a and c. Then, as $b \cos \alpha$ is the vertical component of the oblique force, $R_1 - A - b \cos \alpha = 0$; but $R_1 - A$ is equal to the vertical shear in the panel AB. Hence, shear $- b \cos \alpha = 0$, and transposing, $- b \cos \alpha = -$ shear, or $b \cos \alpha =$ shear. Then $b = \frac{\text{shear}}{\cos \alpha} = \text{shear} \times \sec \alpha$, and the following rule may be deduced:

Rule.—*The stress in any oblique web member is equal to the vertical shear in the panel multiplied by the secant of the angle that the member makes with the vertical.*

It is assumed that the angle a is equal to 45°. After obtaining the secant of a, as explained in Art. **46**, and applying this rule to determine the stresses in the remaining web members, it is found that

Stress in CG = (7,000 − 2,000 − 2,000) × sec a = 3,000 × 1.4142 = 4,242.6 pounds.

Stress in DH = (7,000 − 2,000 − 2,000 − 2,000) × sec a = 1,000 × 1.4142 = 1,414.2 pounds.

Stress in AE = 7,000 × sec a = 7,000 × 1.4142 = 9,899.4 pounds.

46. The same results may be obtained by the following method: The load at the center of the truss or the point D is equal to 2,000 pounds and half of this load goes to each of the compression members DH and DJ, since DI does not take any of the load, but is simply a rod to prevent the chord HJ from sagging, or to assist in supporting any load placed on the lower chord. Therefore, the stress in each of these members is equal to one-half of the load at D multiplied by the secant of the angle HDI. The secant of the angle may be found from a table of secants, if the angle is known. If not, it may be calculated by considering HDI as a right triangle and ascertaining the ratio between the hypotenuse DH and the side DI adjacent to the angle, which ratio is equal to the secant of the angle a. Therefore, $\sec a = \frac{DH}{DI}$; but $DH = \sqrt{DI^2 + HI^2}$; therefore, $\sec a = \frac{\sqrt{DI^2+HI^2}}{DI}$. This formula, so as to apply to any of the other panels, may be stated as follows:

$$\sec a = \frac{\sqrt{(\text{height of panel})^2 + (\text{width of panel})^2}}{\text{height of panel}}$$

As the width and height in this instance are equal, the angle a is 45°, the secant of which is 1.4142. The horizontal thrusts of the members HD and DJ are resisted by the tension member HJ. The vertical thrusts must also be resisted at the points H and J by the tension members CH

and KJ, which transmit the downward forces to the points C and K. At the point C, there is applied a load of 2,000 pounds in addition to the force of 1,000 pounds transmitted by the tension bar CH from the member HD. Consequently, the stress in CG is $1{,}000 + 2{,}000 = 3{,}000$ pounds multiplied by the secant of the angle HDI, or α, since the compression web members are all slanted at the same angle. The stresses in the other oblique web members may be found in a similar manner and be expressed in a tabulation as follows:

$$
\begin{aligned}
DH &= 2{,}000 \times \tfrac{1}{2} \times \sec\alpha = 1{,}000 \times 1.4142 = 1{,}414.2 \text{ pounds}\\
CG &= 2{,}000 \times 1\tfrac{1}{2} \times \sec\alpha = 3{,}000 \times 1.4142 = 4{,}242.6 \text{ pounds}\\
BF &= 2{,}000 \times 2\tfrac{1}{2} \times \sec\alpha = 5{,}000 \times 1.4142 = 7{,}071.0 \text{ pounds}\\
AE &= 2{,}000 \times 3\tfrac{1}{2} \times \sec\alpha = 7{,}000 \times 1.4142 = 9{,}899.4 \text{ pounds}
\end{aligned}
$$

Theoretically, there is no stress in the vertical web DI, but practically there would be a slight tensile stress. In the web CH, there is a tensile stress equal to the vertical component of the stress in HD, or 1,000 pounds, which stress may be found by means of the formula $CH = \frac{DH}{\sec\alpha} = \frac{1{,}414.2}{1.4142} = 1{,}000$. The stress in BG is equal to the vertical component of the stress in CG and is found, by means of the preceding formula, to be 3,000 pounds. The other stresses may be found in a similar manner and are as given in the following tabulation:

$$
\begin{aligned}
DI &= 0\\
CH &= 2{,}000 \times \tfrac{1}{2} = 1{,}000 \text{ pounds}\\
BG &= 2{,}000 \times 1\tfrac{1}{2} = 3{,}000 \text{ pounds}\\
AF &= 2{,}000 \times 2\tfrac{1}{2} = 5{,}000 \text{ pounds}
\end{aligned}
$$

47. The stresses in the horizontal members, or chords, may be obtained when the stresses in the web members are known. Consider the compression member AE separately, as shown in Fig. 45. Then the forces acting on it are the reaction R_1 of 7,000 pounds and a vertical load at A of 2,000 pounds. The stress in AF was found to be 5,000 pounds, so that the total downward force acting on the end of the strut

AE is 2,000 + 5,000 = 7,000 pounds, which just counteracts the reaction, or upward force. The stress in AF equals the vertical component of that in AE less the load at A, while the stress in EF is the horizontal component of AE. It may also be observed that in order to keep the end of the strut from being pushed upwards by the reaction, the force EF must act away from the joint E, or in the direction of the arrow; this indicates a tensile stress in the chord. In order that the strut may be in equilibrium, it is necessary to have a horizontal force at the point A equal and opposite in direction to the force EF. Therefore, the stress in AB is equal to that in EF, but it is a compressive stress, as indicated by the arrow. This stress is equal to the horizontal component of the stress in AE, which may be expressed as (AF + load at A) tan α; similarly, the stress in BC or FG, Fig. 44, is equal to the stress in EF plus the horizontal component of the stress in BF, or EF + (BG + load at B) tan α. The tangent of α is obtained from the formula $\tan \alpha = \frac{\text{width of panel}}{\text{height of panel}}$.

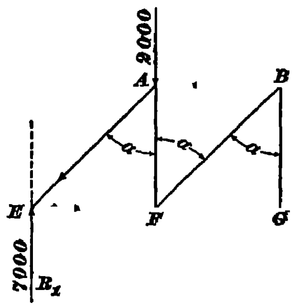


FIG. 45

Substituting these values for the problem under consideration, the value of the tangent is $\frac{10}{10} = 1$. The following notation of the chord stresses may now be made:

$AB = EF = 7{,}000 \times \tan \alpha = 7{,}000 \times 1 = 7{,}000$ pounds.

$BC = FG = (7{,}000 + 5{,}000) \times \tan \alpha = 12{,}000 \times 1 = 12{,}000$ pounds.

$CD = GH = (7{,}000 + 5{,}000 + 3{,}000) \times \tan \alpha = 15{,}000 \times 1 = 15{,}000$ pounds.

$HI = IJ = (7{,}000 + 5{,}000 + 3{,}000 + 1{,}000) \times \tan \alpha = 16{,}000 \times 1 = 16{,}000$ pounds.

It may be noticed that the stress in the outside panel of the lower chord, or EF, is equal to the shear in that panel multiplied by the tangent of the angle EAF, that is, equal to the reaction $\times$ tan α, while in the next panel, or FG, the

stress is equal to the sum of the shears in EF and FG multiplied by tan α. In GH, the stress is equal to shear in EF + shear in FG + shear in GH multiplied by tan α. Hence, the following rule may be given for the stress in a chord member of a Howe truss:

Rule.—*To find the stress in any chord member, multiply the sum of the shears in all panels from the reaction up to and including the one in question, by the tangent of the angle that the inclined web member makes with the vertical.*

The portions of the upper chord are all in compression, while those of the lower chord are all in tension. If it is desired to check the results obtained by either of the preceding methods, the stress diagram may be drawn for the truss and the stresses measured on it.

MAXIMUM STRESSES PRODUCED BY LIVE LOAD

48. The live and dead loads are generally figured separately in structures where it is necessary to provide for a moving load, such as a bridge. The greatest chord stress is produced when the live load covers the entire truss, while the greatest stress is created in the web member when the live load covers the portion of the truss from the web member in question to the remote abutment and the portion to the other abutment is unloaded. This fact may be demonstrated by considering the following example: The live load per panel in Fig. 44 is assumed to be 5,000 pounds. When the live load covers the entire truss, the reactions are 17,500 pounds and the chord and web stresses are as follows:

WEB STRESSES

$AF = 17{,}500 - 5{,}000 = 12{,}500$ pounds

$BG = 17{,}500 - 5{,}000 - 5{,}000 = 7{,}500$ pounds

$CH = 17{,}500 - 5{,}000 - 5{,}000 - 5{,}000 = 2{,}500$ pounds

$DI = 0$

$AE = 17{,}500 \times \sec\alpha = 17{,}500 \times 1.4142 = 24{,}748.5$ pounds

$BF = 12{,}500 \times \sec\alpha = 12{,}500 \times 1.4142 = 17{,}677.5$ pounds

$CG = 7{,}500 \times \sec\alpha = 7{,}500 \times 1.4142 = 10{,}606.5$ pounds

$DH = 2{,}500 \times \sec\alpha = 2{,}500 \times 1.4142 = 3{,}535.5$ pounds

CHORD STRESSES

$AB = EF = 17{,}500 \times \tan\alpha = 17{,}500 \times 1 = 17{,}500$ pounds

$BC = FG = (17{,}500 + 12{,}500) \times \tan\alpha = 30{,}000 \times 1 = 30{,}000$ pounds

$CD = GH = (17{,}500 + 12{,}500 + 7{,}500) \times \tan\alpha = 37{,}500 \times 1 = 37{,}500$ pounds

$HI = (17{,}500 + 12{,}500 + 7{,}500 + 2{,}500) \times \tan\alpha = 40{,}000 \times 1 = 40{,}000$ pounds

49. Assuming the truss to be loaded at every panel point except A, as shown in Fig. 46, the left-hand reaction R_1 is found by taking moments about R_2. Considering the width of the panel as the unit of measurement, the moments of the loads are $(5{,}000 \times 1) + (5{,}000 \times 2)$, etc., and as the load is the same at each panel point and the lever arms increase

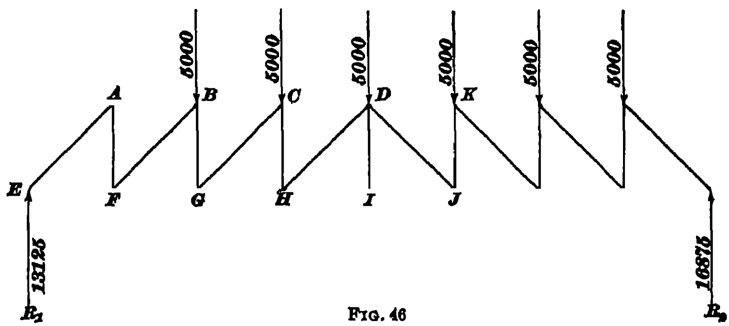


FIG. 46

in arithmetical progression, the sum of the lever arms, or the terms of the progression, is equal to the sum of the first and last terms multiplied by half the number of terms; therefore, the sum of the moments about R_2 is $5{,}000 \times \frac{6(1+6)}{2} = 105{,}000$, and the reaction at R_1 is equal to this amount divided by the number of panels, or $105{,}000 \div 8 = 13{,}125$ pounds. The stresses now existing are as follows:

WEB STRESSES

$AF = 13{,}125$ pounds

$BG = 13{,}125 - 5{,}000 = 8{,}125$ pounds

$CH = 13{,}125 - 5{,}000 - 5{,}000 = 3{,}125$ pounds

$DI = 0$

$AE = 13{,}125 \times \sec\alpha = 13{,}125 \times 1.4142 = 18{,}561$ pounds
$BF = 13{,}125 \times \sec\alpha = 13{,}125 \times 1.4142 = 18{,}561$ pounds
$CG = 8{,}125 \times \sec\alpha = 8{,}125 \times 1.4142 = 11{,}490$ pounds
$DH = 3{,}125 \times \sec\alpha = 3{,}125 \times 1.4142 = 4{,}419$ pounds

CHORD STRESSES

$AB = EF = 13{,}125 \times \tan\alpha = 13{,}125 \times 1 = 13{,}125$ pounds
$BC = FG = (13{,}125 + 13{,}125) \times \tan\alpha = 26{,}250 \times 1 = 26{,}250$ pounds
$CD = GH = (13{,}125 + 13{,}125 + 8{,}125) \times \tan\alpha = 34{,}375 \times 1 = 34{,}375$ pounds
$HI = (13{,}125 + 13{,}125 + 8{,}125 + 3{,}125) \times \tan\alpha = 37{,}500 \times 1 = 37{,}500$ pounds

50. Consider the load as covering all points except A and B. The reaction at R_1 is equal to $\left[\frac{5(1+5)}{2} \times 5{,}000\right] \div 8 = 9{,}375$ pounds, and the stresses are as shown in the following tabulation:

WEB STRESSES

$AF = 9{,}375$ pounds
$BG = 9{,}375$ pounds
$CH = 9{,}375 - 5{,}000 = 4{,}375$ pounds
$DI = 0$
$AE = 9{,}375 \times \sec\alpha = 9{,}375 \times 1.4142 = 13{,}258$ pounds
$BF = 9{,}375 \times \sec\alpha = 9{,}375 \times 1.4142 = 13{,}258$ pounds
$CG = 9{,}375 \times \sec\alpha = 9{,}375 \times 1.4142 = 13{,}258$ pounds
$DH = 4{,}375 \times \sec\alpha = 4{,}375 \times 1.4142 = 6{,}187$ pounds

CHORD STRESSES

$AB = EF = 9{,}375 \times \tan\alpha = 9{,}375 \times 1 = 9{,}375$ pounds
$BC = FG = (9{,}375 + 9{,}375) \times \tan\alpha = 18{,}750 \times 1 = 18{,}750$ pounds
$CD = GH = (9{,}375 + 9{,}375 + 9{,}375) \times \tan\alpha = 28{,}125 \times 1 = 28{,}125$ pounds
$HI = (9{,}375 + 9{,}375 + 9{,}375 + 4{,}375) \times \tan\alpha = 32{,}500 \times 1 = 32{,}500$ pounds

The results of the preceding calculations, arranged in a convenient form, are given below:

	Fully Loaded	All Joints Loaded Except A	All Joints Loaded Except A and B
$AE =$	24,748.5	18,561	13,258
$BF =$	17,677.5	18,561	13,258
$CG =$	10,606.5	11,490	13,258
$DH =$	3,535.5	4,419	6,187
$AB = EF =$	17,500.0	13,125	9,375
$BC = FG =$	30,000.0	26,250	18,750
$CD = GH =$	37,500.0	34,375	28,125
$HI =$	40,000.0	37,500	32,500

It may be observed from these results that the greatest stress is produced in the chord members when the entire truss is loaded; also, that the greatest stress in AE is when the truss is fully loaded. In BF the greatest stress is created when the truss is loaded to the right of the panel, including this member, that is, all joints are loaded except A, while CG has the greatest stress when all joints except A and B are loaded. If loads are considered at every point except A, B, and C, it will be found that the greatest stress occurs in DH under that condition.

51. Counterbracing is required in the panels where the shear created by the dead load is exceeded by the shear of opposite kind produced by the live load. This may be readily ascertained by drawing the shear diagrams; for instance, Fig. 47 (a) represents the frame diagram of the truss having only the dead load on it. The live load is 5,000 pounds at each panel point and will be considered first as being applied at only the first panel point to the left, or AB. Then the reaction for the live load is $\frac{5,000 \times 70}{80}$ $= 4,375$ pounds at R_1 and $5,000 - 4,375 = 625$ pounds at R_2. In the shear diagram (b), the shaded portion represents the shear, to scale, created by the dead load along the truss; the portion above the line will be called positive and

that below negative. The heavy line designates the shear produced by the live load, which for the first panel is equal to the left reaction, while for the remainder of the truss it is equal to the right reaction, or 625 pounds. It is observed from this diagram that the negative shear from the live load does not exceed the positive shear produced by the dead load and, consequently, no counterbraces are required.

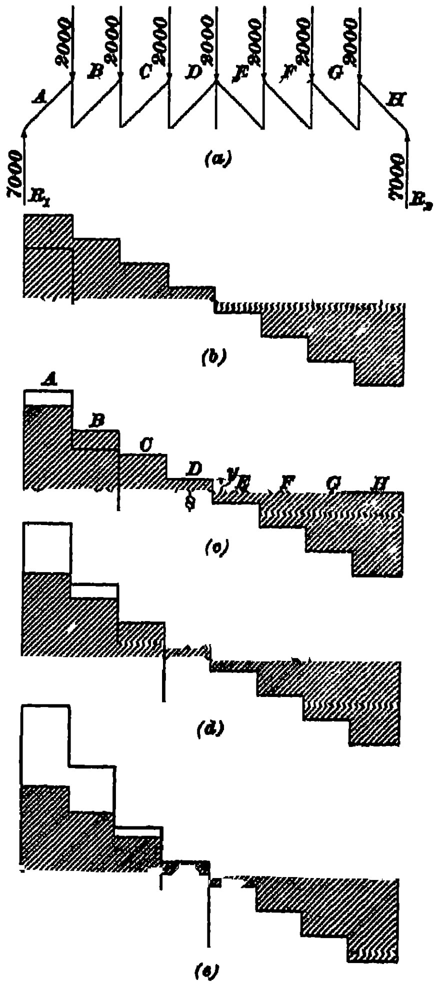


FIG. 47

The shear shown by the heavy line in the diagram (*c*) is produced by placing live loads on two panel points of the truss, at *A B* and *B C* in (*a*). As will be observed, the negative shear in the panel *D*, created by the live load and represented by the heavy line, exceeds the positive shear produced by the dead load, that is, x is greater than y; therefore, it will be necessary to counterbrace this panel. Also, the diagrams (*d*) and (*e*) show that counterbracing is required in the panel *D*, but not in any other, though in practice the panel *C* would be braced as an additional precaution. Then, theoretically, the only panels to be braced are those at *D* and *E*

THE PRATT TRUSS

52. The **Pratt truss** has its vertical members in compression and the inclined members in tension, while the top chord is in compression and the lower one in tension. This fact may be readily determined by considering a portion of the truss as being separated from the remainder by a line, as at *de*, Fig. 48, cutting three members of the truss. The kind of stress in each of these members and its amount may then be found by taking moments about different points, as x, y, and z. To ascertain the stress in the member c, moments will be taken about the point x as a center. The positive moment is $7{,}000 \times 24 = 168{,}000$ foot-pounds, while the negative moment is $2{,}000 \times 12 = 24{,}000$ foot-pounds.

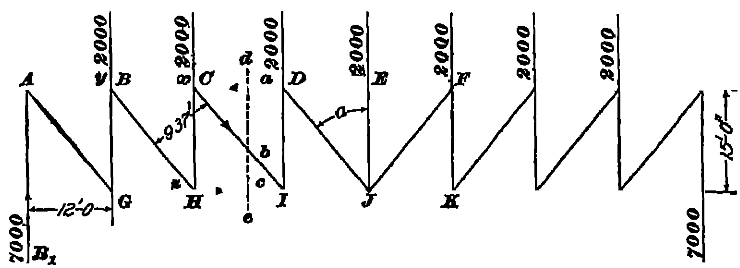


FIG. 48

The algebraic sum is $168{,}000 - 24{,}000 = 144{,}000$ foot-pounds, which is the moment of the stress in c. The lever arm of this force is 15 feet; hence, the stress is equal to $144{,}000 \div 15 = 9{,}600$ pounds. The surplus of the moment $R_1 \times 24$ is therefore equal to 144,000 foot-pounds and is positive. To create equilibrium, the moment of c should equal this amount, but be of opposite sign, therefore negative, and must act in the direction shown by the arrow, or away from the joint, which indicates that it is a tensile stress.

The stress in b is determined by considering the moments about the point y as a center. The positive moments are:

$$\begin{array}{lrl} 7{,}000 \times 12 = & 84000 & \text{foot-pounds} \\ 2{,}000 \times 12 = & \underline{24000} & \text{foot-pounds} \\ \text{Total,} & 108000 & \text{foot-pounds} \end{array}$$

The negative moment of *c* is 9,600 × 15 = − 144,000 foot-pounds; hence, the excess of this moment over the positive one is 108,000 − 144,000 = − 36,000 foot-pounds. To produce equilibrium, the moment of *b* should be of opposite sign, that is, positive, and as its lever arm is 9.37 feet, its stress is equal to 36,000 ÷ 9.37 = 3,842 pounds. The arrow points away from the joint, and the stress is therefore tensile.

The stress in *a* may be obtained by considering *z* as the center of moments. The positive moments are:

7,000 × 24	=	168000	foot-pounds
3,842 × 9.37	=	36000	foot-pounds
Total,		204000	foot-pounds

The negative moment is 2,000 × 12 = 24,000 foot-pounds. As the positive moments exceed the negative by 204,000 − 24,000 = 180,000 foot-pounds, the moment of *a* must equal this amount in order to establish equilibrium and it must be of opposite sign, or negative. As its lever arm is 15 feet the stress is equal to 180,000 ÷ 15 = 12,000 pounds. In order to produce a negative moment, this force must act in the direction of the arrow shown at *a*, or toward the joint, and consequently the stress is compressive. The stress in any member may be obtained by this method.

53. The method of determining the stresses by a notation, as explained in connection with the Howe truss, may also be applied to this form of truss. The stress in EJ is compressive and equal to the amount of the load at E, or 2,000 pounds. One-half of this stress must be taken care of by each of the rods, DJ and FJ; thus, 1,000 pounds is transmitted to the point D by DJ, so that the compression in DI is 2,000 + 1,000 = 3,000 pounds. This amount is carried up the oblique member IC, and consequently the stress in CH is 2,000 + 3,000 = 5,000 pounds. Hence, the stress in the vertical web members may be tabulated as follows:

Stress in EJ = 2,000 pounds
Stress in DI = 3,000 pounds
Stress in CH = 5,000 pounds
Stress in BG = 7,000 pounds

The stress in DJ is equal to 1,000 multiplied by the secant of the angle DJE, or α; in CI it is equal to 3,000 multiplied by sec α, etc. Hence, the following notation may be made:

Stress in $DJ = 1{,}000 \times \sec \alpha = 1{,}000 \times 1.28075 = 1{,}281$ pounds.

Stress in $CI = 3{,}000 \times \sec \alpha = 3{,}000 \times 1.28075 = 3{,}842$ pounds.

Stress in $BH = 5{,}000 \times \sec \alpha = 5{,}000 \times 1.28075 = 6{,}404$ pounds.

Stress in $AG = 7{,}000 \times \sec \alpha = 7{,}000 \times 1.28075 = 8{,}965$ pounds.

The stress in the chord members may be obtained as explained in connection with the Howe truss. For instance, the stress in AB is equal to the horizontal component of that in AG and is a compressive stress, while the stress in GH is equal to the same amount, but is tensile. In this problem, the horizontal component of AG is equal to 7,000 $\times \tan \alpha = 7{,}000 \times .800196 = 5{,}601$ pounds. The stress in the members BC and HI is equal to $(7{,}000 + 5{,}000) \times \tan \alpha = 12{,}000 \times .800196 = 9{,}602$ pounds. Then the chord stresses may be written as follows:

$AB = GH = 7{,}000 \times \tan \alpha = 7{,}000 \times .800196 = 5{,}601$ pounds.

$BC = HI = (7{,}000 + 5{,}000) \times \tan \alpha = 12{,}000 \times .800196 = 9{,}602$ pounds.

$CD = IJ = (7{,}000 + 5{,}000 + 3{,}000) \times \tan \alpha = 15{,}000 \times .800196 = 12{,}003$ pounds.

$DE = (7{,}000 + 5{,}000 + 3{,}000 + 1{,}000) \times \tan \alpha = 16{,}000 \times .800196 = 12{,}803$ pounds.

THE WARREN TRUSS

54. The **Warren truss** is usually built of iron or steel, and while it is not much used in this country, in England it is frequently employed for comparatively short spans. The stresses in the members may be obtained analytically, as explained in connection with the Howe and Pratt trusses. The chord stresses may also be determined by the *method of chord increments*, which is explained as follows: Referring

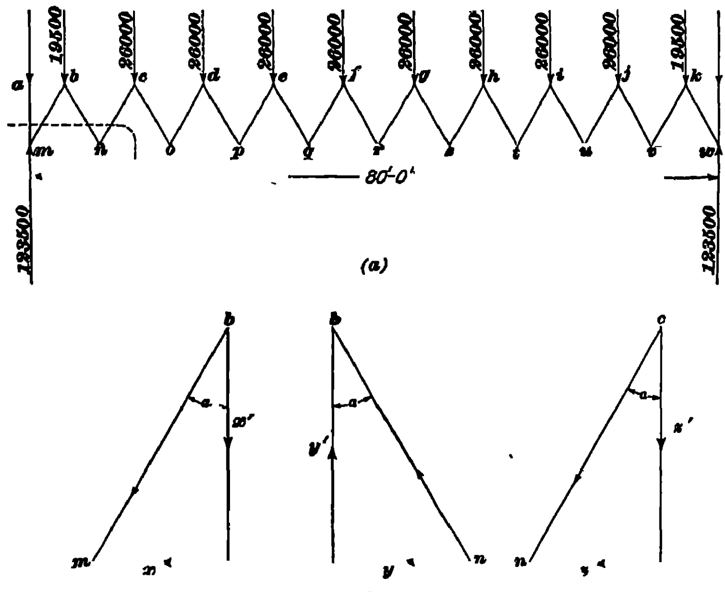


FIG. 49

to Fig. 49 (*a*), assume that it is desired to find the stress in the chord member *n o*. Draw a curved section, as shown by the dotted line, cutting this member and all the web members to the left. The vertical shears in the web members may be designated by s_1, s_2, s_3, etc. Then in order to have the truss in equilibrium, the sum of the horizontal components must equal zero. The action of the stresses in the web members *b m*, *b n*, and *c n* may be observed from Fig. 49 (*b*),

which shows the horizontal components of these stresses. The members $b\,m$ and $c\,n$ are in compression; hence, their vertical components x' and z' act downwards and their horizontal components x and z toward the left, as shown by the arrows. The member $b\,n$ is in tension and its vertical component y' is an upward force, while its horizontal component y acts toward the left. Therefore, the sum of the horizontal components, since all act in the same direction, is $x + y + z$, and the stress in $n\,o$ must be equal to this sum and act in the opposite direction. Then, as the horizontal component of any web member is equal to the vertical shear in that member multiplied by the tangent of the angle which that member makes with the vertical, the following rule may be stated:

Rule.—*The stress in any chord member is equal to the sum of the products of the vertical shear in each web member between the chord in question and the nearer reaction, multiplied by the tangent of the angle that the web member makes with the vertical.*

Hence, the stress in $n\,o$ is equal to $s_1 \tan \alpha_1 + s_2 \tan \alpha_2 + s_3 \tan \alpha_3$, but as α_1, α_2, and α_3 are all equal, the equation becomes, stress in $n\,o = \tan \alpha(s_1 + s_2 + s_3)$.

55. The stress in the web member is equal to the vertical component of its stress multiplied by the secant of the angle that the web makes with the vertical, or, referring to Fig. 49 (b) $x' \times \sec \alpha$; but as the vertical component of the oblique force represents the shear in that member, its stress is equal to the shear multiplied by the secant of its angle, or $s \times \sec \alpha$. Hence, the following rule may be given:

Rule.—*The stress in any web member is equal to the shear in that member multiplied by the secant of its angle with the vertical.*

Assume a Warren truss to be of the dimensions given in Fig. 49; the sum of the live and dead loads is taken at 26,000 pounds per panel. It will be noticed that at the points b and k there is a load of only 19,500 pounds. This may be explained by supposing that the extent of a panel is that part of the truss included between two adjoining apexes and that between each of these there is a distributed load of 26,000

pounds. Between the points a and b there is only one-half panel with a load of 13,000 pounds, one-half of which is supported directly at the point m through the member $a\,m$. The other half, or 6,500 pounds, is supported at the point b, which point also supports one-half of the total panel load $b\,c$, or 13,000 pounds. Adding these loads, 13,000 + 6,500, gives a total load of 19,500 pounds, as indicated in the diagram. The small load at a is disregarded and, in fact, the members $a\,m$ and $a\,b$ are not considered in determining the stresses in the members of the truss. Applying the rule given in this article, the stress in the member $b\,m$ is equal to the shear in that member, which is 123,500 pounds, since no load is applied until the point b is reached, multiplied by the secant of α; as this angle is 30°, its secant is 1.1547. Hence,

Stress in $b\,m$ is $123{,}500 \times 1.1547 = 142{,}605$ pounds.

Shear in $b\,n$ is $123{,}500 - 19{,}500 = 104{,}000$ pounds.

Stress in $b\,n$ is $104{,}000 \times 1.1547 = 120{,}089$ pounds.

The shear in $c\,n$ is the same as in $b\,n$ and consequently the stress is the same amount, but a different kind, the stress in $c\,n$ being compressive, while that in $b\,n$ is tensile. The stresses in the remaining web members may be obtained similarly. The chord stress, as previously stated, is equal to the sum of the shears in all the web members to the left, multiplied by the tangent of α. Then,

Stress in $m\,n = s_1 \times \tan\,\alpha = 123{,}500 \times .57735 = 71{,}303$ pounds.

Stress in $n\,o = (s_1 + s_2 + s_3) \times \tan\,\alpha = (123{,}500 + 104{,}000 + 104{,}000) \times .57735 = 191{,}391$ pounds.

Stress in $o\,p = (s_1 + s_2 + s_3 + s_4 + s_5) \times \tan\,\alpha = (123{,}500 + 104{,}000 + 104{,}000 + 78{,}000 + 78{,}000) \times .57735 = 281{,}458$ pounds.

Considering the upper chord, the stress in $b\,c = (s_1 + s_2) \times \tan\,\alpha = (123{,}500 + 104{,}000) \times .57735 = 131{,}347$ pounds.

Stress in $c\,d = (s_1 + s_2 + s_3 + s_4) \times \tan\,\alpha = 123{,}500 + 104{,}000 + 104{,}000 + 78{,}000) \times .57735 = 236{,}425$ pounds.

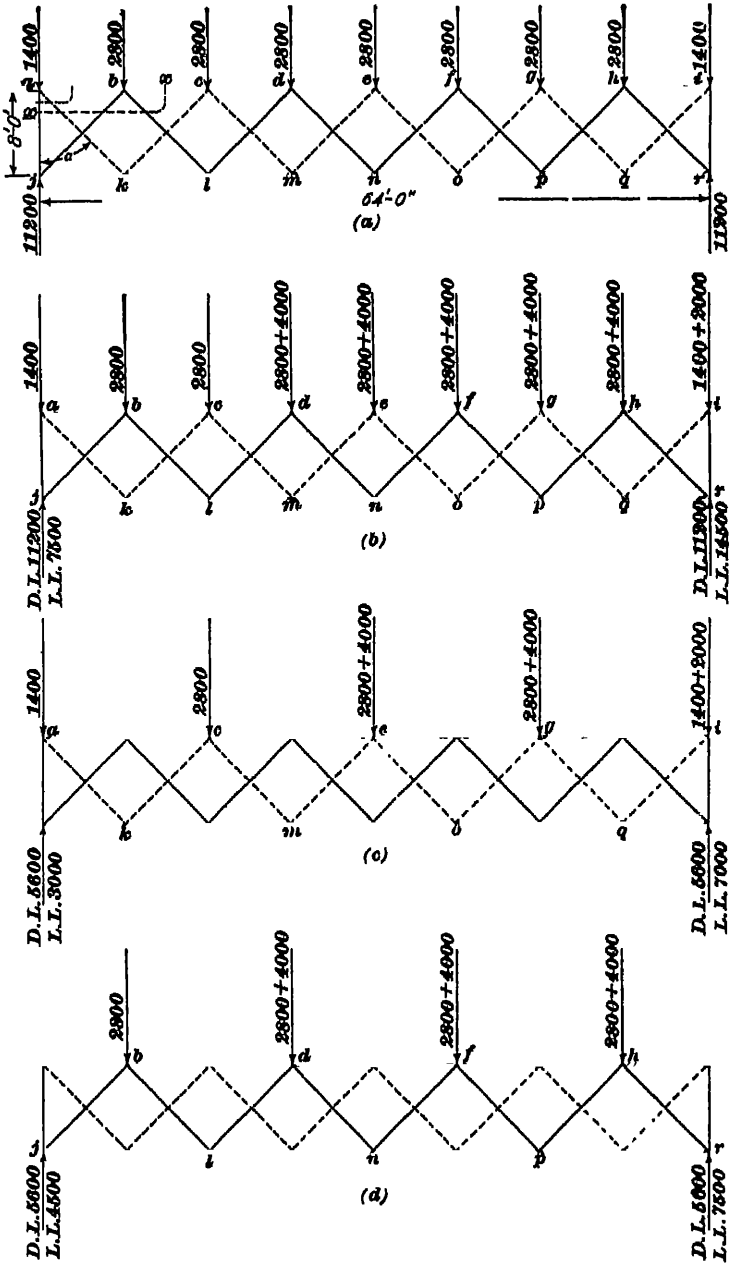
The stresses in all the chord members may be obtained similarly, and those in the lower chord are always in tension while those in the upper are in compression.

THE LATTICE TRUSS

56. The **lattice truss,** or **double Warren,** as it is sometimes called, is shown in Fig. 50. It consists of two systems of triangular bracing, one of which is represented in the figure by full lines and the other by dotted lines. The truss is equivalent to the two trusses welded into one and the loads at c, e, and g, in view (a), are considered as being carried to the abutments by the diagonals shown dotted, while the full-line diagonals transfer the loads at b, d, f, and h to the abutments. The chord stresses are found by the method of chord increments, as in the Warren truss. For instance, the stress in $a\,b$ is equal to the vertical shear in the web $a\,k$ multiplied by the tangent of the angle $j\,a\,k$, or α, which in this case is 45°, and consequently its tangent is 1. Taking the dead load at 350 pounds per foot, the panel loads are $350 \times 8 = 2{,}800$ pounds. The system of diagonals to which $a\,k$ belongs receives three panel loads; then half of the center load will be carried to the left reaction down the diagonal $e\,m$ and up $m\,c$, at which point another load is added, so that one and one-half panel loads are carried down to k and up to a. Hence, the vertical shear in $a\,k$ is $1\frac{1}{2} \times 2{,}800 = 4{,}200$ pounds, and the stress in the chord $a\,b$ is shear in $a\,b \times \tan\,\alpha = (1\frac{1}{2} \times 2{,}800) \times 1 = 4{,}200$ pounds.

The stress in the chord $b\,c$ is equal to the sum of the shears in all the web members cut by the curved section $x\,x$, multiplied by $\tan\,\alpha$. The whole load at d supported by the full-line diagonals is transferred to the left abutment by passing down $d\,l$ and up $l\,b$, where it encounters another panel load; hence, the vertical shear in $b\,j$ is equivalent to two panel loads, while that in $b\,l$ is equal to one panel load. As $a\,k$ is equal to one and one-half panel loads, the stress in the chord $b\,c$ equals $(1\frac{1}{2} + 2 + 1) \times 2{,}800 \times 1 = 12{,}600$ pounds. In the same manner, the stresses in all the chord members may be determined, those in the upper chord always being compressive and in the lower tensile.

The stress in any web member is equal to the vertical shear in that member multiplied by the secant of its angle with



(a)
64'-0"
8'-0"
11200
11200
1400
2800
(b)
D.L. 11200
L.L. 7500
D.L. 11200
L.L. 14500
1400+2000
2800+4000
(c)
D.L. 5600
L.L. 3000
D.L. 5600
L.L. 7000
(d)
D.L. 5600
L.L. 4500
D.L. 5600
L.L. 7500

FIG. 50

the vertical, which, in this truss, is 1.4142; hence, the stress in ak produced by the dead load is $1\frac{1}{2} \times 2{,}800 \times 1.4142 = 5{,}940$ pounds. To find the maximum stresses in the webs, it is necessary to consider the truss as only partly covered by the live load; therefore, it will be assumed that the live load extends over the entire portion of the truss to the right of the panel c, as shown in Fig. 50 (*b*). As stated previously, the two systems of diagonals must be considered separately in determining the web stresses, the loads supported by the dotted diagonals and the resulting reactions being shown in Fig. 50 (*c*). The dead-load reaction is marked *D. L.*, while that produced by the live load is designated by *L. L.* The stress in ak, under the condition of loading shown in (*c*), is $(3{,}000 + 5{,}600 - 1{,}400) \times \sec\alpha = 7{,}200 \times 1.4142 = 10{,}182$ pounds. The same stress exists in kc, while in cm and me it is $(3{,}000 + 5{,}600 - 1{,}400 - 2{,}800) \times \sec\alpha = 4{,}400 \times 1.4142 = 6{,}222$ pounds. In eo and og, the stress is $(3{,}000 + 5{,}600 - 1{,}400 - 2{,}800 - 2{,}800 - 4{,}000) \times \sec\alpha = -2{,}400 \times 1.4142 = -3{,}394$ pounds. The stress in gq and qi is $(3{,}000 + 5{,}600 - 1{,}400 - 2{,}800 - 2{,}800 - 4{,}000 - 2{,}800 - 4{,}000) \times \sec\alpha = -9{,}200 \times 1.4142 = -13{,}011$ pounds.

The $-$ sign in the last two results does not denote a tensile stress, but simply indicates that the shear in these members is negative, when the shear at the left-hand reaction is considered as positive; in other words, the shear changes sign at the point e, for the system of dotted diagonals.

Referring to Fig. 50 (*d*), the stresses in the full-line diagonals are determined as follows:

Stress in $jb = (5{,}600 + 4{,}500) \times \sec\alpha = 10{,}100 \times 1.4142 = 14{,}283$ pounds.

Stress in bl and $ld = (5{,}600 + 4{,}500 - 2{,}800) \times \sec\alpha = 7{,}300 \times 1.4142 = 10{,}324$ pounds.

Stress in dn and $nf = (5{,}600 + 4{,}500 - 2{,}800 - 2{,}800 - 4{,}000) \times \sec\alpha = 500 \times 1.4142 = 707$ pounds.

Stress in fp and $ph = (5{,}600 + 4{,}500 - 2{,}800 - 2{,}800 - 4{,}000 - 2{,}800 - 4{,}000) \times \sec\alpha = -6{,}300 \times 1.4142 = -8{,}909$ pounds.

Stress in hr = (5,600 + 4,500 − 2,800 − 2,800 − 4,000 − 2,800 − 4,000 − 2,800 − 4,000) × sec α = − 13,100 × 1.4142 = − 18,526 pounds.

The same method may be pursued to determine the web stresses under the different conditions of loading, and the maximum stress in each member may thus be ascertained. The members ak, cm, go, iq, and bl, dn, fn, hp are in tension, while ck, em, eo, gq, and bj, dl, fp, hr are in compression.

57. In Fig. 51, the stress diagrams for the different conditions of loading are shown. The stresses may be scaled from these diagrams, computed by the method of sections, or determined analytically, as explained below.

When the dead load only is considered, the reactions produced by the loads on the full-line diagonals are each 5,600 pounds; then the stress in hr, Fig. 51 (a), is compressive and is equal to 5,600 × sec α. At the point h, a load of 2,800 pounds counteracts half of the force, thus leaving 5,600 − 2,800 = 2,800 pounds of vertical force that creates tension in the member hp. The same force is carried up the web fp, producing a compressive stress in that member, and at f it meets the load of 2,800 pounds, which is just equal to it and consequently entirely neutralizes it. Hence, under a uniform load, there is no stress in fn and dn; that is, the truss would be stable without these members. This condition is changed, however, when the live load of 4,000 pounds is applied at HI. The right-hand reaction is then equal to 9,100 pounds and the left-hand reaction to 6,100 pounds. The stress in hr is 9,100 × sec α; at h, the load of 4,000 + 2,800 = 6,800 pounds is encountered and a force of 9,100 − 6,800 = 2,300 pounds is left to pass down hp and up pf. The load of 2,800 pounds at f must be taken care of, but as fp can take only 2,300 pounds, the remaining 500 pounds must be resisted by fn; hence, the stress in fn, which is compressive, is equal to 500 × sec α. The same amount of stress is created in dn, but it is of opposite character, or tensile. At d, another load of 2,800 pounds is applied, so that dl must carry 2,800 + 500 = 3,300 pounds.

The stress in this member is, of course, compressive and the same amount of tension exists in $b\,l$. At b, the load of 2,800 pounds is added, thus producing a force of (3,300 + 2,800) = 6,100 pounds to be resisted by $b\,j$; therefore, the compression in this member is 6,100 × sec a. If the live load is applied at the points h and f, the right-hand reaction is 11,600 pounds and the left-hand, 7,600 pounds. Then the stress in $h\,r$ is 11,600 × sec a, while in $h\,p$ and $f\,p$ the stress is (11,600 − 6,800) × sec a. At f, another load of 6,800 pounds is to be resisted and as $f\,p$ can take care of 4,800 pounds, the remainder, or 2,000 pounds, must be resisted by $f\,n$. Hence, the compression in $f\,n$ and the tension in $d\,n$ are equal to 2,000 × sec a. In $d\,l$ and $b\,l$, the stress is (2,000 + 2,800) × sec a, while the compression in $b\,j$ is (2,000 + 2,800 + 2,800) × sec a. In this manner the web stresses may be determined for all conditions of loading. It will be observed that when the live load comes from the right, as considered in the calculations given above, the stress in $f\,n$ is compressive, while in $d\,n$ it is tensile, but when the live load is applied at the left-hand end of the truss, the stresses in these members will be reversed. Therefore, it will be necessary to make $d\,n$ and $f\,n$ capable of resisting both tension and compression. In the dotted diagonals, the kind of stress does not change under the different conditions of loading.

LATERAL BRACING

58. When a trussed girder is used to support a tramway between two buildings or in any similar position, it is necessary to take into consideration the wind pressure on the girder, and to provide a system of **lateral bracing** to resist this pressure. When the girder is not enclosed, as in a bridge, the wind acts on both sides of it, that is, on both trusses forming the girder; but the area to be regarded as exposed to the wind is a question that the designer must decide. If the girder is enclosed, of course the surface exposed to the wind is equal to the entire area of one covered side.

In the following example the stresses in the members of the trusses may be determined as previously explained; therefore, only the bracing required to resist the wind pressure will be considered.

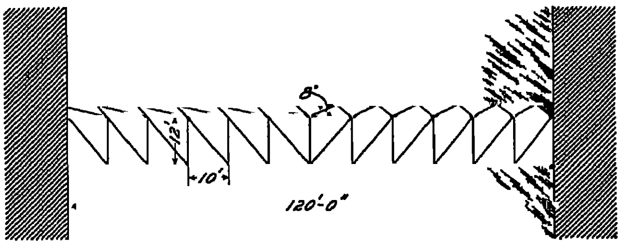


FIG 52

EXAMPLE.—A covered runway, or bridge, connecting two wings of a large factory at the fifth story has a span of 120 feet and a depth of 12 feet. The trusses are composed of twelve panels and the distance between trusses is 8 feet. The floor of the bridge, which is supported on the lower chords of the trusses, is composed of two layers of $2\frac{1}{2}$-inch tongued-and-grooved spruce plank laid diagonally, one layer crossing the other, and a 1-inch finished floor; this construction furnishes sufficient bracing against the wind at the lower chords, but it is necessary to introduce a system of diagonal bracing between the upper chords, as shown in Fig. 52. Considering the wind pressure as 30 pounds per square foot, what will be the stresses created in the system of lateral bracing?

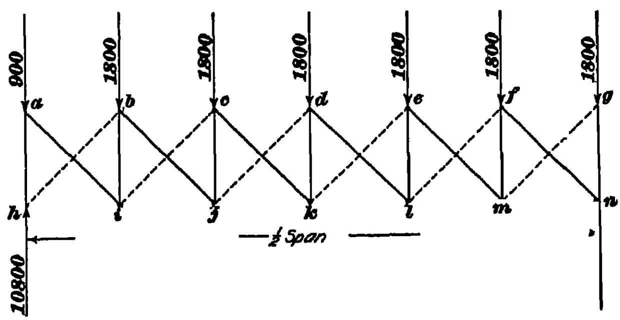


FIG. 53

SOLUTION.—As the ends of the trusses are to be built into the walls, the horizontal reactions due to the wind load are transmitted to the masonry instead of being resisted by the transverse strength of the end struts. (The transverse strength of columns or struts against

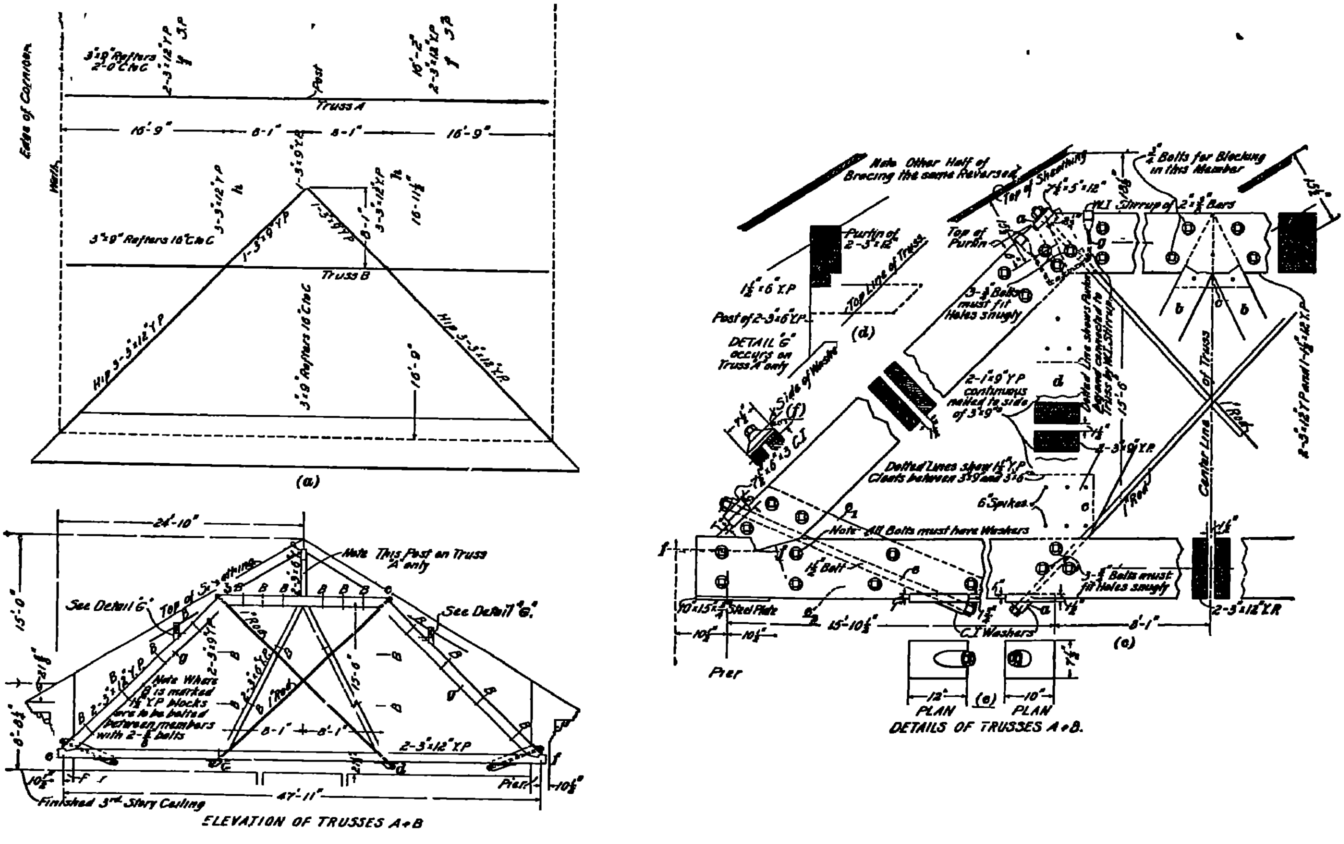
lateral forces is treated in *Wind Bracing*, and consequently it is not necessary to consider it here) The area of one panel is $12 \times 10 = 120$ sq. ft. and therefore the wind load per panel is $120 \times 30 = 3{,}600$ lb., but as one-half of this amount is to be resisted by the floor, the panel loads acting on the system of bracing between the upper chords are 1,800 lb. each. A plan of the top of the girder showing the wind loads applied is given in Fig. 53. The members ah, bi, cj, etc. are heavy wooden beams and hence should always be in compression. The tension rods designated by the heavy lines are in use when the load is applied as shown, but when the wind comes from the other side the dotted diagonals provide the required tension and the heavy diagonals are not in use. The stress in gn is 1,800 lb. and half of this force reacts on fn, while the other half goes to the tension member on the right of gn. Then the tensile stress in fn is $900 \times \sec fng = 900 \times 1.601 = 1{,}441$ lb. The stress in fm is $900 + 1{,}800 = 2{,}700$ lb , compression, and the tension in em is $2{,}700 \times 1.601 = 4{,}323$ lb. The remaining stresses may be analyzed and the following tabulation made:

$gn = 1{,}800+$	$ck = 6{,}300 \times 1.601 = 10{,}086-$
$fn = 900 \times 1.601 = 1{,}441-$	$cj = 6{,}300 + 1{,}800 = 8{,}100+$
$fm = 1{,}800 + 900 = 2{,}700+$	$bj = 8{,}100 \times 1.601 = 12{,}968-$
$em = 2{,}700 \times 1.601 = 4{,}323-$	$bi = 8{,}100 + 1{,}800 = 9{,}900+$
$el = 2{,}700 + 1{,}800 = 4{,}500+$	$ai = 9{,}900 \times 1.601 = 15{,}850-$
$dl = 4{,}500 \times 1.601 = 7{,}205-$	$ah = 9{,}900 + 900 = 10{,}800+$
$dk = 4{,}500 + 1{,}800 = 6{,}300+$	Ans.

DETAILS OF TRUSS CONSTRUCTION

WOODEN TRUSSED GIRDERS

59. It is frequently desirable to support roofs on trussed girders; that is, trusses with horizontal top and bottom chords. A truss of this kind constructed of timber is illustrated in Fig. 54 by a plan (*a*), an elevation (*b*), and with details (*c*), (*d*), (*e*), and (*f*). The elevation shows that the timber trusses *A* and *B* have a theoretical span of 47 feet 11 inches and the plan shows that they are placed 16 feet 11½ inches between centers. The elevation, Fig. 54 (*b*), shows the general outline of the truss. Its structural elements consist of a rigid central panel counterbraced with 1-inch diagonal rods that insure this portion of the frame against any distortion produced by unsymmetrical loads. and the two triangles



Edge of Cornice
Wall
Truss A
Truss B
Post
16'-9"
8'-1"
3"x9" Rafters 2'-0" C to C
2-3"x12" Y.P
3"x9" Rafters 16" C to C
Hip 3-3"x12" Y.P
(a)
24'-10"
15'-0"
47'-11"
Note This Post on Truss "A" only
See Detail "G"
Top of Sheathing
1" Rod
2-3"x12" Y.P
Note Where B is marked 1½" Y.P blocks are to be bolted between members with 2-⅝" bolts
Finished 3rd Story Ceiling
Pier
ELEVATION OF TRUSSES A+B
(b)
Note Other Half of Bracing the same Reversed
¾" Bolts for Blocking in this Member
W.I. Stirrup of 2"x⅜" Bars
Top of Purlin
Purlin of 2-3"x12"
Top Line of Truss
1½"x6" Y.P
Post of 2-3"x6" Y.P
DETAIL "G" occurs on Truss "A" only
Side of Washer
(d)
(f)
3-⅞" Bolts must fit Holes snugly
2-1"x9" Y.P continuous nailed to side of 3"x9"
Dotted Lines show 1½" Y.P Cleats between 3"x9" and 3"x6"
6" Spikes
Note All Bolts must have Washers
1½" Bolt
Center Line of Truss
10"x15"x¼" Steel Plate
15'-10½"
C.I. Washers
3-⅞" Bolts must fit Holes snugly
2-3"x12" Y.P
PLAN
(e)
DETAILS OF TRUSSES A+B.

FIG. 54

eba and *dcf* form a rigid frame incapable of distortion. The chord members and end posts of the truss *g*, *g*, Fig. 54 (*b*), are each composed of two 3″ × 12″ pieces of yellow pine, placed side by side and separated by yellow-pine blocks 1½ inches thick. The position of these blocks is shown in the elevation (*b*) at *B*, *B*, *B*.

Referring to the details shown in Fig. 54 (*c*), it will be seen that the diagonal counterbracing, consisting of 1-inch rods, extends through the space left between the timbers making up the upper and lower chords and that they are secured at the ends by especially designed check-washers *a*, *a*. The strut members *b*, *b* in the middle panel of the truss are each made up of two 3″ × 6″ yellow-pine timbers. These are cut to the shape shown in the detail at the ends and are secured in place by being spiked to cleats *c*, *c* 1½ inches thick that fit between the middle space of the connecting members and form what would be termed in steel construction, a gusset plate. The vertical members *d*, as designated in the drawing of the details, are composed of two 3″ × 9″ yellow-pine pieces fitted tightly between the upper and lower chord members and further held in place and reenforced by two 1″ × 9″ yellow-pine pieces nailed on each side of the heavier pieces of the member. These side pieces extend over the top and bottom chords and are bolted to them with ¾-inch bolts. The required strength in the heel connection of the truss is obtained by the introduction of the 1½-inch through bolt *e* and by 3″ × 9″ and 3″ × 6″ cleats e_1 and e_2, one being placed above the bolt and the other below it, both being inserted between the pieces composing the end post and the lower chord of the truss. The presence of the lower cleat e_2 is not quite evident in the drawing, because its outline coincides with that of the lower chord and end post. The connection is secured to the cleats by through bolts, and the 1½-inch bolt forming the main resistance of the joint is provided with special cast-iron washers, details of which are shown at (*e*) and (*f*). Additional resistance is obtained at this important joint by notching the end post into the lower chord member and thus

obtaining the shearing resistance of the timber along the line *ff*. From the plan, Fig. 54 (*a*), it is noticed that purlins *h*, *h* composed of 3″ × 12″ pieces of yellow pine extend between the trusses *A* and *B*. These purlins are supported from the truss by wrought-iron stirrup irons *g*, as shown in (*c*), and carry the bulk of the weight of the roof between the trusses *A* and *B*. The other purlins, marked *f* in the plan, are supported on the end posts of the truss in the manner shown in (*d*), which is so clear as to need no explanation.

60. Another type of timber truss, which is frequently used in mill construction or temporary work, is that shown

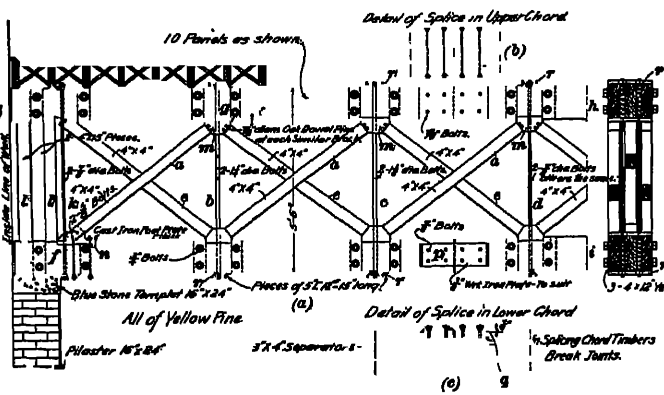


FIG. 55

in Fig. 55. This trussed girder was designed for a span of 45 feet and a total live and dead load of 50,000 pounds, uniformly distributed. It is constructed of Georgia yellow pine and is 5 feet 6 inches in depth. The members and details of construction are designed for a factor of safety of at least 4. Both chords of the trusses are composed of three pieces of timber placed side by side and separated with 3″ × 4″ separators composed of the same material as the truss. These separators are gained into the chord timbers about ½ inch. While this is apt to weaken the chord in tension somewhat, the rigidity gained by the insertion of the

separators in this manner, laterally and vertically, more than compensates for the loss in strength. From the figure, it will be noticed that the web compression members are all composed of 4″ × 4″ pieces. Those marked *a*, *a*, *a* are doubled, for with the rods *b*, *c*, and *d*, they form the principal members of the web, while the members *e*, *e*, *e* are composed of single pieces of timber. These pieces would not be necessary if it were certain that the load would always be uniformly distributed. The possibilities are, however, that with a girder of this span and supporting the great floor area that it necessarily must, the floor load will frequently be concentrated on portions of the girder, and consequently the counterbraces are necessary to insure the truss against distortion. The structural outline of this truss extends along the lines *fghi* and the upper chord is extended from *g* to *j* only in order to support the portion of the floor adjacent to the wall. It is evident from this that the frame diagram of the truss is as shown in Fig. 56, or in the form of a Howe truss. Consequently, the rods *k* are of little use except to rigidly tie the extended portion of the upper chord to the lower one at the abutments, while the struts *l*, *l* are introduced to take up the reaction at the end *j* of the extended portion of the upper chord. The timber strut members of the web are held in place on bearing blocks by means of oak dowels, as shown at *m*, special care being taken to secure a rigid and adequate connection at *n* by the introduction of a cast-iron foot-plate.

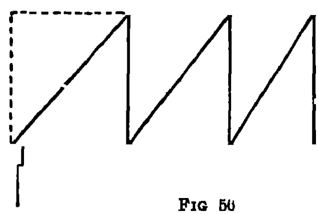


FIG 56

61. Should it be necessary to splice the timbers making up both the top and bottom chord, the splices should be kept as near as possible to the abutments, for, in the chords, the stresses decrease toward the supports. For the splice in the upper, or compression, chord it is only necessary to observe

that the spliced timbers are cut true and square and abut, their alinement being secured by pieces of timber bolted on each side. Such a detail as described is shown in view (*b*). The splice for the chord in tension is shown at *p* in view (*a*). It is necessary to use wrought-iron splice plates having a lug *q*, Fig. 55 (*c*), on each end. All such splice connections should be analyzed for shear parallel with the grain and for bearing on the end wood, and, if possible, a strength greater by 25 per cent. than the strength of the net section of the member spliced should be realized. The washers *r*, *r* are subjected to considerable bending stress and are therefore made of pieces of 5-inch channel iron equal in length to the width of the chord members. When channels are thus used for washers it is generally necessary to provide a socket wrench to tighten up the bolts.

62. In Fig. 57 is shown a trussed girder having a span of over 50 feet. At the ends, it is built in a manner similar to a plate girder with a web *a* extending between the flange angles. Toward the center, however, for economy and also because it was desired to provide a construction that would allow the diffusion of light from a skylight, the girder was made open, as shown. The girders are stiffened laterally at the lower flange by I beams extending to the walls and at the top flange by a brace, secured at *c*, extending to a horizontal steel beam member resting on the walls. At the center of the span, the girder is further reenforced laterally by the beams for the support of the diffusing sash shown at *d*. There are no peculiarities of the design requiring detailed explanation, as the drawing is sufficiently clear with reference to the constructive details.

63. In Fig. 58 is shown a detail design of a trussed through bridge, such as would be used to connect two wings or two buildings of a manufacturing plant. It must be designed to carry, besides its own weight, the truck loads of the heaviest merchandise that would be likely to be sent across it. The bridge has a span of 120 feet, is 10 feet from center to center of trusses across, and is 15 feet 6 inches

from out to out of chord members in height or depth. The two main views are elevations supposed to join along the lines *a a*. The chord members of the truss are composed of two $6'' \times 6'' \times \frac{1}{2}''$ angles riveted to a $\frac{1}{2}'' \times 18''$ web-plate. By the adoption of a built-up **T** section of this kind the necessity of gusset plates is obviated, as the web members of the truss can be riveted through the vertical parts of the chord members. Owing to the fact that it is impossible to obtain such angles and plates as are shown in the drawing longer than 50 or 60 feet, the upper and lower chord members must be spliced as shown at *a, b*.

In the upper splice, it is only necessary to provide sufficient splice plates and rivets to hold the ends of the upper chord member in alinement, but in the design of the splice *b* for the lower chord, long splice plates must be used and sufficient rivets to realize, by their shearing resistance, the strength of the net section of the chord member. To stiffen the lower chord laterally at the splice, it is advisable to use cover angles that fit into the chord angles, as at *c*. Where possible, it would be well to splice the angles of the lower chord at a different place from the splice of the web plate. The central panels are counterbraced so that there will be no distortion of the frame from a moving load. The end posts of the truss, as at *d*, should be strongly constructed so as to withstand the bending stress produced by the gusset-plate brace *e*, shown in the view placed in the lower right-hand corner. This gusset-plate brace transmits considerable stress from the upper lateral wind bracing, and the only way to resist the stress is to provide a sufficient section modulus in the end members so as to supply the necessary resisting moment. While the floor system will offer sufficient lateral resistance at the bottom of the truss, it is good practice to provide lateral bracing at the botton as well as at the top. As the truss is of steel, and of considerable span, it is necessary to provide for the expansion and contraction of the frame. For this purpose the steel bearing plate *f* is supplied and the anchor bolts are passed through slotted holes in the heel plate *g*.